α-divergence appeared as rate function in large deviation estimate

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The α-divergence in α-geometry is shown to be appeared as rate function in large deviation estimate through the following mathematical structure.

	exponential function family	power function family
fundamental equation	$\frac{dy}{dx} = y$	$\frac{dy}{dx} = y^q$
fundamental function	exponential function	q-exponential function
multiplication	$\times$ (product)	$\otimes_q$ (q-product)
q	$q = 1$ (stochastic independence)	$q \neq 1$ (q in generalized dimension D_q)
law of error	Gauss' law of error (Gaussian distribution)	generalized Gauss' law of error [2] (q-Gaussian distribution including Cauchy distribution and t-distribution and others as special cases)
approximation of $n!$	Stirling's formula	q-Stirling's formula [3]
multinomial coefficient	multinomial coefficient	q-multinomial coefficient [3]
fundamental information	Shannon entropy	Tsallis entropy
axiom for entropy	Shannon-Khinchin axioms	generalized Shannon-Khinchin axioms [1]
additivity	Shannon additivity	generalized Shannon additivity
fundamental operator	differential operator	Jackson's q-differential operator
divergence	KL divergence	α-divergence [4] $\left(q = \frac{1-\alpha}{2} \right)$
fundamental distribution	binomial distribution	q-binomial distribution [4]
statistical mechanics	Boltzmann-Gibbs statistics	Tsallis statistics

References

[1] H. Suyari, *Generalization of Shannon-Khinchin axioms to nonextensive systems and the uniqueness theorem for the nonextensive entropy*, IEEE Trans. Inform. Theory, vol.50, 1783-1787 (2004).

[2] H. Suyari and M. Tsukada, *Law of Error in Tsallis Statistics*, IEEE Trans. Inform. Theory, vol.51, 753-757 (2005).

[3] H. Suyari, *Mathematical structure derived from the q-multinomial coefficient in Tsallis statistics*, Physica A, vol.368, 63-82 (2006).

[4] H. Suyari and A.M. Scarfone, α-divergence derived as the generalized rate function in a power-law system, Proceedings of the 2006 International Symposium on Information Theory and its Applications (ISITA2014), pp.130-134 (2014).

[5] J. Naudts and H. Suyari, Large deviation estimates involving deformed exponential functions, Physica A, vol.436, pp.716–728 (2015).