Unrolling versus bilevel optimization in the context of learning variational models

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Abstract

In image and signal processing and beyond, quantities of interest are often reconstructed from potentially noisy observations by means of suitably parameterized energy minimization problems. An unknown ground truth is then approximately recovered in terms of an optimization problem where the energy to be minimized depends on an observation and parameters. While purely model based appproaches rely on the assumption that a suitable energy function and associated parameters are a priori known or can be hand-crafted from domain knowledge, data driven approaches are commonly used in the case where choosing an appropriate energy is not obvious or infeasible and thus, the parameters or parts of them shall be learned from data. This gives rise to bilevel optimization problems in which the original convex problem (hereafter referred to as the lower level problem) appears as a constraint. Those bilevel problems are hard to solve. Moreover, in many cases one cannot even assume that the lower level problem has a unique solution. An approach to avoid solving the lower level problem explicitly is the so-called unrolling. Thereby, one replaces the minimizer with an iterate of an algorithm known to be capable of solving the problem. In this contribution we consider the approach to unroll a fixed number of iterations of gradient descent to the lower level problem of a tractable toy model. We compare this approach to solving the lower level problem explicitly by investigating the expressivity and computing the corresponding risks in a fixed framework.