Numerical Treatment of Implicit Singular BVPs in ODEs

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We deal with boundary value problems for systems of ordinary differential equations with singularities. Typically, such problems have the form

$$z'(t) = \frac{M(t)}{t^\alpha} z(t) + f(t, z(t)), \quad t \in (0, 1],$$
$$B_0 z(0) + B_1 z(1) = \beta,$$

where $\alpha \geq 1$. B_0 and B_1 are constant matrices which are subject to certain restrictions for a well-posed problem which is said to feature a *singularity of the first kind* for $\alpha = 1$, while for $\alpha > 1$ the problem has a *singularity of the second kind*, also commonly referred to as *essential singularity*. We briefly recapitulate the analytical properties of the above problem with a special focus on the most general boundary conditions which guarantee its well-posedness.

To compute the numerical approximation for z we use polynomial collocation, because the method retains its high order even in case of singularities. The usual high-order superconvergence at the mesh points does not hold in general, however, the uniform superconvergence is preserved (up to logarithmic factors). We are now working on the MATLAB code `bvpsuite 2.0`, an updated version of `bvpsuite 1.1`. For higher efficiency, we provide an estimate of the global error and adaptive mesh selection. The code can be applied to mixed order ODEs in implicit formulation. We present the current work on the path-following strategy for parameter dependent ODEs. We discuss some numerical experiments to illustrate the scope and the performance of the software.

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