MOSER ITERATION APPLIED TO ELLIPTIC PROBLEMS WITH CRITICAL GROWTH ON THE BOUNDARY

We study the boundedness of weak solutions to the following problem

$$\begin{aligned} -\operatorname{div} \mathcal{A}(x,u,\nabla u) &= \mathcal{B}(x,u,\nabla u) \qquad && \text{in } \Omega,\\ \mathcal{A}(x,u,\nabla u)\cdot \nu &= \mathcal{C}(x,u) && \text{on } \partial\Omega. \end{aligned} \tag{1}$$

Here $\Omega \subset \mathbb{R}^N$, $N > 1$, is a bounded domain with a Lipschitz boundary $\partial\Omega$, while $\nu(x)$ denotes the outer unit normal of Ω at $x \in \partial\Omega$. Moreover, $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ are Carathéodory functions that have a critical growth. Using a modified version of Moser's iteration which in turn is based on the books [1] and [4] we prove that every solution $u \in W^{1,p}(\Omega)$ of (1) is actually in $L^\infty(\overline{\Omega})$.
In the last part we generalize problem (1) to the following system

$$\begin{aligned} -\operatorname{div} \mathcal{A}_1(x,u,\nabla u) &= \mathcal{B}_1(x,u,v,\nabla u,\nabla v) \qquad && \text{in } \Omega,\\ -\operatorname{div} \mathcal{A}_2(x,v,\nabla v) &= \mathcal{B}_2(x,u,v,\nabla u,\nabla v) && \text{in } \Omega,\\ \mathcal{A}_1(x,u,\nabla u)\cdot \nu &= \mathcal{C}_1(x,u,v) && \text{on } \partial\Omega,\\ \mathcal{A}_2(x,v,\nabla v)\cdot \nu &= \mathcal{C}_2(x,u,v) && \text{on } \partial\Omega, \end{aligned} \tag{2}$$

in which the functions $\mathcal{A}_i, \mathcal{B}_i$, and $\mathcal{C}_i$ still have critical growth. Through the same technique as before we are able to prove that any weak solution to (2) is in $L^\infty(\overline{\Omega}) \times L^\infty(\overline{\Omega})$.
The talk is based on the works [2,3].

References

[1] P. Drábek, A. Kufner, F. Nicolosi, "Quasilinear Elliptic Equations with Degenerations and Singularities" Walter de Gruyter & Co., Berlin, 1997. 1

[2] G. Marino and P. Winkert, *Moser iteration applied to elliptic equations with critical growth on the boundary*, Nonlinear Anal. **180** (2019), 154–169. 1

[3] G. Marino and P. Winkert, *Global a priori bounds for weak solutions of quasilinear elliptic systems with nonlinear boundary condition*, preprint. 1

[4] M. Struwe, "Variational Methods", Springer-Verlag, Berlin, 2008. 1