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Reversal Modes in Magnetic Nanowires

by

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Abstract

The aim of this paper is to explain phenomena in the reversal of the magnetization in nanowires. In numerical simulations two different reversal modes have been found, one occurring for very thin, the other for thicker wires. We study the two modes analytically and investigate the reasons why they occur.

Introduction

In the last years several groups have succeeded in the production and investigation of magnetic wires with less than 100 nm diameter, e.g. [16, 15, 17]. Arrays of such nanowires are in consideration as future high density storage devices [2]. The time necessary to change the magnetization of a nanowire is directly related to the writing and reading speed of such a device. Therefore it is important to understand their reversal process. It is known that the reversal of the magnetization starts at one end of the wire and then a domain wall separating the already reversed part from the not yet reversed part is propagating through the wire. However, because of technical difficulties related to the small size of the wires, there are few experimental results about the speed of the wall, e.g. [1, 3, 7, 14], and there are no experimental results about the shape of the wall.

In numerical simulations of this process, several groups [5, 6, 19], have observed two different reversal modes. These modes depend on the wire thickness and correspond to different switching speeds. For thin wires the transverse mode is observed: the magnetization is constant on each cross section, rotating and moving along the wire (Figure 1). For thicker wires the vortex mode is observed: the magnetization is approximately tangential to the boundary and forms a vortex which moves along the wire. (Figure 2). It is the purpose of this paper to explain the different behaviors by extracting the conclusions that are relevant for physical applications from the mathematical papers [8, 10, 9, 11].

The micromagnetic model

We work in the framework of micromagnetism. This is a mesoscopic continuum theory that assigns a nonlocal, non-convex energy to each magnetization m from the domain Σ to the sphere $\mathbb{S}^2 \subset \mathbb{R}^3$. Experimentally observed ground states of the magnetization correspond to minimizers of

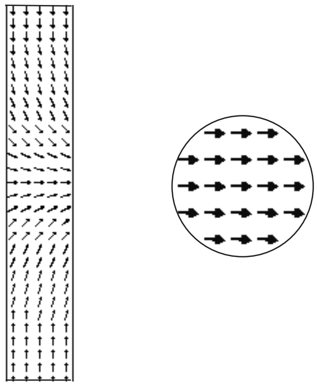

Figure 1: Transverse Mode: longitudinal section and cross section

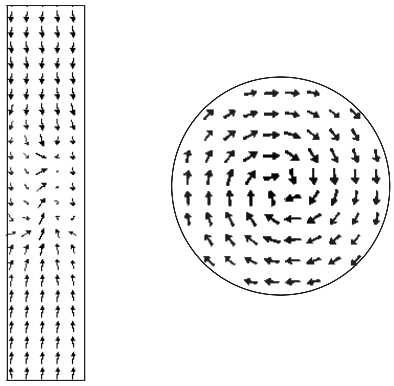

Figure 2: Vortex Mode: longitudinal section and cross section

the micromagnetic energy functional

$$E(m) \;:=\; \underbrace{\int_\Sigma A_{\mathrm{ex}}\,|\nabla m|^2}_{\text{exchange energy}} + \underbrace{\int_{\mathbb{R}^3} K_{\mathrm{d}}\,|\nabla u|^2}_{\text{stray field energy}} - \underbrace{\int_\Sigma J_s\,h\cdot m.}_{\text{external field energy}}$$

Here h is an external magnetic field, and u is the weak solution of $\Delta u = \operatorname{div} m$ in $\mathbb{R}^3$. A_{ex}, K_d and J_s are material constants.

In the micromagnetic model the evolution of the magnetization is described by the Landau-Lifshitz-Gilbert (LLG) equation:

$$\partial_t m \;=\; -\gamma m \times H_{\mathrm{eff}} + \alpha m \times (m \times H_{\mathrm{eff}}), \quad \text{where } H_{\mathrm{eff}} = \delta_m E.$$

The first term describes the precession of the magnetization around the effective field, and the second term describes a change of the magnetization in direction of the effective field. The number γ is the gyromagnetic ratio, and α is a phenomenological damping constant.

Static domain walls

Forster and al. [5] suggest that the reversal modes correspond to domain walls that minimize the static energy functional. We make this idea rigorous establishing a cross over of two scaling regimes for the energy in dependence of the radius.

Let $E_{\mathcal{M}_l}$ be the energy of the optimal wall profile, let $E_{\mathcal{T}_l}$ be the energy of the optimal domain wall profile for transverse walls, i.e., for walls that are constant on the cross section, and let $E_{\mathcal{V}_l}$ be the energy of the optimal wall profile for vortex walls, i.e., for walls with a corotational symmetry as depicted in Figure 2.

$E_{\mathcal{M}_l}$ scales like $E_{\mathcal{T}_l}$ if the radius R goes to zero and that it scales like $E_{\mathcal{V}_l}$ if R tends to infinity [8]: There exist constants c, C such that

$$\begin{array}{lrcccccc} \text{for } R \le 2: & cR^2 & \le & E_{\mathcal{M}_l}(R) & \le & E_{\mathcal{T}_l}(R) & \le & CR^2, \\ \text{for } R > 2: & cR^2\sqrt{\ln(R)} & \le & E_{\mathcal{M}_l}(R) & \le & E_{\mathcal{V}_l}(R) & \le & CR^2\sqrt{\ln(R)}. \end{array}$$

Neither $E_{\mathcal{T}_l}$ nor $E_{\mathcal{V}_l}$ has the optimal scaling in the opposite regime. This shows that the transverse wall is energetically favorable for small radii and the vortex wall is energetically favorable for big radii.

To capture the essence of the energy minimizing problem for small radii, we use the notion of Γ-convergence as defined in [4]. We rescale the energy E by a factor of $\frac{1}{R^2}$ and rescale the magnetization m such that that the domain of definition is the wire with radius 1.

In the limit we get a reduced problem, where the admissible functions are maps from $\mathbb{R}$ to $\mathbb{S}^2$, and where the energy simplifies to

$$E_{\mathrm{red}}(m) = \pi\|\partial_x m\|^2_{L^2(\mathbb{R})} + \frac{\pi}{2}\|m_y\|^2_{L^2(\mathbb{R})}.$$

The minimizer m^{red} of the reduced problem exists and is unique up to translation and rotation. Its energy is $\sqrt{8}\pi$, and its profile is that of a Bloch wall i.e.,

$$m^{\mathrm{red}} = \left(\tanh\left(\frac{x}{\sqrt{2}}\right), \frac{1}{\cosh\left(\frac{x}{\sqrt{2}}\right)}, 0\right). \tag{1}$$

Since Γ-convergence implies the convergence of minimizers as well as the convergence of the minimal energies, we can conclude for small radii, that minimizers of E are almost constant on the cross section, that they have a profile similar to m^{red} and that their energy is approximately $\sqrt{8}\pi R^2$.

The functional E_{red} has been used before to to approximate the micromagnetic energy functional in nanowires [18]. Our convergence result clarifies why and in which sense this approximation is valid. They also perform numerical calculations of domain walls in nanowires that show that for thin wires the profile is indeed close to m^{red}.

For large R there are explicit examples of functions in $\mathcal{V}_l$ whose energies have the optimal scaling. They have a square root type singularity, and the width of their transition regions scales like $R^2\sqrt{\ln(R)}$. The latter is in contrast to the regime of small R, where the thickness of the transition region of the optimal walls is of order one.

Our results regarding the form of the domain walls match numerical simulations of [18] very well. Having solved the static problem we turn our attention dynamics and investigate the two modes separately.

The transverse mode

We study the transverse mode via a perturbation argument from the static case. For thin wires and weak external magnetic fields there exist traveling wave solutions of the over-damped limit of the LLG equation [9]. Because of the continuous pertrubation, the profile of the travelling wave solutions is close to that of the static domain wall, which for small radius has approximately the profile (1).

For implicit function theorem used for the perturbation argument it is crucial that the static domain walls are smooth. This can only be expected for thin wires since in thick wires the examples of low energy domain walls are vortex walls that have a singularity. In the limit $R \to 0$ the domain walls have the smooth profile (1). However, Γ convergence implies convergence only in a norm related to the energy, so the regularity does not automatically carry over to the case of finite radii. In [10] the Morrey-Campanato approach is used to obtain the regularity results and convergence in strong norms.

These results are a step towards understanding the transverse mode. The final goal is to show that for thin magnetic nanowires there exist, possibly rotating, traveling wave solutions of the full LLG equation, and to describe them with an effective theory. We expect that, for thin wires and weak external field, the existence of traveling wave solutions to the full LLG equation can be proved with the methods of [12].

The vortex mode

We model the vortex mode by harmonic map heat flow under an additional external field. This is a simplified model, which captures the highest order terms with respect to the derivatives. Moreover, we assume that the magnetization in each point is tangential to the closest boundary. This ensures that we have a magnetization without surface charges.

We have the following picture in mind: The *existence* of the singularity in the vortex mode is due to the strong influence of the stray field energy, which prevents surface charges, but the properties of the *evolution* of a magnetization with a singularity is mainly determined by the highest order terms with respect to the derivatives.

Because of the singularity usual PDE methods are not applicable. Using variational methods developed in [13] we show the existence of corotationally symmetric traveling wave solutions with a moving vortex. For weak and strong external fields, the traveling waves connect the original state anti-parallel to the external magnetic field with the fully reversed state in direction of the external field [11].

Since the other terms in the energy are lower order, it is plausible that such solutions exist also for the full gradient flow equation and even for the LLG equation.

This is the first analytic model that takes into account the three dimensional structure of moving domain walls in nanowires. Since numerical calculations have problems resolving the singularity in the vortex mode, analytic considerations are especially important [18].

Conclusion

Altogether, we have explored the reversal modes in magnetic nanowires from different angles, studying one static and two dynamic models. Using the static model we have proved that different reversal modes correspond to different types of domain walls and to different regimes of scaling of the energy. With the first dynamic model we have investigated the transverse mode via a perturbation argument from the static case. The second dynamic model was used to study the role of the singularity in the vortex mode. Our investigations contribute to an understanding why these modes occur and lead to predictions that can be compared to numerical simulations.

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