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Abstract

We investigate the magnetization dynamics in soft ferromagnetic films with small damping. In this case, the gyrotropic nature of Landau-Lifshitz-Gilbert dynamics and the shape anisotropy effects from stray-field interactions effectively lead to a wave-type dynamics for the in plane magnetization. We apply this result to study the motion of Néel walls in thin films and prove the existence of a traveling wave solution under a small constant forcing.

1 Introduction

The motion of a domain wall is a fundamental mechanism in the dynamics of ferromagnetic patterns. If, for instance, a small particle is first saturated in a high field and the field is then reduced (or even reversed) it will in general switch at some point to a state of opposite magnetization. In this typical switching process, magnetic domains nucleate and evolve by propagation of domain walls. As proposed in [11] and [7], the dynamics of a magnetization distribution in a ferromagnetic material is governed by the Landau-Lifshitz-Gilbert equation (LLG)

$$\mathbf{m}_t + \alpha\, \mathbf{m} \wedge \mathbf{m}_t - \gamma\, \mathbf{m} \wedge \mathcal{H}_{\text{eff}} = 0$$

that prescribes a damped gyromagnetic precession of the (unit) magnetization vector field $\mathbf{m}$ about the effective field. In this model $\alpha > 0$ is a dimensionless damping coefficient called Gilbert factor and $\gamma > 0$ is the gyromagnetic ratio giving the typical precession frequency. The effective field $\mathcal{H}_{\text{eff}} = \mathbf{h} - \nabla \mathbb{E}(\mathbf{m})$ consists of the applied field and the (negative) first variation of the micromagnetic energy $\mathbb{E}(\mathbf{m})$. Since $\alpha > 0$ is a small parameter, LLG inherently sets a slow relaxation and a fast precession time scale.

According to the micromagnetic model and as confirmed experimentally, magnetic domain walls extend to finite layers that exhibit diverse transition patterns depending on the particular parameter regime, see [8, 4, 10]. The internal structure of domain walls in thin films and its impact on magnetic pattern formation have long been a subject of intense study. A particular mathematical feature of the thin–film geometry is the predominance of the shape anisotropy effect that strongly favors in-plane magnetization. In terms of an electrostatic analogy this is interpreted as the appearance of surface magnetic charges that lead to strong dipolar interaction. Appropriate local descriptions of such transition layers rely on flat domain walls that separate two domains of opposite magnetization at infinity. Among the simplest wall configurations is the Néel wall, an in-plane rotation of the magnetization vector, that is stable in extremely thin films.

In certain regimes, ranging from thin film to bulk geometries, the interplay of gyromagnetic precession and energetic dipolar interactions yields effective equations for the magnetization dynamics. In the context of thin films, gyromagnetic precession competes with stray-field shape anisotropy that leads to an in-plane magnetization. Dynamically, one expects fast but small oscillations of the vertical magnetization component. Consequently, an effective description of the domain wall motion has to account for the separation of time scales in LLG when the relative thickness tends to zero. One of these regimes has already been considered in [6, 9], where the authors have studied the limit of relative thickness going to zero, with the Gilbert damping α held fixed of order one. In this case the effective equation obtained for the in-plane magnetization is an overdamped limit of LLG.

In the first part of the present paper we consider a thin–film regime, where the Gilbert damping is comparable to the relative thickness of the film. We show (Theorem 1) that in this regime, LLG effectively reduces to a damped geometric wave equation for the in-plane magnetization components $m = (m_1, m_2)$,

$$\left[\partial_t^2 m + \nu\, \partial_t m + \nabla E(m)\right] \perp T_m \mathbb{S}^1 \tag{1.1}$$

where $\nabla E(m)$ is the L^2 gradient of a reduced energy functional acting on the in-plane components only and ν is an effective damping constant. Expressed in polar coordinates $m = (\cos\theta, \sin\theta)$ with $\mathcal{E}(\theta) = E(m)$ this yields

$$\partial_t^2 \theta + \nu\, \partial_t \theta + \nabla \mathcal{E}(\theta) = 0 \tag{1.2}$$

a damped wave equation for the phase function. With this type of dynamics the oscillatory features of LLG that give rise to spin waves and resonances are potentially preserved in such a limit.

In the second part we investigate domain wall motion in thin films in the presence of a constant applied field. For the bulk geometry, valuable information can be drawn from LLG by means of explicit solutions with constant speed of propagation, known as *Walker's exact solutions*, [19]. For sufficiently small applied fields, this construction reveals not only the shape of a moving domain wall but also its kinematic properties. In particular, its speed of propagation is almost proportional to the applied field, with a constant of proportionality β know as the wall mobility. These same features are expected, and observed, for the Néel wall in thin films. Theorem 2 contains the construction of traveling wave solutions for (1.1) as a perturbation of the Néel wall's static profile. The result also gives a first order expansion for the wall mobility β, the infinitesimal rate of change of velocity at small fields. Back to physical units we obtain

$$\beta = \gamma\, d^2/\alpha\, \mathbb{E}_{\mathrm{ex}}$$

where d is the exchange length and $\mathbb{E}_{\mathrm{ex}}$ is the exchange contribution to the energy of the static Néel wall. A crucial ingredient in the proof of the latter result is a spectral analysis for the static Néel wall, that particularly proves stability up to translations and a spectral gap for the linearization. This result has its own interest.

The paper is organized in the following way. Section 2 gives a brief summary of the micromagnetic model, including the reduction to the thin–film approximation for Néel walls. Here, we also discuss the main features of Walker's solution and the formal asymptotic limit that leads to the damped wave-type equation (1.1) in the bulk and heuristically explain our choice of regime. In Sections 3 we present the scaling and asymptotics in the thin–film regime and prove Theorem 1. The spectral analysis for the Néel wall is given in Section 4. In Section 5 we prove Theorem 2.

2 Mathematical framework for domain walls

2.1 The micromagnetic model

The continuum description of ferromagnetism by Landau and Lifshitz [11] is based on a direction field (magnetization) $\mathbf{m} : \Omega \to \mathbb{S}^2$ that represents the local average of magnetic moments on a magnetic sample $\Omega \subset \mathbb{R}^3$ and a variational principle in terms of the micromagnetic energy. In the absence of external fields it reads

$$\mathbb{E}(\mathbf{m}) = \frac{1}{2}\left(d^2 \int_\Omega |\nabla \mathbf{m}|^2 \, dx + \int_{\mathbb{R}^3} |\nabla u|^2 \, dx + Q \int_\Omega \Phi(\mathbf{m}) \, dx \right).$$

The Dirichlet part is called exchange energy with exchange length d, and has its origin in the nearest neighbor approximation for the Heisenberg spin interaction.

This term penalizes the spatial variations of $\mathbf{m}$ and sets a finite length scale. The stray-field $\nabla u : \mathbb{R}^3 \to \mathbb{R}^3$ is uniquely determined by the static Maxwell equations

$$\int_{\mathbb{R}^3} \nabla u \cdot \nabla \varphi \, dx = \int_{\Omega} \mathbf{m} \cdot \nabla \varphi \, dx \quad \text{for all} \ \ \varphi \in C_0^\infty(\mathbb{R}^3).$$

Accordingly, the stray-field energy portion is nonlocal and favors vanishing distributional divergence, that is $\nabla \cdot \mathbf{m} = 0$ in Ω and $\mathbf{m} \cdot \mathbf{n} = 0$ on $\partial\Omega$ where $\mathbf{n}$ is the outward normal. The last energy term models crystalline anisotropy where penalty function Φ is an even polynomial on $\mathbb{S}^2$, and the parameter Q measures the relative strength of anisotropy penalization versus stray-field interaction.
This combination of nonlocality (through stray-field interaction) and nonconvexity (through the saturation constraint $|\mathbf{m}| = 1$) gives rise to the formation of typical magnetic patters consisting in domains, where the magnetization changes slowly. Magnetic domains are separated by thin transition layers, called domain walls, which may form a complex network. Domain walls are of major interest in micromagnetic theory since their internal structure and mutual interaction heavily influence the coarser magnetic microstructure.

2.2 Reduction of the stray-field energy

Let us consider an infinitely extended uniaxial magnetic film $\Omega = \mathbb{R}^2 \times (0, \delta)$ with the easy axes oriented in the $\mathbf{e_2}$ direction. Since we are mostly interest in thin films, it is safe to assume that $\mathbf{m}$ does not depend on the thickness variable x_3. We consider parameterized transitions along the $\mathbf{e_1}$ direction (that we call transition axis), that connect antipodal states on the easy axis

$$\mathbf{m} : \mathbb{R}^2 \to \mathbb{S}^2 \quad \text{with} \quad \mathbf{m}(\pm\infty, x_2) = (0, \pm 1, 0) \quad \text{for any} \quad x_2 \in \mathbb{R}$$

and that are l-periodic in the $\mathbf{e_2}$ direction

$$\mathbf{m}(x_1, x_2 + l) = \mathbf{m}(x_1, x_2) \quad \text{for any} \quad x = (x_1, x_2) \in \mathbb{R}^2.$$

We identify a global magnetization field as given by $\mathbf{m}(\mathbf{x}) = \mathbf{m}(x)\chi_{(0,\delta)}(z)$, and defined for $\mathbf{x} = (x, z) \in \mathbb{R}^3$. Then $\mathbf{m} = \mathbf{m}(\mathbf{x})$ induces the stray-field ∇u determined by the potential equation $\Delta_{\mathbb{R}^3} u = \mathrm{div}_{\mathbb{R}^3} \mathbf{m}$ in the sense of distributions on $\mathbb{R}^3$. We observe that the stray-field potential $u = u(x, z)$ is l-periodic in x_2 as well, and the z dependence only stems from shape anisotropy. Then by Green's formula

$$\int_{\mathbb{R}} \int_0^l \int_{\mathbb{R}} |\nabla u|^2 \, dx \, dz = \int_0^\delta \int_0^l \int_{\mathbb{R}} \nabla u \cdot \mathbf{m} \, dx \, dz.$$

In case of uniaxial anisotropy with $\Phi(\mathbf{m}) = 1 - m_2^2$, the micromagnetic energy induces the following (averaged) domain wall energy per unit area:

$$\mathbb{E}(\mathbf{m}) = \frac{1}{2} ⨍_0^\delta ⨍_0^l \int_{\mathbb{R}} \left(d^2 |\nabla \mathbf{m}|^2 + \nabla u \cdot \mathbf{m} + Q(1 - m_2^2) \right) \, dx \, dz. \tag{2.1}$$

Fourier representation

For the exchange and anisotropy contribution the average over the film thickness is redundant. For the stray-field contribution, however, a calculation in frequency space with the partially discrete Fourier transform

$$\hat{\mathbf{m}}(\xi) = \mathcal{F}(\mathbf{m})(\xi) = \frac{1}{\sqrt{2\pi l}} \int_{\mathbb{R}\times(0,l)} \mathbf{m}(x)\, e^{-i\xi\cdot x}\, dx \quad \text{for} \quad \xi \in \mathbb{R}\times(2\pi/l)\mathbb{Z}$$

yields a representation in terms of Fourier multipliers

$$⨍_0^\delta ⨍_0^l \int_{\mathbb{R}} \nabla u \cdot \mathbf{m}\, dx\, dz = \frac{1}{2}\int_{\mathbb{R}^2} \left[\sigma(\delta\xi)\, \frac{(\xi\cdot\hat{m}(\xi))^2}{|\xi|^2} + \big(1-\sigma(\delta\xi)\big)\, |\hat{m}_3(\xi)|^2\, d\xi\right],$$

where $d\xi$ denotes the partially discrete integration measure on $\mathbb{R}^2$

$$d\xi = d\xi_1 \otimes \sum_{k\in\mathbb{Z}} \delta_{\eta_k} \quad \text{where} \quad \eta_k = 2\pi k/l.$$

The Fourier multiplier associated with stray-field interaction (see [5, 13]) reads

$$\sigma(\xi) = 1 - \frac{1-\exp(-|\xi|)}{|\xi|}.$$

It exhibits the following small and high frequency asymptotics

$$\sigma(\xi) = \frac{1}{2}\,|\xi| + \mathcal{O}\left(|\xi|^2\right) \quad \text{as} \quad |\xi|\to 0 \quad \text{and} \quad \sigma(\xi) = 1 + \mathcal{O}(1/|\xi|) \quad \text{as} \quad |\xi|\to\infty$$

that serve to deduce thin-film and bulk approximations, respectively. In the thin-film regime we suppose that δ is small compared to the typical wave length, that is $\delta\,|\xi| \ll 1$: Collecting the leading order terms for the in-plane components m and the vertical component m_3 yields the following thin-film approximation for the stray-field energy

$$⨍_0^\delta ⨍_0^l \int_{\mathbb{R}} \nabla u \cdot \mathbf{m}\, dx\, dz \sim \frac{\delta}{2}\int_{\mathbb{R}^2} \frac{(\xi\cdot\hat{m}(\xi))^2}{|\xi|}\, d\xi + \int_{\mathbb{R}^2} |\hat{m}_3(\xi)|^2\, d\xi. \tag{2.2}$$

The m_3 contribution can be interpreted as the residual surface charge interaction having the form of an additional anisotropy that penalizes vertical magnetizations (shape anisotropy). The term in m corresponds to residual volume charge interactions.

Note that the matrix multiplier $\hat{\mathcal{H}}(\xi) = \frac{\xi}{|\xi|}\otimes\frac{\xi}{|\xi|}$ corresponds to the Helmholtz transform $\mathcal{H}$, the L^2 projection onto gradient fields, that is formally $\mathcal{H}(m) = \nabla\Delta^{-1}\mathrm{div}\, m$. With this notation we have

$$\int_{\mathbb{R}^2} \frac{(\xi\cdot\hat{m}(\xi))^2}{|\xi|}\, d\xi = \|\mathcal{H}(m)\|^2_{\dot{H}^{1/2}}$$

with the homogeneous fractional $H^{1/2}$ Sobolev norm. More generally for $s \in \mathbb{R}$

$$\|F\|_{\dot{H}^s}^2 = \int_{\mathbb{R}^2} |\xi|^{2s} |\hat{F}(\xi)|^2 \, d\xi = \fint_0^l \int_{\mathbb{R}} \big||\nabla|^s F\big|^2 \, dx,$$

where

$$|\nabla|^s F \equiv (-\Delta)^{s/2} F = \mathcal{F}^*\big(|\xi|^s \hat{F}(\xi)\big).$$

The full Sobolev norms are given by

$$\|F\|_{H^s}^2 = \int_{\mathbb{R}^2} \big(1 + |\xi|^2\big)^s |\hat{F}(\xi)|^2 \, d\xi,$$

and we denote the corresponding function spaces by $H^s = H^s(\mathbb{R} \times \mathbb{T}_l)$, where $\mathbb{T}_l = \mathbb{R}/l\mathbb{Z}$ denotes the 1-torus of length l.

Using fractional Sobolev norms the approximate stray-field energy for $\mathbf{m} = (m, m_3)$ can be written in the following compact form

$$\mathbb{E}_{\text{stray}}(\mathbf{m}) = \frac{\delta}{4} \, \|\mathcal{H}(m)\|_{\dot{H}^{1/2}}^2 + \frac{1}{2} \, \|m_3\|_{L^2}^2.$$

An important remark concerning further dimensional reduction is that if $F = F(x_1)$ is independent of x_2 then $\hat{F}(\xi_1, \xi_2) = 0$ unless $\xi_2 = 0$, thus

$$\|F\|_{\dot{H}^s}^2 = \int_{\mathbb{R}} |\xi_1|^{2s} |\hat{F}(\xi_1)|^2 \, d\xi_1 = \int_{\mathbb{R}} \big||\nabla|^s F\big|^2 \, dx_1.$$

Therefore we do not distinguish dimension $n = 1, 2$ when considering (fractional) Sobolev norms and Fourier multiplication operators.

2.3 Néel walls in soft thin films

From a variational point of view the leading order stray-field contribution in (2.2), determines asymptotically a geodesic magnetization path. Whereas in the bulk situation the stray field interaction can be eliminated completely by choosing a path perpendicular to the transition axis, that is $m_1 = 0$ (Bloch walls), the penalty on the vertical component as $\delta \to 0$ enforces in-plane rotations, that is $m_3 = 0$ (Néel walls), taking into account stray fields that typically appear to the leading order. In view of (2.2) we approximate the domain wall energy per unit area (2.1) by

$$\mathbb{E}(\mathbf{m}) = \frac{1}{2} \fint_0^l \int_{\mathbb{R}} \left(d^2 |\nabla \mathbf{m}|^2 + \frac{\delta}{2} \big||\nabla|^{\frac{1}{2}} \mathcal{H}(m)\big|^2 + Q\,(1 - m_2^2) + m_3^2 \right) dx$$

where $\mathbf{m} = (m, m_3)$.

We consider a parameter regime of a soft thin film so that anisotropy and relative thickness are balanced, more precisely

$$Q \ll 1, \quad \kappa \equiv d/\delta \gg 1 \quad \text{while} \quad \mathcal{Q} \equiv 4\,\kappa^2\, Q. \tag{2.3}$$

Then we introduce the small parameter $\varepsilon = \sqrt{Q}$. Rescaling x by $w = \delta/(2Q)$, the typical tail width, and energy by $\delta/2$ yields the interaction energy

$$E_\varepsilon(\mathbf{m}) = \frac{1}{2} \fint_0^L \int_{\mathbb{R}} \Big(\mathcal{Q}\,|\nabla \mathbf{m}| + \big||\nabla|^{\frac{1}{2}} \mathcal{H}(m)\big|^2 + (1 - m_2^2) + \Big(\frac{m_3}{\varepsilon}\Big)^2 \Big)\, dx \tag{2.4}$$

where $L = l/w$ and we assume

$$\varepsilon \ll \mathcal{Q} \ll 1.$$

If in addition $m = m(x_1)$ then $\mathcal{H}(m) = m_1 \mathbf{e_1}$ is independent of x_2 as well. In this case we set for simplicity of notation $x = x_1$ and $\xi = \xi_1$ the corresponding frequency variable. With this notation

$$\int_{\mathbb{R}} ||\nabla|^{\frac{1}{2}} m_1|^2\, dx = \int_{\mathbb{R}} |\xi| |\hat{m}_1(\xi)|^2\, d\xi.$$

Accordingly, the reduced variational principle for one dimensional domain wall transitions reads

$$E_\varepsilon(\mathbf{m}) = \frac{1}{2} \int_{\mathbb{R}} \Big(\mathcal{Q}|\mathbf{m}'|^2 + \big||\nabla|^{\frac{1}{2}} m_1\big|^2 + (1 - m_2^2) + \Big(\frac{m_3}{\varepsilon}\Big)^2 \Big)\, dx \to \min \tag{2.5}$$
$$\mathbf{m} : \mathbb{R} \to \mathbb{S}^2 \quad \text{with} \quad \mathbf{m}(\pm\infty) = (0, \pm 1, 0),$$

where $m' = \frac{dm}{dx_1}$. The translation invariance of the latter transition energy induces a certain lack of compactness. Nevertheless, one can show (see [5]) that for $\varepsilon_k \to 0$ there exists a sequence of minimizers $\mathbf{m}_{\varepsilon_k}$ of the variational principle above and a subsequence that locally converges to $\mathbf{m} = (m, 0)$ where m satisfies a reduced variational principle:

$$E_0(m) = \frac{1}{2} \int_{\mathbb{R}} \Big(\mathcal{Q}\,|m'|^2 + \big||\nabla|^{\frac{1}{2}} m_1\big|^2 + |m_1|^2 \Big)\, dx \to \min \tag{2.6}$$
$$m : \mathbb{R} \to \mathbb{S}^1 \quad \text{with} \quad m(\pm\infty) = (0, \pm 1).$$

A minimizer is called a Néel wall. The Néel wall energy can conveniently be expressed as

$$E_0(m) = \frac{1}{2} \Big(\mathcal{Q}\|m'\|_{L^2}^2 + \|m_1\|_{\dot{H}^{1/2}}^2 + \|m_1\|_{L^2}^2 \Big) \to \min$$
$$m : \mathbb{R} \to \mathbb{S}^1 \quad \text{with} \quad m(\pm\infty) = (0, \pm 1).$$

Since $|m'|^2 = (m_1')^2/(1-m_1^2)$ the Néel wall energy is a strictly convex functional in m_1. Thus the variational principle has for any $\mathcal{Q} > 0$ indeed a minimizer that is unique up to translations and the sign of m_1.

The Néel wall exhibits an internal two-scale structure: it decomposes into a core region nearby the wall center with typical width $w_{\rm core} = d^2/\delta$ and a tail region $w_{\rm core} \lesssim |x| \lesssim w_{\rm tail}$ with $w_{\rm tail} = w = \delta/(2Q)$ the scaling factor we have used above. Then the number $\mathcal{Q} \lesssim 1$ has an interpretation as twice the quotient $w_{\rm core}/w_{\rm tail}$. In the regime $\mathcal{Q} \ll 1$ the typical features of Néel walls are logarithmic energy scaling, see [5, 4],

$$\min E_0 = \pi \ln(1/\mathcal{Q})\big(1 + o(1)\big) \quad \text{as} \quad \mathcal{Q} \to 0 \tag{2.7}$$

and the extremely slow logarithmic decay of wall profiles,

$$m_1(x) \sim \ln(1/|x|)/\ln(1/\mathcal{Q}),$$

see [13, 14], giving rise to far range interaction of neighboring walls, see [3]. It is remarkable that, in case of finite Néel walls, the above energy asymptotic (2.7) holds true when the infimum in taken over L-periodic transitions $m = m(x_1, x_2)$, see [2]. The result proves particularly strong stability of the one-dimensional Néel wall with respect to two-dimensional variations, a result yet unknown for infinite Néel walls.

In Section 4 we show that up to translations Néel walls are indeed strict minimizers. For the purpose of spectral analysis we introduce a phase θ so that $m = (\cos\theta, \sin\theta)$. Then the Néel wall problem (2.6) reads

$$\begin{aligned} \mathcal{E}(\theta) = \frac{1}{2}\Big(\mathcal{Q}\int_{\mathbb{R}} |\theta'|^2 dx\ + \|\cos\theta\|^2_{\dot{H}^{1/2}} + \|\cos\theta\|^2_{L^2}\Big) \to \min \\ \theta : \mathbb{R} \to (-\pi/2, \pi/2) \quad \text{with} \quad \theta(\pm\infty) = \pm\pi/2. \end{aligned} \tag{2.8}$$

2.4 Landau-Lifshitz-Gilbert dynamics

In the presence of an external field $\mathbf{h}$, the equilibrium condition reads $\mathbf{m}\wedge\mathcal{H}_{\rm eff} = 0$, where the effective field is given by $\mathcal{H}_{\rm eff} = \mathbf{h} - \nabla\mathbb{E}(\mathbf{m})$, that is the difference of the external field and the L^2-gradient of the energy. Again the nonconvex character of the energy leads to a large variety of (local) minimizers and metastable states, and the hysteresis phenomenon. If otherwise a torque exists, that is $\mathbf{m} \wedge \mathcal{H}_{\rm eff} \neq 0$, the magnetization performs a damped precession dynamics described by the Landau-Lifshitz-Gilbert equation [12, 7]

$$\mathbf{m}_t + \alpha\, \mathbf{m} \wedge \mathbf{m}_t - \gamma\, \mathbf{m} \wedge \mathcal{H}_{\rm eff} = 0.$$

Here, $\alpha > 0$ is the dimensionless Gilbert damping factor and $\gamma > 0$ is the gyromagnetic ratio. For the local dynamic picture we consider a single oriented spin $\mathbf{m} = \mathbf{m}(t)$ under the influence of a constant external field $\mathbf{h}$. If a torque $\mathbf{m} \wedge \mathbf{h}$ exists and damping is neglected (that is $\alpha = 0$) then $\mathbf{m}$ precesses about $\mathbf{h}$ with frequency $\omega = \gamma\,|\mathbf{h}|$. If damping is switched on, that is $\alpha > 0$, the spin vector $\mathbf{m}$ spirals down to the constant spin, parallel to $\mathbf{h}$. The typical relaxation time is $1/(\alpha\,\omega)$. Typically, $\alpha > 0$ is a small parameter that reflects the fact that relaxation happens on a time scale that is considerably slower than precession.
Gilbert's description of damping in magneto-dynamics is a phenomenological one, mainly justified by thermal effects. In a realistic situation, however, other damping mechanisms (e.g. eddy current damping, material defects) that cannot be captured by a local term might be relevant.

Gyrotropic domain wall motion

In the presence of an applied field $\mathbf{h} = H(t)\,\mathbf{e_2}$ that points towards one of the equilibrium states, a domain wall starts to move. If $\mathbb{E}(\mathbf{m})$ is the associated internal domain wall energy, the averaged effective field is given by $\mathcal{H}_{\text{eff}} = -\nabla\mathbb{E}(\mathbf{m}) + \mathbf{h}$. Hence, the associated Landau-Lifshitz-Gilbert dynamics reads

$$\begin{gathered}\mathbf{m}_t + \alpha\,\mathbf{m} \wedge \mathbf{m}_t + \gamma\,\mathbf{m} \wedge \nabla\mathbb{E}(\mathbf{m}) = \gamma\,\mathbf{m} \wedge \mathbf{h} \\ \mathbf{m}(t) : \mathbb{R} \to \mathbb{S}^2 \quad \text{with} \quad \mathbf{m}(t, \pm\infty) = (0, \pm 1, 0).\end{gathered}$$

This evolution problem inherits a detailed description including kinematic properties and shape of a moving domain wall. In contrast to the static case the dynamic magnetization path for a fixed time t is rather determined by the balance of energetic and dynamic forces as prescribed by LLG. Indeed, precession pushes the magnetization vector away from its energetically optimal path taking into account a gain in stray-field energy.

Walker's explicit solution in the bulk

An important illustration is provided by Walker's construction of explicit solutions with constant propagation speed (traveling wave), see [19, 8, 18, 15]. We consider the bulk geometry and the associated domain wall energy

$$\mathbb{E}(\mathbf{m}) = \frac{1}{2}\int_{\mathbb{R}} \Big(d^2|\mathbf{m}'|^2 \ + Q\,(1 - m_2^2) + m_1^2\Big)\,dx.$$

If the optimal path $m_1 \equiv 0$ (Bloch wall) perpendicular to the transition axis is chosen, the stray-field is fully eliminated, and the resulting variational principle is essentially equivalent to the optimal profile problem for Cahn-Hilliard. Standard methods yield an optimal profile $m_2(x) = \tanh(x/w_0)$ with typical wall width $w_0 = d/\sqrt{Q}$ and energy $\mathbb{E}_0 = 2d\sqrt{Q}$, a calculation first performed to Landau and

Lifshitz. Subject to a constant applied external field $\mathbf{h} = H\,\mathbf{e_2}$, and in a moving frame $\xi = x + c\,t$ where c represents the speed of propagation, LLG reads

$$c\,\mathbf{m}' + \alpha\,c\,\mathbf{m} \wedge \mathbf{m}' + \gamma\,\mathbf{m} \wedge \nabla\mathbb{E}(\mathbf{m}) = \gamma\,\mathbf{m} \wedge \mathbf{h}. \tag{2.9}$$

Under the geometric assumption that the wall moves with constant polar inclination φ, the balance of driving force and dissipation yields standard profile $m_2(\xi) = \tanh(\xi/w)$ but with a decrease in wall width $w = d/\sqrt{Q + \sin^2\varphi}$. The inclination angle and the propagation speed c are determined by

$$\frac{1}{2}\sin(2\varphi) = \frac{H}{\alpha} \quad \text{and} \quad c = \frac{d\,\gamma\sin(2\varphi)}{2\sqrt{Q + \sin^2\varphi}}. \tag{2.10}$$

Beyond a peak velocity that is reached for finite field-strength H this construction breaks down. On the other side, the wall mobility, that is the infinitesimal rate of change of propagation speed c for small field H, is given by

$$\beta = \lim_{H\to 0}\frac{c(H)}{H} = \frac{\gamma w_0}{\alpha} = \frac{2d^2\gamma}{\alpha\,\mathbb{E}_0} = \frac{d^2\gamma}{\alpha\,\mathbb{E}_{\text{ex}}} \tag{2.11}$$

where we have used energy equi-partition in the last equation.

2.5 Formal wave asymptotics for the Bloch wall

In case of small crystalline anisotropy the residual anisotropy interaction stemming from stray-field dominates, and results in a singular competition with gyromagnetic forces in the context of dynamics. Let us show that skew-symmetry leads a wave-type dynamics along the Bloch wall pass perpendicular to the transition axis. To this end we introduce the small parameter $\varepsilon = \sqrt{Q}$ and rescale space by the Bloch wall width $d/\sqrt{Q}$, time by $1/(\gamma\varepsilon)$ and renormalizing energy by $d\sqrt{Q}$. We obtain

$$E_\varepsilon(\mathbf{m}) = \frac{1}{2}\int_{\mathbb{R}}\left(|\mathbf{m}'|^2 + (1 - m_2^2) + \left(\frac{m_1}{\varepsilon}\right)^2\right)dx.$$

In this spatial scale the effective field reads $\mathcal{H}_{\text{eff}} = -\varepsilon^2\nabla E_\varepsilon(\mathbf{m})$. Accordingly LLG becomes

$$\mathbf{m}_t + \alpha\,\mathbf{m} \wedge \mathbf{m}_t + \varepsilon\,\mathbf{m} \wedge \nabla E_\varepsilon(\mathbf{m}) = 0. \tag{2.12}$$

We investigate the rescaled equation in the following regime of low anisotropy and low damping

$$\varepsilon \to 0 \quad \text{while} \quad \nu_\varepsilon = \alpha(\varepsilon)/\varepsilon \to \nu.$$

A convenient choice of coordinate system in this situation is the following spherical coordinate system as proposed by Enz, see [17]

$$m_1 = \sin\varphi, \quad m_2 = \cos\varphi\sin\theta, \quad m_3 = \cos\varphi\cos\theta, \tag{2.13}$$

with limiting conditions $\theta(\pm\infty) = \pm\pi/2$ and $\varphi(\pm\infty) = 0$. Then the associated energy functional $\mathcal{E}_\varepsilon(\theta, \varphi) = E_\varepsilon(\mathbf{m})$ is given by

$$\mathcal{E}_\varepsilon(\theta, \varphi) = \frac{1}{2} \int_{\mathbb{R}} \left(|\varphi'|^2 + \cos^2 \varphi \, |\theta'|^2 + 1 - \cos^2 \varphi \, \sin^2 \theta + \frac{1}{\varepsilon^2} \sin^2 \varphi \right) dx,$$

and the rescaled LLG (2.12) reads

$$R(\varphi) \begin{pmatrix} \partial_t \theta \\ \partial_t \varphi \end{pmatrix} + \begin{pmatrix} \partial_\theta \mathcal{E}_\varepsilon(\theta, \varphi) \\ \partial_\varphi \mathcal{E}_\varepsilon(\theta, \varphi) \end{pmatrix} = 0 \tag{2.14}$$

with

$$R(\varphi) = \begin{pmatrix} \alpha & -\cos \varphi \\ \cos \varphi & \alpha \, \cos^2 \varphi \end{pmatrix}.$$

Nearby the static Bloch wall path $m_1 = 0$, that is for $\varphi \ll 1$, the energy decomposes into its leading order contributions

$$\mathcal{E}_\varepsilon(\theta, \varphi) \approx \mathcal{E}_0(\theta) + \frac{1}{2\varepsilon^2} \int_{\mathbb{R}} \varphi^2 \, dx \quad \text{with} \quad \mathcal{E}_0(\theta) = \frac{1}{2} \int_{\mathbb{R}} \left(|\theta'|^2 + \cos^2 \theta \right) dx$$

thus

$$\nabla \mathcal{E}_\varepsilon(\theta, \varphi) \approx \begin{pmatrix} \nabla \mathcal{E}_0(\theta) \\ \varphi/\varepsilon \end{pmatrix} \quad \text{and} \quad R(\varphi) \approx \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix}.$$

Observe that if (θ, φ) is considered as a complex function the approximate $R(\varphi)$ gives rise to a damped Schrödinger dynamics. More precisely (2.14) becomes

$$\begin{cases} -\partial_t \varphi + \alpha \, \partial_t \theta + \varepsilon \nabla \mathcal{E}_0(\theta) = 0, \\ \\ \partial_t \theta + \alpha \, \partial_t \varphi + (\varphi/\varepsilon) = 0. \end{cases} \tag{2.15}$$

Setting $\psi = \varphi/\varepsilon$ we obtain from (2.15) as $\varepsilon \to 0$ that

$$\begin{cases} \partial_t \psi + \nu \, \partial_t \theta + \nabla \mathcal{E}_0(\theta) = 0, \\ \\ \partial_t \theta - \psi = 0. \end{cases}$$

Differentiating the second equation with respect to time and substituting it into the first one we obtain the wave-type limit equation

$$\partial_t^2 \theta + \nu \, \partial_t \theta + \nabla \mathcal{E}_0(\theta) = 0.$$

3 Wave-type dynamics in thin films

Gyromagnetic precession is geometrically incompatible with the asymptotic constraint of in-plane magnetization that is imposed by stray-filed interaction. In other words, the competition between energetic and dynamic forces becomes singular in a thin-film limit. Here we investigate a suitable effective limit for LLG

$$\mathbf{m}_t + \alpha\, \mathbf{m} \wedge \mathbf{m}_t + \gamma\, \mathbf{m} \wedge \nabla \mathbb{E}(\mathbf{m}) = 0 \tag{3.1}$$

as the relative thickness δ/d tends to zero, where as in Section 2.2, $\mathbf{m}(t) : \mathbb{R}^2 \to \mathbb{S}^2$ is l-periodic in the second variable and the interaction energy is given by

$$\mathbb{E}(\mathbf{m}) = \frac{1}{2} \fint_0^l \int_{\mathbb{R}} \Big(d^2 |\nabla \mathbf{m}|^2 + \frac{\delta}{2} \big| |\nabla|^{\frac{1}{2}} \mathcal{H}(m) \big|^2 + Q(1 - m_2^2) + m_3^2 \Big)\, dx.$$

We recall that Gilbert's damping factor α is a small parameter as well, that is to say, precession proceeds much faster than relaxation. Prior work on thin-film reductions for LLG, leading to enhanced dissipation, see [6, 9, 16], considers the regime when $\delta/d \ll \alpha$. In order to preserve the oscillatory features of LLG dynamics, we take into account small Gilbert damping as well. As it turns out, the effective dynamics depends on asymptotic regime as α and the relative thickness δ/d tend to zero.

Scaling and asymptotic regime

We set $\varepsilon = \sqrt{Q}$ and consider the regime (2.3) when $\varepsilon \ll \mathcal{Q}$ while $\mathcal{Q} = (2\varepsilon d/\delta)^2 \lesssim 1$ is moderately small but uniformly bounded from below. In other words the dimensionless parameter $\varepsilon \sim \delta/d$ can be considered as a relative thickness. We rescale space and time

$$x \mapsto wx \quad \text{where} \quad w = \delta/(2\varepsilon^2) \quad \text{(tail width)}, \quad t \mapsto t/(\gamma\varepsilon). \tag{3.2}$$

In this spatial scale the averaged effective field $\mathcal{H}_{\text{eff}} = -\nabla \mathbb{E}(\mathbf{m})$ reads

$$\mathcal{H}_{\text{eff}} = -\varepsilon^2 \nabla E_\varepsilon(\mathbf{m}) \quad \text{where} \quad E_\varepsilon = (2/\delta)\mathbb{E} \tag{3.3}$$

Recall that $E_\varepsilon(\mathbf{m})$ is the rescaled domain wall energy given by (2.4). Under the above assumptions the exchange portion of the energy does not drop out asymptotically. Let us fix the dimensionless periodicity scale $L = l/w \gtrsim 1$ (for the argument we will assume that $L = 1$). The underlying L^2-scalar product is given by

$$\langle F, G \rangle = \fint_0^L \int_{\mathbb{R}} F \cdot G\, dx \quad \text{for} \quad F, G \in L^2(\mathbb{R} \times \mathbb{T}_L; \mathbb{R}^n), \quad n = 2, 3.$$

If $F \in H^{-1}(\mathbb{R} \times \mathbb{T}_L; \mathbb{R}^n)$ and $G \in H^1(\mathbb{R} \times \mathbb{T}_L; \mathbb{R}^n)$ we use the same notation for the corresponding dual pairing. Accordingly, the L^2-gradient of $E_\varepsilon = E_\varepsilon(\mathbf{m})$ at $\mathbf{m} = (m, m_3)$ defined by

$$\langle \nabla E_\varepsilon(\mathbf{m}), \mathbf{u} \rangle = \frac{d}{ds} E_\varepsilon(\mathbf{m} + s\mathbf{u}) \Big|_{s=0} \quad \text{for any } \mathbf{u} \in H^1(\mathbb{R} \times \mathbb{T}_L; \mathbb{R}^3),$$

is an L-periodic distribution and reads

$$\nabla E_\varepsilon(\mathbf{m}) = \Big[\mathcal{Q}(-\Delta) - \mathbf{e_2} \otimes \mathbf{e_2} \Big] \mathbf{m} + \Big((-\Delta)^{1/2} \mathcal{H}(m), m_3 \Big),$$

where $(-\Delta)^{1/2} = |\nabla|$ corresponds to the Fourier multiplier $|\xi|$.

According to (3.2) and (3.3) the Landau-Lifshitz-Gilbert equation (3.1) becomes

$$\mathbf{m}_t + \alpha\, \mathbf{m} \wedge \mathbf{m}_t + \varepsilon\, \mathbf{m} \wedge \nabla E_\varepsilon(\mathbf{m}) = 0. \tag{3.4}$$

We investigate this rescaled equation in the asymptotic regime when

$$\varepsilon \to 0 \quad \text{while} \quad \alpha(\varepsilon)/\varepsilon \to \nu \tag{3.5}$$

for some positive ν. Fixing $\mathcal{Q} \lesssim 1$ we implicitly assume that $\varepsilon \ll \mathcal{Q}$. For the argument we will assume that $\mathcal{Q} = 1$ for any $\varepsilon > 0$. In order to derive an effective equation for the in-plane magnetizations we decompose $E_\varepsilon(\mathbf{m})$ into a portion that only involves the in-plane components $m = (m_1, m_2)$ independent of ε, and one that only contains the vertical component m_3, that is for $\mathbf{m} = (m, m_3)$

$$E_\varepsilon(\mathbf{m}) = E_0(m) + \frac{1}{2} \fint_0^L \int_{\mathbb{R}} \left(\mathcal{Q}\, |\nabla m_3|^2 + \Big(\frac{m_3}{\varepsilon}\Big)^2 \right) dx. \tag{3.6}$$

The in-plane portion of the energy $E_0(m)$ is given by

$$E_0(m) = \frac{1}{2} \fint_0^L \int_{\mathbb{R}} \Big(\mathcal{Q}\, |\nabla m|^2 + ||\nabla|^{\frac{1}{2}} \mathcal{H}(m)|^2 + (1 - m_2^2) \Big)\, dx. \tag{3.7}$$

For in-plane magnetizations, it agrees with the reduced Néel wall energy (2.6) and turns out to be the effective interaction energy for the dynamic problem as well.

Theorem 1. *Let $T > 0$ and $\mathbf{m}_\varepsilon : \mathbb{R}^2 \times (0, T) \to \mathbb{S}^2$ be a family of smooth solutions of* (3.4) *L-periodic in the second spatial direction and with uniformly bounded initial energy* (3.6)*, that is $E_\varepsilon(\mathbf{m}_\varepsilon(0)) \le C$. Suppose that $\alpha(\varepsilon)/\varepsilon \to \nu$ and the in-plane components $m_\varepsilon \rightharpoonup m$ converge weakly in $L^2_{\text{loc}}(\mathbb{R} \times \mathbb{T}_L \times (0, T); \mathbb{R}^2)$. Then $m \in H^1_{\text{loc}}(\mathbb{R} \times \mathbb{T}_L \times (0, T); \mathbb{S}^1)$, and it is a weak solution of*

$$\big[\partial_t^2 m + \nu\, \partial_t m + \nabla E_0(m) \big] \perp T_m \mathbb{S}^1. \tag{3.8}$$

By an in-plane magnetization $m \in H^1_{\rm loc}(\mathbb{R}\times\mathbb{T}_L\times(0,T);\mathbb{S}^1)$ satisfying (3.8) we mean

$$\int_0^T \fint_0^L \int_{\mathbb{R}} \Big(-\partial_t m\cdot\partial_t\phi + \nu\,\partial_t m\cdot\phi\Big)\,dxdt + \int_0^T \langle \nabla E_0(m),\phi\rangle\,dt = 0 \qquad (3.9)$$

for any admissible test function $\phi \in L^\infty\cap H^1(\mathbb{R}\times\mathbb{T}_L\times(0,T);\mathbb{R}^2)$ with compact support so that $\phi\cdot m = 0$, that is ϕ is a tangential vector field along m. Observe that $L^\infty\cap H^1$ forms an algebra and that all admissible test functions are of the form $\phi = m^\perp\varphi$ for some compactly supported $\varphi\in L^\infty\cap H^1(\mathbb{R}\times\mathbb{T}_L\times(0,T))$ and $m^\perp = (-m_2,m_1)$. Thus, by weak* density it is enough to verify (3.9) for $\phi = m^\perp\varphi$ with $\varphi\in C_0^\infty(\mathbb{R}\times\mathbb{T}_L\times(0,T))$.

Proof of Theorem 1. Let us assume for notational convenience that $\mathcal{Q}=1$ and $L=1$, and set $\mathbb{T}=\mathbb{T}_1=\mathbb{R}/\mathbb{Z}$. We blow-up the vertical component by ε, that is we set $m_3=\varepsilon v$. Then, for $\mathbf{m}=(m,\varepsilon v)$, the energy can be written as

$$E_\varepsilon(\mathbf{m}) = E_0(m) + G_\varepsilon(v)$$

where $E_0(m)$ is the in-plane portion (3.7) and

$$G_\varepsilon(v) = \frac{1}{2}\int_{\mathbb{R}\times\mathbb{T}} \left(\varepsilon^2\,|\nabla v|^2 + |v|^2\right)\,dx.$$

We introduce the rescaled damping factor $\nu_\varepsilon = \alpha(\varepsilon)/\varepsilon$. Using the multiplier $\mathbf{m}\wedge\mathbf{m}_t$ for the rescaled LLG (3.4), we find the energy inequality:

$$\nu_\varepsilon\int_0^\infty \Big[\|\partial_t m_\varepsilon\|^2_{L^2_x} + \varepsilon^2\,\|\partial_t v_\varepsilon\|^2_{L^2_x}\Big]\,dt + \sup_{T>0}\Big[E_0(m_\varepsilon(T)) + G_\varepsilon(v_\varepsilon(T))\Big] \le C. \quad (3.10)$$

Hence, we have for a subsequence $\varepsilon=\varepsilon_k\searrow 0$ as $k\to\infty$

$$m_\varepsilon \overset{*}{\rightharpoonup} m \qquad \text{weakly} * \text{ in } L^\infty_t\dot H^1_x\cap\dot H^1_t L^2_x \quad (3.11)$$

$$v_\varepsilon \overset{*}{\rightharpoonup} v \qquad \text{weakly} * \text{ in } L^\infty_t L^2_x \quad (3.12)$$

$$(\varepsilon\, v_\varepsilon) \qquad \text{is uniformly bounded in } L^\infty_t H^1_x\cap\dot H^1_t L^2_x \quad (3.13)$$

where $|m|=1$ almost everywhere in $\mathbb{R}\times\mathbb{T}\times(0,T)$.

In view of Aubin's Lemma, (3.11) also implies that

$$m_\varepsilon\to m \text{ strongly in } L^q_{\rm loc}(\mathbb{R}\times\mathbb{T}\times(0,T);\mathbb{R}^2) \quad\text{for any}\quad 2\le q<\infty. \quad (3.14)$$

Equation (3.4) can be written as

$$\begin{pmatrix}\partial_t m_{\varepsilon,1}\\ \partial_t m_{\varepsilon,2}\\ \varepsilon\partial_t v_\varepsilon\end{pmatrix} = \begin{bmatrix} 0 & \varepsilon\, v_\varepsilon & -m_{\varepsilon,2}\\ -\varepsilon\, v_\varepsilon & 0 & m_{\varepsilon,1}\\ m_{\varepsilon,2} & -m_{\varepsilon,1} & 0\end{bmatrix}\left\{\varepsilon\,\nu_\varepsilon\begin{pmatrix}\partial_t m_{\varepsilon,1}\\ \partial_t m_{\varepsilon,2}\\ \varepsilon\partial_t v_\varepsilon\end{pmatrix} + \begin{pmatrix}\varepsilon\,(\nabla E_0)_1\\ \varepsilon\,(\nabla E_0)_2\\ -\varepsilon^2\,\Delta\, v_\varepsilon + v_\varepsilon\end{pmatrix}\right\}.$$

Using the notation $V^\perp = (-V_2, V_1)$ for $V \in \mathbb{R}^2$, we split the system into

$$\partial_t m_\varepsilon - v_\varepsilon m_\varepsilon^\perp = \varepsilon\, R_\varepsilon(m_\varepsilon, \varepsilon\, v_\varepsilon)^\perp, \tag{3.15}$$

$$\partial_t v_\varepsilon + m_\varepsilon^\perp \cdot \Big[\nu_\varepsilon\, \partial_t m_\varepsilon + \nabla E_0(m_\varepsilon)\Big] = 0, \tag{3.16}$$

where

$$R_\varepsilon(m_\varepsilon, \varepsilon\, v_\varepsilon) = \nu_\varepsilon\, \big(m_\varepsilon\, \varepsilon\, \partial_t v_\varepsilon - \varepsilon\, v_\varepsilon\, \partial_t m_\varepsilon\big) - \big(\varepsilon\, v_\varepsilon\, \nabla E_0(m_\varepsilon) + m_\varepsilon \varepsilon\, \Delta v_\varepsilon\big).$$

We first consider equation (3.15). According to the energy inequality the family $R_\varepsilon = R_\varepsilon(m_\varepsilon, \varepsilon\, v_\varepsilon)$ has uniform bounds as a distribution. Indeed, for any compactly supported $\phi \in L^\infty \cap H^1(\mathbb{R} \times \mathbb{T} \times (0,T); \mathbb{R}^2)$ the pairing $\int_0^T \langle R_\varepsilon, \phi\rangle\, dt$ is bounded by

$$\int_0^T |\langle R_\varepsilon, \phi\rangle|\, dt \le \nu_\varepsilon \int_0^T \Big(|\langle \varepsilon\, v_\varepsilon, \partial_t m_\varepsilon \phi\rangle| + |\langle \varepsilon\, \partial_t v_\varepsilon, m_\varepsilon\, \phi\rangle|\Big)\, dt$$
$$+ \int_0^T \Big(|\langle \nabla E_0(m_\varepsilon), \varepsilon\, v_\varepsilon\, \phi\rangle| + |\langle \varepsilon\, \nabla v_\varepsilon, \nabla(m_\varepsilon\, \phi)\rangle|\Big)\, dt.$$

Since for $m = m_\varepsilon(t)$ one has $\|(-\Delta)m\|_{H_x^{-1}} \le \|\nabla m\|_{L_x^2}$ and

$$\|(-\Delta)^{\frac{1}{2}}\mathcal{H}(m)\|_{H_x^{-1}} \le \|(-\Delta)^{\frac{1}{2}}\mathcal{H}(m)\|_{L_x^2} \le \|\nabla m\|_{L_x^2}$$

we deduce from the energy inequality (3.10) that the energy gradient

$$\nabla E_0(m_\varepsilon(t)) = \Big[(-\Delta) + (-\Delta)^{1/2}\mathcal{H} - \mathbf{e_2} \otimes \mathbf{e_2}\Big] m_\varepsilon(t) \in (H^{-1} + L^\infty)(\mathbb{R} \times \mathbb{T}; \mathbb{R}^2)$$

has a bound in this space that is uniform in $\varepsilon > 0$ and $t > 0$. Using this, (3.13), and Cauchy-Schwarz we find

$$\int_0^T |\langle R_\varepsilon, \phi\rangle|\; dt \le C \left(\int_0^T \Big(\|\phi(t)\|_{H_x^1}^2 + \|\phi(t)\|_{L_x^\infty}^2\Big)\; dt\right)^{1/2} \tag{3.17}$$

with a constant C that only depends on the initial energy (that is uniformly bounded by assumption) and the size of the support of ϕ.

Estimate (3.17) is valid for test functions of the form $\phi = m_\varepsilon^\perp\, \partial_t \varphi$ for any $\varphi \in C_0^\infty(\mathbb{R} \times \mathbb{T} \times (0,T))$. Thus, passing to the limit as $\varepsilon \to 0$ in (3.15) while using (3.11) and (3.12) yields a distributional equation

$$\int_0^T \int_{\mathbb{R}\times\mathbb{T}} (m^\perp \cdot \partial_t m)\, \partial_t \varphi\, dx\, dt = \int_0^T \int_{\mathbb{R}\times\mathbb{T}} v\, \partial_t \varphi\, dx\, dt, \tag{3.18}$$

for any $\varphi \in C_0^\infty(\mathbb{R} \times \mathbb{T} \times (0,T))$, that means formally $\partial_t v = \partial_t^2 m \cdot m^\perp$ in the sense of distributions.

For the second equation (3.16), integration by parts yields for the highest order term

$$-\int_0^T \int_{\mathbb{R}\times\mathbb{T}} (m_\varepsilon^\perp \cdot \Delta m_\varepsilon)\, \varphi \, dx\, dt = \int_0^T \int_{\mathbb{R}\times\mathbb{T}} (m_\varepsilon^\perp \cdot \nabla m_\varepsilon)\, \nabla\varphi \, dx\, dt$$

for any $\varphi \in C_0^\infty(\mathbb{R}\times\mathbb{T}\times(0,T))$. Using (3.11), (3.12), and (3.14) we can pass to the limit is every term. We get

$$\int_0^T \int_{\mathbb{R}\times\mathbb{T}} v\, \partial_t\varphi \, dx\, dt = \nu \int_0^T \int_{\mathbb{R}\times\mathbb{T}} \partial_t m \cdot m^\perp \varphi \, dx\, dt + \int_0^T \left\langle \nabla E_0(m), m^\perp \varphi \right\rangle \, dt \tag{3.19}$$

for any $\varphi \in C_0^\infty(\mathbb{R}\times\mathbb{T}\times(0,T))$. Substituting (3.18) into (3.19) equation yields the result. □

Application to moving Néel walls

We apply the latter asymptotic limit in the one-dimensional context of Néel wall transitions:

$$E_0(m) = \frac{1}{2}\int_{\mathbb{R}} \Big(\mathcal{Q}\,|m'|^2 + \big||\nabla|^{\frac{1}{2}} m_1\big|^2 + |m_1|^2 \Big)\, dx \tag{3.20}$$
$$m : \mathbb{R} \to \mathbb{S}^1 \quad \text{with} \quad m(\pm\infty) = (0, \pm 1).$$

The theorem suggests the following dynamic model for the evolution of Néel walls in thin films subject to a constant applied field $h = H\hat{\mathbf{e}}_2$ that points towards one of the end-states determined by anisotropy

$$\Big(\partial_t^2 m + \nu\, \partial_t m + \nabla E_0(m) \Big) \cdot m^\perp = h \cdot m^\perp \tag{3.21}$$
$$m : \mathbb{R}\times(0,T) \to \mathbb{S}^1 \ \text{ with } \ m(\pm\infty, t) = (0, \pm 1).$$

In terms of the phase function the reduced dynamic equation (3.21) reads

$$\partial_t^2 \theta + \nu\, \partial_t \theta + \nabla\mathcal{E}(\theta) = H(t)\, \cos\theta \tag{3.22}$$
$$\theta : \mathbb{R}\times(0,T) \to \mathbb{R} \ \text{ with } \ \theta(\pm\infty, t) = (0, \pm\pi/2).$$

For constant applied fields $H = \text{const.}$ we are mainly interested in solutions $\theta = \theta(x + ct)$ of constant propagation speed c (traveling waves) and the typical dependence of c on H. In this context we show the existence of traveling wave solution with a profile nearby the static Néel wall θ_0 for small H. A crucial ingredient is the following stability analysis.

4 Stability for static Néel walls

We have seen in Section 2.2 that the phase θ_0 of a minimizing Néel wall is unique up to a translation. In this section we show that, modulo translation invariance, Néel walls are stable minimizers. This is merely a consequence of the fact that the Néel wall energy is a uniformly convex functional of m_1 subject to the constraint $m_1(0) = 1$. The corresponding non-degeneracy result for the linearization serves to construct special solutions for the dynamic problem close to the ground state. Throughout this section we set $\mathcal{Q} = 1$ for notational convenience, but all estimates carry over to arbitrary $\mathcal{Q} > 0$ with only a change of constants. The Euler-Lagrange equation for (2.8) reads

$$\theta'' + \sin\theta\big[1 + (-\Delta)^{1/2}\big]\cos\theta = 0. \tag{4.1}$$

Note that, the left hand side of (4.1) is just the negative of the $L^2(\mathbb{R})$–gradient of the Néel wall energy $\mathcal{E}(\theta)$ given by

$$\langle \nabla\mathcal{E}(\theta), u\rangle = \frac{d}{ds}\mathcal{E}(\theta + su)\Big|_{s=0}$$

for any variation $u \in H^1(\mathbb{R})$.

In [13] it is proved that the phase θ_0 of the Néel wall is nondecreasing, and θ_0' only may vanish in an interval centered at the origin. Here we rule out this possibility.

Lemma 1. *The phase θ_0 of a centered Néel wall is monotone with $\theta_0'(0) > 0$.*

Proof. Due to Proposition 1 in [13] we only have to show that $\theta_0'(0) = 0$ is impossible. Observe that $\theta = \theta_0$ is a solution of the ordinary differential equation

$$\theta'' + b(x)\,\sin\theta = 0 \tag{4.2}$$

where $b(x) = \big[1 + (-\Delta)^{1/2}\big](\cos\theta_0)(x)$. Since $\theta_0' \in L^2(\mathbb{R})$ we have $(\cos\theta_0)' = \sin\theta_0\theta_0' \in L^2(\mathbb{R})$, and due to the estimate $\|(1 + (-\Delta)^{1/2})u\|_{L^2} \leq C\|u\|_{H^1}$ we conclude that $b(x) \in L^2(\mathbb{R})$. Hence, by (4.2) $\theta_0'' \in L^2(\mathbb{R})$. Next, using the interpolation inequality

$$\|\theta_0'\|_{L^4} \leq \|\theta_0\|_{L^\infty}^{1/2}\|\theta_0''\|_{L^2}^{1/2}$$

we have $(\cos\theta_0)' = -\sin\theta_0\theta_0'' - \cos\theta_0|\theta_0'|^2 \in L^2(\mathbb{R})$. Thus, $b(x) \in H^1(\mathbb{R})$. Then, Sobolev embedding theorem in one dimension imply that $b(x)$ is continuous and bounded. If we assume that $\theta(0) = \theta'(0) = 0$, the uniqueness theorem for ordinary differential equations implies that $\theta(x) = 0$, a contradiction. □

Observe that the regularity assertions in the proof can be bootstrapped, and we find that Néel wall profiles are indeed smooth.

4.1 A lower bound for the Hessian

We introduce the bilinear form

$$B(f,g) = \operatorname{Re} \int_{\mathbb{R}} \big(1+|\xi|\big) \hat{f}(\xi)\, \bar{\hat{g}}(\xi)\, d\xi,$$

that is equivalent to the $H^{1/2}(\mathbb{R})$ inner product. In particular, $B(f,f) \geq \|f\|_{L^2}^2$.

Lemma 2. *Suppose that θ_0 minimizes $\mathcal{E}(\theta)$ subjected to center and boundary conditions $\theta(0) = 0$ and $\theta(\pm\infty) = \pm\pi/2$. Then the Hessian exhibits a lower bound*

$$\operatorname{Hess} \mathcal{E}(\theta_0)\langle u, u\rangle \geq \|u\,\theta_0'\|_{L^2}^2 + B(u \sin\theta_0, u \sin\theta_0)$$

for any admissible variation $u \in H^1(\mathbb{R})$ with $u(0) = 0$.

Proof. We first consider functions u that vanish in a neighborhood of 0. Then

$$\operatorname{Hess} \mathcal{E}(\theta_0)\langle u, u\rangle = \int_{\mathbb{R}} |u'|^2\, dx - B(\cos\theta_0, u^2 \cos\theta_0) + B(u \sin\theta_0, u \sin\theta_0).$$

In order to estimate the middle term, we deduce from the Euler-Lagrange equation with the admissible test function $u^2 \cot\theta_0$

$$\int_{\mathbb{R}} \theta_0' \left(u^2 \cot\theta_0\right)'\, dx = B(\cos\theta_0, u^2 \cos\theta_0).$$

Recalling that $-d(\cot\theta)/d\theta = 1 + \cot^2\theta$, we find with Young's inequality

$$\int_{\mathbb{R}} \left(|u'|^2 - \theta_0'(u^2 \cot\theta)'\right)\, dx \geq \int_{\mathbb{R}} |\theta_0'|^2 |u|^2\, dx$$

and the claim follows. If we only assume that $u(0) = 0$, then we approximate by $u_\delta \in H^1(\mathbb{R})$ defined by $u_\delta(x) = 0$ in $(-\delta, \delta)$ and $u_\delta(x) = u(x - \delta)$ for $x > \delta$ and analog for $x < \delta$. Then u_δ is admissible for the above estimate and converges to u strongly in H^1, so that the claim follows in the limit $\delta \to 0$. □

4.2 The linearized operator

We consider the linearization $\mathcal{L}_0$ of the mapping $\nabla\mathcal{E} = \nabla\mathcal{E}(\theta)$ at θ_0 given by

$$\mathcal{L}_0 u = \frac{d}{ds}\Big|_{s=0} \nabla\mathcal{E}(\theta_0 + su). \tag{4.3}$$

With the bounded and smooth coefficients

$$s(x) = \sin\theta_0(x) \quad \text{and} \quad C(x) = \cos\theta_0(x)\big[1 + (-\Delta)^{1/2}\big](\cos\theta_0)(x) \tag{4.4}$$

the operator reads

$$\mathcal{L}_0 u = -u'' + s(x)\big[1 + (-\Delta)^{1/2}\big](s(x)\, u) - C(x) u. \tag{4.5}$$

Proposition 1. *For all $f \in L^2(\mathbb{R})$ such that $f \perp \theta_0'$ in $L^2(\mathbb{R})$ there exists a solution $u \in H^2(\mathbb{R})$ of*

$$\mathcal{L}_0\, u = f. \tag{4.6}$$

The solution is unique up to a constant multiple of θ_0'.

Proposition 1 is a consequence of the following lemmata. Lemma 3 provides a spectral bound for $\mathcal{L}_0$ on the complementary space $\{u \perp \theta_0'\}$. Lemma 4 provides existence and (partial) uniqueness of weak solutions by the Riesz representation theorem. Finally we show H^2-regularity.

Lemma 3. *There is a constant $\Lambda > 0$ with the following property: If $u \in H^1(\mathbb{R})$ such that $u \perp \theta_0'$ in $L^2(\mathbb{R})$, then*

$$\operatorname{Hess}\mathcal{E}(\theta_0)\langle u,u\rangle \geq \Lambda\, \|u\|_{L^2}^2. \tag{4.7}$$

Recall that $\langle \mathcal{L}_0 u, u\rangle = \operatorname{Hess}\mathcal{E}(\theta_0)\langle u,u\rangle$. Note that as a consequence of this lemma and that θ_0' annihilates the Hessian, the operator $\mathcal{L}_0$ has zero as a simple eigenvalue with eigenspace spanned by θ_0'.

Proof of Lemma 3. We first show that the claim of Lemma 2 holds for $u \in H^1(\mathbb{R})$ with $u \perp \theta_0'$ in $L^2(\mathbb{R})$. Indeed, θ_0' annihilates $\operatorname{Hess}\mathcal{E}(\theta_0)$. In view of Lemma 1 the function $v = u - u(0)/\theta_0'(0)\theta_0'$ is admissible with

$$\begin{aligned}
\operatorname{Hess}\mathcal{E}(\theta_0)\langle u,u\rangle &= \operatorname{Hess}\mathcal{E}(\theta_0)\langle v,v\rangle \\
&\geq \|\theta_0' v\|_{L^2}^2 + B(v\sin\theta_0, v\sin\theta_0) \\
&\geq \|\theta_0' v\|_{L^2}^2 + \|\sin\theta_0\, v\|_{L^2}^2 \\
&\geq \Lambda\, \|v\|_{L^2}^2
\end{aligned}$$

where we have used that $|\theta_0'|$ and $|\sin\theta_0|$ are bounded from below for small and for large $|x|$, respectively. But by orthogonality $\|v\|_{L^2}^2 \geq \|u\|_{L^2}^2$ and (4.7) follows. □

Lemma 4. *Let $f \in L^2(\mathbb{R})$ such that $f \perp \theta_0'$ in $L^2(\mathbb{R})$. Then problem* (4.6) *has a weak solution $u \in H^1(\mathbb{R})$, unique up to a multiple of θ_0'. Moreover, if $u \perp \theta_0'$ in $L^2(\mathbb{R})$ then*

$$\|u\|_{H^1} \leq C\|f\|_{H^{-1}}$$

for a universal constant $C > 0$.

Proof. Let $H^1_\perp := \{u \in H^1(\mathbb{R}) \,:\, u \perp \theta_0' \text{ in } L^2(\mathbb{R})\}$ and

$$a(u,v) := \operatorname{Hess}\mathcal{E}(\theta_0)\langle u,v\rangle = \int_{\mathbb{R}} u'v'\,dx + B(\sin\theta_0\, u, \sin\theta_0\, v) - \int_{\mathbb{R}} C(x)uv\,dx.$$

We claim that $a(\cdot,\cdot) : H^1_\perp \times H^1_\perp \to \mathbb{R}$ is a symmetric bilinear form, that induces a norm on $H^1_\perp$ equivalent to the H^1–norm. Indeed, since

$$B(\sin\theta_0\, u, \sin\theta_0\, u) \leq \|\sin\theta_0\, u\|_{H^{1/2}}^2 \leq \|\sin\theta_0\, u\|_{H^1}^2 \leq C\|u\|_{H^1}^2$$

for some constant C that only depends on $\|\theta_0\|_{H^1}$, we only need to show that $a(\cdot,\cdot)$ is coercive. From (4.7) and the boundedness of $C(x)$ (see (4.4) and comment after Lemma 1) we infer

$$\begin{aligned}(1+C)\,a(u,u) &= \int_{\mathbb{R}} |u'|^2\,dx + B(u\sin\theta_0, u\sin\theta_0) \\ &\qquad - \int_{\mathbb{R}} C(x)u^2\,dx + C\,\mathrm{Hess}\,\mathcal{E}(\theta_0)\langle u,u\rangle \\ &\geq \int_{\mathbb{R}} |u'|^2\,dx - \int_{\mathbb{R}} C(x)\,u^2\,dx + C\,\Lambda \int_{\mathbb{R}} u^2\,dx \\ &\geq \min\{1, C\,\Lambda - \|C(x)\|_{L^\infty}\}\|u\|_{H^1}^2 \end{aligned} \tag{4.8}$$

where we choose the constant so that $C\,\Lambda > \|C(x)\|_{L^\infty}$.

Observe that $\mathcal{L}_0$ commutes with the L^2 projection onto the complement of θ_0', and let f be as in the assumptions of the Lemma. Then, $u \in H^1_\perp$ is a weak solutions of(4.6) means

$$a(u,v) = \langle f, v\rangle \quad \text{for any} \quad v \in H^1_\perp.$$

Then existence and uniqueness in the space $H^1_\perp$ follows from the Riesz representation theorem. Moreover, if $u \in H^1_\perp$ is a weak solution of (4.6) then we get by (4.8)

$$C\|u\|_{H^1}^2 \leq a(u,u) = \langle u, f\rangle \leq \|u\|_{H^1}\|f\|_{H^{-1}}$$

and the estimate follows. □

Lemma 5. *Let $f \in L^2(\mathbb{R})$ with $f \perp \theta_0'$ in $L^2(\mathbb{R})$ and $u \in H^1(\mathbb{R})$ be a weak solution of* (4.6)*, then $u \in H^2(\mathbb{R})$. Moreover, if $u \perp \theta_0'$ in $L^2(\mathbb{R})$, there exists a constant $C > 0$ such that*

$$\|u\|_{H^2} \leq C\,\|f\|_{L^2}. \tag{4.9}$$

Proof. Since $\theta_0' \in H^2(\mathbb{R})$ we can, in view of Lemma 4, assume that $u \in H^1_\perp$. We have

$$u'' = s(x)(1+(-\Delta)^{1/2})(s(x)\,u) - C(x)u + f$$

where $C(x)$ and $s(x)$, as in (4.4), are smooth with bounded derivatives. Using the estimate in Lemma 4 we find

$$\|u''\|_{L^2}^2 \leq C\,(\|f\|_{L^2}^2 + \|u\|_{H^1}^2) \leq C\,\|f\|_{L^2}^2$$

and the result follows. □

5 Traveling wave solutions for the Néel wall

The analysis performed in the last section allows to construct a traveling wave solution for the dynamic problem near the static Néel wall, by means of the implicit function theorem. Similar arguments have been used in the context of convolution models for phase transitions [1].

Theorem 2. *For sufficiently small field strength H there is a traveling wave for the reduced Landau-Lifshitz-Gilbert dynamics*

$$c^2\,\theta'' + c\,\nu\,\theta' + \nabla\mathcal{E}(\theta) = H\cos\theta$$

that connects antipodal states at infinity $\theta(\pm\infty) = \pm\pi/2$ near the static Néel wall θ_0. Moreover, the propagation speed has an expansion $c = \beta H + o(H)$ where the wall mobility is given by $\beta = 1/(M\nu)$ with $M = \frac{1}{2}\int_{\mathbb{R}} |\theta_0'|^2\,dx$.

Mobility of a the Néel wall in physical units

In order to extract the wall mobility in physical units, we have to account for all changes of scale. Recall that we rescaled time $\tilde{t} = t/(\gamma\sqrt{Q})$ and space $\tilde{x} = w\,x$ where w is the tail width of the static Néel wall $w = \delta/(2Q)$. Therefore, since in physical units the mobility β^* has the dimension of velocity, we have

$$\beta^* = \frac{\gamma\delta}{2\varepsilon}\beta.$$

The exchange coefficient reads $\mathcal{Q} = 4Q(d/\delta)^2$. Rescaling to physical units

$$\mathbb{E}_{\text{ex}} = \frac{d^2}{2}\int_{\mathbb{R}} |\mathbf{m}'|^2 dx = \frac{\delta}{2}\mathcal{Q}M.$$

The number $\mathbb{E}_{\text{ex}}$ is the exchange part of the energy per unit of area that remains implicit in the thin film situation. Then we deduce from Theorem 2

$$\beta^* = \frac{\delta^2\mathcal{Q}\gamma}{4\,\varepsilon\,\nu\,\mathbb{E}_{\text{ex}}} = \frac{d^2\gamma}{\alpha\mathbb{E}_{\text{ex}}}.$$

Taking into account energy equi-partition for the Bloch wall we observe that this expression formally agrees with (2.11) deduced from the Walker solution. With the energy asymptotic (2.7) we have the upper bound, $\mathbb{E}_{\text{ex}} \lesssim \delta/\ln(1/\mathcal{Q})$ for $\mathcal{Q} \ll 1$, giving a lower bound for the mobility.

Proof of Theorem 2. We let

$$G\big((\theta,c),H\big) = c^2\,\theta'' + c\,\nu\,\theta' + \nabla\mathcal{E}(\theta) - H\cos\theta.$$

This functional inherits the translation invariance of the Néel wall, in the sense that, if $G\big((\theta(x), c), H\big) = 0$ then $G\big((\theta(x+x_0), c), H\big) = 0$ for all real x_0. To factor out this invariance we consider the extended functional equation

$$\mathcal{G}\big((\theta, c), H\big) = \big[G\big((\theta, c), H\big), \theta(0)\big] = (0, 0).$$

Step 1: We claim that, for a static Néel wall θ_0 and perturbations ϕ, the mapping

$$H^2(\mathbb{R}) \times \mathbb{R} \times \mathbb{R} \ni \big((\phi, c), H\big) \mapsto \mathcal{G}\big((\theta_0 + \phi, c), H\big) \in L^2(\mathbb{R}) \times \mathbb{R}$$

is continuously Fréchet differentiable. First, since $H^2(\mathbb{R}) \hookrightarrow BC(\mathbb{R})$, the linear mapping $\theta \to \theta(0)$ is well defined and bounded. Next, we consider the mapping

$$((\phi, c), H) \to G((\theta_0 + \phi, c), H).$$

The dependence on c and H of this functional is obviously smooth. Regarding ϕ dependence we only need to consider the nonlinear part

$$\phi \mapsto \sin(\theta_0 + \phi)\Big[1 + (-\Delta)^{1/2}\Big]\cos(\theta_0 + \phi) - H\cos(\theta_0 + \phi)$$

that is C^1 regular from $H^2(\mathbb{R}) \hookrightarrow H^1(\mathbb{R})$ into $L^2(\mathbb{R})$. Indeed, the mappings

$$\phi \mapsto \cos(\theta_0 + \phi) \quad \text{and} \quad \phi \mapsto \sin(\theta_0 + \phi) - \sin\theta_0$$

are bounded and continuous on $H^1(\mathbb{R})$. Moreover, since $H^1(\mathbb{R})$ forms a smooth multiplicative algebra, one infers continuous differentiability of these mappings from $H^1(\mathbb{R})$ into itself. Since $(-\Delta)^{1/2} : H^1(\mathbb{R}) \to L^2(\mathbb{R})$ smoothly, we deduce from the product rule the continuous differentiability of

$$\phi \mapsto \Big(\sin(\theta_0 + \phi) - \sin\theta_0\Big)\Big[1 + (-\Delta)^{1/2}\Big]\cos(\theta_0 + \phi)$$

as a mapping from $H^1(\mathbb{R})$ into $L^2(\mathbb{R})$ and clearly for

$$\phi \mapsto \sin\theta_0\Big[1 + (-\Delta)^{1/2}\Big]\cos(\theta_0 + \phi) - H\ \cos(\theta_0 + \phi).$$

Step 2: The linearization with respect to the first two components (θ, c) at the (stationary) Néel wall $(\theta_0, 0)$ reads

$$\begin{bmatrix} \mathcal{L}_0 & \nu\,\theta_0' \\ \delta_0 & 0 \end{bmatrix}.$$

We show that, as a mapping $H^2(\mathbb{R}) \times \mathbb{R} \to L^2(\mathbb{R}) \times \mathbb{R}$, it has a bounded inverse. We only need to show invertibility. This means that, for every $(f, b) \in L^2(\mathbb{R}) \times \mathbb{R}$ there is a unique $(v, c) \in H^2(\mathbb{R}) \times \mathbb{R}$ so that

$$\mathcal{L}_0 v + c\,\nu\,\theta_0' = f \quad \text{and} \quad v(0) = b. \tag{5.1}$$

Indeed, according to Proposition 1, the first equation is solvable provided

$$(f - c\nu\theta_0') \perp \theta_0' \quad \text{that is} \quad c\,\nu \int_{\mathbb{R}} |\theta_0'|^2\, dx = \int_{\mathbb{R}} f\,\theta_0'\, dx.$$

This fixes c and determines v up to a multiple of θ_0', that is $v + \lambda\theta_0'$. But due to Lemma 1 we have $\theta_0'(0) > 0$, and the second equation in (5.1) fixes λ as well. Regarding uniqueness we have to show that

$$\mathcal{L}_0 w + c\,\nu\,\theta_0' = 0 \quad \text{and} \quad w(0) = 0$$

only admits the zero solution $(w, c) = (0, 0)$. Indeed, since $\mathcal{L}_0$ is (formally) self-adjoint we deduce $c \int_{\mathbb{R}} |\theta_0'|^2\, dx = 0$ whence $c = 0$. But then, according to Lemma 4, $w = 0$ as well.

Step 3: The implicit function theorem implies the existence of a differentiable branch

$$H \mapsto (\theta[H], c[H]), \quad \text{so that} \quad G\big((\theta[H], c[H]), H\big) = (0, 0)$$

for small enough H and $(\theta[0], c[0]) = (\theta_0, 0)$. We let

$$\psi = \frac{d\theta}{dH}\Big|_{H=0} \quad \text{and} \quad \beta = \frac{dc}{dH}\Big|_{H=0}.$$

Then

$$\begin{aligned} 0 &= \left\langle \theta_0', \frac{d}{dH}\Big|_{H=0} G((\theta[H], c[H]), H) \right\rangle \\ &= \nu\,\beta \int_{\mathbb{R}} |\theta_0'|^2\, dx + \langle \theta_0', \mathcal{L}_0\psi - \cos\theta_0 \rangle \\ &= \nu\,\beta \int_{\mathbb{R}} |\theta_0'|^2\, dx - 2, \end{aligned}$$

gives the formula for the wall mobility and the theorem follows. □

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