On Asymptotic Optimality of ML-type Detectors in Quantum Hypothesis Testing

One is given $n$ systems all prepared i.i.d. in some particular state chosen uniformly at random from some finite set $\Sigma$, and must identify which state it is with minimum probability of error. One asks for the asymptotic behaviour of the error probability as $n$ grows. In general there is an upper bound for the exponential rate of decay of the average error probability known as the "(multiple) quantum Chernoff bound" (QCB) notified by $\xi_{QCB}(\Sigma)$. This is simply the minimal pairwise quantum Chernoff distance (as defined in [3]) between any two states in the set $\Sigma$.

It is however generally unknown whether there are sequences of measurements whose minimum error exhibits exponential decay at the same rate i.e. whether the QCB is achieved. In [1] an algorithm for a sequence of measurements called maximum likelihood type tests (ML-type tests) was laid out for which QCB is achieved in the following cases:

- All the states commute; (classical result)
- All the states are pure; ([2])
- Each pair of states have disjoint support; ([1])

In this presentation I will be talking about how we demonstrated that these tests also achieve the QCB when the eigenbases of the density operators in $\Sigma$ are mutually unbiased, thus extending the set of sets of states for which we know that the QCB can be achieved. This result was obtained by deriving a lower bound on the minimal eigenvalue of a Gram matrix that is assigned to a subset of linearly independent unit vectors of the union set of mutually unbiased bases. A series of lemmas with elementary proofs on the maximum eigenvalue, Euclidean norm and finally the determinant of the Gram matrix leads us to the non-trivial lower bound on the minimum eigenvalue.

References
