Hessian structures on deformed exponential families II

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A deformed exponential family is a generalization of exponential family, and it plays an important role in anomalous statistics [8], [9]. For example, the set of all q-Gaussian distributions is included in this family.

In information geometry, such a set of probability density distributions is regarded as a Riemannian manifold with dual affine connections (cf. [1], [2], [3]). In particular, it is known that a deformed exponential family naturally has two kinds of dualistic Hessian structures (cf. [4], [5], [6]). In the recent study by Mori [7], it is showed that the set of all q-Gaussian distributions admits the e- and m-connection in the sense of the standard Gaussian family. Namely, the q-Gaussian family has invariant dually flat affine connections, however these connections are not the (± 1)-connection on the q-Gaussian family.

In this talk, after reviewing the Hessian structure constructed by Mori [7], we consider relations among the three different Hessian structures on the set of q-Gaussian distributions.

Keywords: Hessian manifold, statistical manifold, deformed exponential family, information geometry, anomalous statistics

References

1. S. Amari, *Information Geometry and Its Applications*; Springer; Tokyo, 2016.
2. S. Amari and H. Nagaoka, *Method of Information Geometry*, Amer. Math. Soc., Providence, Oxford University Press, Oxford, 2000.
3. N. Ay, J. Jost, H.V. Lê and L.J. Schwachhöfer, *Information Feometry*, Springer, Cham, 2017.
4. H. Matsuzoe, *A sequence of escort distributions and generalizations of expectations on q-exponential family*, Entropy, **19**(2017), no. 1, 7
5. H. Matsuzoe and M. Henmi, *Hessian structures on deformed exponential families*, Lecture Notes in Comp. Sci., **8085**(2013), 275–282.
6. H. Matsuzoe and M. Henmi, *Hessian Structures and Divergence Functions on Deformed Exponential Families*, Geometric Theory of Information, Signals and Communication Technology, Springer, (2014), 57-80.
7. A. Mori, *Symplectic geometry of Student-t distributions*, preprint.
8. J. Naudts, *Estimators, escort probabilities, and ϕ-exponential families in statistical physics*, J. Inequal. Pure Appl. Math., **52004**, no. 4, 102.
9. J. Naudts, *Generalised Thermostatistics*, Springer-Verlag, 2011.