Lagrangian and Hamiltonian Mechanics for Probabilities on the Statistical Bundle

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We provide an Information-Geometric formulation of Classical Mechanics on the Riemannian manifold of probability distributions, which is an affine manifold endowed with a dually-flat connection. In a non-parametric formalism, we consider the full set of positive probability functions on a finite sample space, and we provide a specific expression for the tangent and cotangent spaces over the statistical manifold, in terms of a Hilbert bundle structure that we call the Statistical Bundle. In this setting, we compute velocities and accelerations of a one-dimensional statistical model using the canonical dual pair of parallel transports and define a coherent formalism for Lagrangian and Hamiltonian mechanics on the bundle. Finally, in a series of examples, we show how our formalism provides a consistent framework for accelerated natural gradient dynamics on the probability simplex, paving the way for direct applications for instance in optimization and game theory.