Behavioral analysis of certain nonlinear diffusion equations via information geometry

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In recent decades *information geometry* [1, 2, 3], which is geometry based on the Legendre structure, has been successfully applied mainly to the fields of statistics, information theory, machine learning theory and so on. Among them generalized thermostatistics [4, 5] is a branch of statistical physics that treats phenomena related to probability distributions in non-exponential family.

The purpose of this talk is to demonstrate that information geometric viewpoints give new insights and interesting ways of understanding into the behaviors of a nonlinear diffusion equations called *porous medium equation* (PME) and an associated Fokker-Planck equation, which is suitable to be regarded as time-evolutions of non-exponential probability distributions called the q-*Gaussian* [4, 5].

We first prepare information geometric structure for the manifold of q-Gaussian probability distributions. We next prove that the manifold is an attracting invariant manifold for solutions to the above equations. For PME we show that projections of its solutions to the manifold follow geodesic curves of information geometry and discuss its constants of motion. For the associated Fokker-Planck equation, we give a convergence rate to the manifold using a generalized divergence function (relative entropy), which can be interpreted as a difference of free energies. Finally, we would like to discuss the related recent topics. The talk is mainly based on the results reported in [6].

References

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