Relating information dynamics and stochastic thermodynamics

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The notion of information as a physically relevant quantity has long captivated the imagination of scientists working in statistical physics, information theory and theoretical computer science. To understand that such a connection exists, one need not look far, as the form of Shannon's entropy quantifying the information content of a data source and the entropy of Gibbs' are virtually identical, indeed following from almost identical axioms. One might say that the connection is 'hiding' in plain sight. However, for a long time, how this apparent coincidence might manifest in a pragmatic sense was not fully clear. Jaynes introduced the idea that the entropy of a physical system might be synonymous with our degree of belief in, or uncertainty about, that system, leading to the well known 'max-ent' arguments. But not until later, with the works of Landauer, Bennett, Szilard and others, did processes, thought of primarily in information-theoretic terms, such as erasure, measurement etc., get linked meaningfully to thermodynamics [1]. Yet these ideas were explored through rarefied thought experiments, rather than through a cohesive thermodynamic theory that explicitly incorporated information.

Much more recently, however, the role of information in thermodynamics is coming into sharper relief. To understand this one may turn to modern advances in non-equilibrium thermodynamics, where progress has been made by specifically describing the microscopic dynamical evolution of small systems, with uncertainty built in as a result of poorly described, or coarse grained, environments [2, 3, 4, 5, 6, 7]. From this framework a description of thermodynamic entropy production emerges in terms of asymmetries in the probabilistic behaviour of the system, systematically studied in frameworks such as *stochastic thermodynamics* [8, 9]. In expectation, these quantities form well known information theoretic objects such as Kullback-Leibler divergences [10, 11]. Building on this framework the field of *information thermodynamics* emerged, where the information theoretic description of processes like measurement and erasure could be introduced into these probabilistic dynamics [12, 13].

Another (and ultimately equivalent) perspective describes the implicit information processing that occurs in the dynamics of physical systems, leading to bounds on thermodynamics based on this information processing [14, 15, 16]. However, this implicit information processing, often termed *computation* or *distributed computation* has been analysed quite separately in the study of complex systems, without discussion of thermodynamics. *Information dynamics* [17] is a framework for describing this information processing and identifies *storage* and *transmission* of information as computational primitives of distributed computation. For a process x characterised by a time series $\{x_0, x_1, \ldots, x_N\}$, in the context of an (arbitrary) extraneous time series $y = \{y_0, y_1, \ldots, y_N\}$ these computational primitives are identified by the *active information storage*

$$A_X = \left\langle \frac{p(x_{n+1}|x_n, x_{n-1}, \ldots)}{p(x_{n+1})} \right\rangle \tag{1}$$

and *transfer entropy*

$$T_{Y\to X} = \left\langle \frac{p(x_{n+1}|x_n, x_{n-1}, \ldots, y_n, y_{n-1}, \ldots)}{p(x_{n+1}|x_n, x_{n-1}, \ldots)} \right\rangle, \tag{2}$$

which have enjoyed great success in understanding of systems that support distributed computation, such as cellular automata [18, 19], amongst other applications, for example in neuroscience [20]. Here we introduce and study the *physical reversibility of distributed computation* and relate it to the entropy production found in stochastic thermodynamics. We do so through the consideration of appropriate measures of the *time reversed computation*, such as the time reversed transfer entropy [21]

$$T^{\dagger}_{Y\to X} = \left\langle \frac{p^{\dagger}(x^{\dagger}_{n+1}|x^{\dagger}_{n}, x^{\dagger}_{n-1}, \ldots, y^{\dagger}_{n}, y^{\dagger}_{n-1}, \ldots)}{p^{\dagger}(x^{\dagger}_{n+1}|x^{\dagger}_{n}, x^{\dagger}_{n-1}, \ldots)} \right\rangle$$

$$= \left\langle \frac{p^{\dagger}(x_{n}|x_{n+1}, x_{n+2}, \ldots, y_{n+1}, y_{n+2}, \ldots)}{p^{\dagger}(x_{n}|x_{n+1}, x_{n+2}, \ldots)} \right\rangle, \tag{3}$$

where $p^{\dagger}$ are the appropriate time reversed dynamics [7], employed appropriately in continuous time [22], along with analogous measures for the active information storage [23]. In doing so we identify physical thermodynamic costs exclusively associated with storage and transfer of information and discuss their behaviour in a variety of scenarios encountered in stochastic thermodynamics. Further we show that the relations that arise in our framework persist in regimes that stochastic thermodynamics has not traditionally been able to describe comprising complex superstructures where individual components experience correlated noise. This opens up the possibility of studying the thermodynamics of computationally significant behaviour in complex, self organising, systems.

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