Learning the optimal regularizer for linear inverse problems

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Machine learning has gained growing importance in the field of inverse problems, inspiring a wide variety of data-driven techniques for the stable reconstruction of unknowns from noisy measurements. A very successful paradigm, which involves the classical framework of variational regularization of inverse problems, consists of learning an optimal regularization functional from sample data.

In this talk, I will consider a linear inverse problem defined on Hilbert spaces, a setting that covers several inverse problems including denoising, deblurring, and X-ray tomography. I will tackle the problem of learning the optimal generalized Tikhonov regularizer with respect to the mean squared error of the reconstruction.

I will first characterize the optimal solution, showing that it is completely independent of the forward operator, and only depends on the data distribution. Then, I will consider the problem of learning the regularizer from a finite training set, in two different frameworks: a supervised one, based on samples of both inputs and outputs, and an unsupervised one, based only on a sample of outputs. In both cases, I will provide theoretical generalization bounds, under some weak assumptions on the data distribution, including the case of sub-Gaussian variables and processes.

The strength of the proposed approach rests on its infinite-dimensional setting, which ultimately shows that finer and finer discretizations do not make this learning problem harder. The results are validated through numerical simulations.

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