Some implications of high-dimensional geometry for neurocomputing

Věra Kůrková

Institute of Computer Science of the Czech Academy of Sciences

Computational difficulties of multidimensional tasks, called the ”curse of dimensionality”, have long been well-known. In particular, the numbers of parameters needed for computation of certain types of tasks grow exponentially with increasing dimension. On the other hand, almost deterministic behaviour of some randomized models and algorithms depending on large numbers of variables can be attributed to the ``blessing of dimensionality''. These phenomena can be explained by rather counter-intuitive properties of geometry of high-dimensional spaces. They imply concentration of values of sufficiently smooth functions of many variables around their mean values and possibilities of reduction of dimensionality of data by random projections.

We will show how properties of high-dimensional geometry can be employed to obtain some estimates of model complexities and approximate measures of sparsity of neural networks. Combining various concentration inequalities (Lévy, Hoeffding, Azuma) with estimates of sizes of dictionaries of computational units and growth of sizes of sets of network input-output functions with sizes of their domains, we will derive probabilistic lower bounds for approximation of random classifiers by neural networks. We will complement probabilistic results with some concrete constructions and discuss connections with the central paradox of coding theory and pseudo-noise sequences.