Leipzig – August 6, 2010

Max Planck Institute for Mathematics in the Sciences

“Third Conference on Information Geometry

and its Applications"

The $f - \tilde{f}$ correspondence

and

its applications in quantum information

Paolo Gibilisco

Università di Roma “Tor Vergata"

gibilisco@volterra.uniroma2.it

History

This work started in 2005 at IGAIA2 in Tokio: searching a link between monotone metrics and the uncertainty principle.

People involved:
Luo, Z. Zhang, Q. Zhang, Kosaki, Yanagi, Furuichi, Kuriyama, Gibilisco, Imparato, Isola, Hansen, Andai, Petz, Hiai, Szabo, Audenaart, Cai

Heisenberg uncertainty principle

$A, B \in \mathcal{M}_{n,sa}(\mathbb{C})$, ρ density matrix

$[A, B] := AB - BA$ $\mathbb{E}_\rho(A) := \mathrm{Tr}(\rho A)$

$$\mathrm{Var}_\rho(\mathrm{A}) := \mathbb{E}_\rho(\mathrm{A}^2) - \mathbb{E}_\rho(\mathrm{A})^2$$

Heisenberg uncertainty principle (1927) reads as

$$\mathrm{Var}_\rho(\mathrm{A}) \cdot \mathrm{Var}_\rho(\mathrm{B}) \geq \frac{1}{4}|\mathrm{Tr}(\rho[\mathrm{A}, \mathrm{B}])|^2.$$

“Classical"’ doubts

In classical probability, let (X, Y) be a r.v. on $(\Omega, \mathcal{G}, p)$.
The covariance matrix of (X, Y) is symmetric and semidefinite positive so its determinant is non-negative and therefore

$$\mathrm{Var}_p(X) \cdot \mathrm{Var}_p(Y) \geq \mathrm{Cov}_p(X, Y)^2.$$

So to have a general bound for $\mathrm{Var}_p(X) \cdot \mathrm{Var}_p(Y)$ does not seems such a “quantum" phenomenon.

Schrödinger – Robertson UP

$$\mathrm{Cov}_\rho(A,B) := \left[\mathrm{Tr}\rho\left(\frac{AB+BA}{2}\right)\right] - \mathrm{Tr}(\rho A)\cdot\mathrm{Tr}(\rho B),$$

$$\mathrm{Var}_\rho(A) := \mathrm{Cov}_\rho(A,A).$$

Schrödinger and Robertson (1929-1930) improved UP

$$\mathrm{Var}_\rho(A)\cdot\mathrm{Var}_\rho(B) - \mathrm{Cov}_\rho(A,B)^2 \geq \frac{1}{4}|\mathrm{Tr}(\rho[A,B])|^2.$$

The standard u. p. is non-trivial whenever A, B are not compatible, that is, $[A,B] \neq 0$.

Robertson general UP (1934)

Let $A_1 \ldots, A_N \in \mathcal{M}_{n,sa}(\mathbb{C})$.

$$\det\left\{\mathrm{Cov}_\rho(A_h, A_j)\right\} \geq \det\left\{-\frac{i}{2}\mathrm{Tr}(\rho[A_h, A_j])\right\},$$

for $h, j = 1, \ldots, N$

Robertson general UP (2nd version)

The matrix $\{-\frac{i}{2}\mathrm{Tr}(\rho[A_h, A_j])\}$ is anti-symmetric. Therefore, the Robertson UP reads as

$$\det\{\mathrm{Cov}_\rho(A_h, A_j)\} \geq \begin{cases} 0, & N \text{ odd} \\ \det\{-\frac{i}{2}\mathrm{Tr}(\rho[A_h, A_j])\}, & N \text{ even} \end{cases}$$

If $N = 2m + 1$, UP says (classically !) that the *generalized variance* is non-negative.

Searching an UP for N odd

- Robertson UP is based on the commutator $[A_h, A_j]$. If $N = 1$ this structure becomes meaningless !
- Intuitively, an UP for N odd should be based on a structure which involves $[\rho, A]$.
- This commutator appears in quantum dynamics.

Quantum dynamics

Let $\rho(t)$ be a curve in $\mathcal{D}_n^1$ and let $H \in M_{n,sa}$; $\rho(t)$ satisfies Schrödinger equation w.r.t. H if

$$\dot{\rho}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = i[\rho(t), H].$$

Equivalently, $\rho_H(t)$, the time evolution of $\rho = \rho_H(0)$ determined by H, evolves according to the formula

$$\rho_H(t) := e^{-itH}\rho e^{itH}.$$

Operator monotone functions

M_n = complex matrices

Definition

$f : (0, +\infty) \to R$ is operator monotone iff
$\forall A, B \in M_n$ and $\forall n = 1, 2, \ldots$

$$0 \leq A \leq B \quad \Longrightarrow \quad 0 \leq f(A) \leq f(B).$$

$\mathcal{F}_{op}$

Usually one consider o.m. functions that are:
i) normalized i. e. $f(1) = 1$;
ii) symmetric i.e. $tf(t^{-1}) = f(t)$.

$\mathcal{F}_{op}$:= family of *standard functions*.

Examples

$$\frac{1+x}{2}, \quad \sqrt{x}, \quad \frac{2x}{1+x}.$$

Operator means

Let $\mathcal{D}_n := \{A \in M_n | A > 0\}$.
A *mean* is a function $m : \mathcal{D}_n \times \mathcal{D}_n \to \mathcal{D}_n$ such that
(i) $m(A, A) = A$,
(ii) $m(A, B) = m(B, A)$,
(iii) $A < B \implies A < m(A, B) < B$,
(vi) $A < A', B < B' \implies m(A, B) < m(A', B')$,
(v) m is continuous,
(vi) $Cm(A, B)C^* \leq m(CAC^*, CBC^*)$, for every $C \in M_n$.
Property (vi) is the *transformer inequality*.

Kubo–Ando theorem

$\mathcal{M}_{op}$:= family of matrix means.

Kubo and Ando (1980) proved the following, fundamental result.

Theorem

There exists a bijection between $\mathcal{M}_{op}$ and $\mathcal{F}_{op}$ given by the formula

$$m_f(A,B) := A^{\frac{1}{2}} f(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}) A^{\frac{1}{2}}.$$

Kubo–Ando inequality

Examples of operator means

$$\frac{A+B}{2}$$

$$A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\frac{1}{2}}A^{\frac{1}{2}}$$

$$2(A^{-1}+B^{-1})^{-1}$$

Fundamental inequality

$$2(A^{-1}+B^{-1})^{-1} \le m_f(A,B) \le \frac{A+B}{2} \qquad \forall f \in \mathcal{F}_{op}$$

Fisher information

$X : \Omega \to \mathbb{R}$ real r.v. with a differentiable strictly positive density ρ

Fisher score

$$J_\rho := \frac{\rho'}{\rho} \qquad \mathbb{E}_\rho(J_\rho) = 0$$

Fisher information

$$I_X := I_\rho = \mathrm{Var}_\rho(J_\rho) = \int_{\mathbb{R}} \frac{(\rho')^2}{\rho}$$

Rao remark 1945

M statistical model (set of densities)

M can be considered as a manifols where the ρ' play the role of tangent vectors.

I_ρ is a Riemannian metric in the sense that

$$g_{\rho,F}(\rho',\rho') := \int_{\mathbb{R}} \frac{(\rho')^2}{\rho} = I_\rho$$

On the simplex

$$\mathcal{P}_n^1 := \{\rho \in R^n | \sum_i \rho_i = 1, \quad \rho_i > 0\}.$$

$$T\mathcal{P}_n^1 = \{u \in R^n | \sum_i u_i = 0\}.$$

$$g_{\rho,F}(u,v) := \sum_i \frac{u_i v_i}{\rho_i}$$

This will be the Fisher-Rao metric

Geodesic distance (Bhattacharya):

$d_F(\rho,\sigma) = 2\arccos\left(\sum_i \rho_i^{\frac{1}{2}} \sigma_i^{\frac{1}{2}}\right)$

The link with entropy

i) Hessian of Kullback-Leibler relative entropy

$$S(\rho,\sigma) := \sum_i \rho_i(\log\rho_i - \log\sigma_i);$$

$$-\frac{\partial^2}{\partial t\partial s} S(\rho + tu, \rho + sv)\Big|_{t=s=0}$$

$$= \sum_{i=1}^n \frac{u_i v_i}{\rho_i + s v_i}\Big|_{t=s=0} = \sum_{i=1}^n \frac{u_i v_i}{\rho_i} = g_{\rho,F}(u,v).$$

The link with the sphere

FI as a spherical geometry (Rao, Dawid)
ii) pull-back of the map

$$\varphi(\rho) = \varphi(\rho_1, \ldots, \rho_n) = 2(\sqrt{\rho_1}, \ldots, \sqrt{\rho_n})$$

$$\begin{aligned} g^{\varphi}_{\rho}(u, v) &= g_{\varphi(\rho)}(D_\rho\varphi(u), D_\rho\varphi(v)) \\ &= \langle M_{\rho^{-1/2}}(u), M_{\rho^{-1/2}}(v)\rangle \\ &= \sum_{i=1}^{n} \frac{u_i v_i}{\rho_i} = g_{\rho,F}(u, v). \end{aligned} \tag{1}$$

FI in the quantum case

Examples of quantum FI

Hessian of Umegaki relative entropy
$\mathrm{Tr}(\rho(\log\rho - \log\sigma))$
$\longrightarrow$ Bogoliubov-Kubo-Mori metric

Pull-back of the immersion $\rho \to 2\sqrt{\rho}$
$\longrightarrow$ Wigner-Yanase metric
(Gibilisco-Isola 2001 IDAQP)

Question: into the quantum realm do we have only a "zoo" of examples of FI?

Chentsov Theorem

Can we have a unified quantum approach?
Yes using the classical **Chentsov theorem**.
On the simplex $\mathcal{P}_n^1$ the Fisher information is the only Riemannian metric contracting under an arbitrary coarse graining T, namely for any tangent vector X at the point ρ we have

$$g^m_{T(\rho)}(TX, TX) \leq g^n_\rho(X, X)$$

Remark

Coarse graining = stochastic map = linear, positive, trace preserving.

Monotone metrics (or QFI)

$D_n^1 := \{\rho \in M_n | \mathrm{Tr}(\rho) = 1 \quad \rho > 0\} =$ faithful states

Definition

A quantum Fisher information is a Riemaniann metric on D_n^1 contracting under an arbitrary coarse graining T, namely

$$g^m_{T(\rho)}(TA, TA) \leq g^n_\rho(A, A).$$

(quantum) coarse graining = linear, (completely) positive, trace preserving map.

Petz theorem

$$L_\rho(A) := \rho A \qquad R_\rho(A) := A\rho$$

Petz theorem

There is bijection among quantum Fisher information and operator monotone functions given by the formula

$$\langle A, B\rangle_{\rho,f} := \mathrm{Tr}(A \cdot m_f(L_\rho, R_\rho)^{-1}(B)).$$

Summary

Kubo-Ando-Petz

$$f$$

$$\Updownarrow$$

$$m_f(A,B) := A^{\frac{1}{2}} f(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}) A^{\frac{1}{2}}.$$

$$\Updownarrow$$

$$\langle A, B \rangle_{\rho,f} := \mathrm{Tr}(A \cdot m_f(L_\rho, R_\rho)^{-1}(B)).$$

Decomposition of tangent space I

$$T_\rho \mathcal{D}_n^1 = \{A \in M_{n,sa} | A = A^*, \quad \mathrm{Tr}(A) = 0\}.$$

$$T_\rho \mathcal{D}_n^1 = (T_\rho \mathcal{D}_n^1)^c \oplus (T_\rho \mathcal{D}_n^1)^o$$

where

$$(T_\rho \mathcal{D}_n^1)^c := \{A \in T_\rho \mathcal{D}_n^1 | \quad [\rho, A] = 0 \quad \}$$

$(T_\rho \mathcal{D}_n^1)^o :=$ orth. compl. of $(T_\rho \mathcal{D}_n^1)^c$ resp. to H-S

Decomposition of tangent space II

For each QFI and for each $A \in (T_\rho \mathcal{D}_n^1)^c$ one has

$$\langle A, A \rangle_{\rho,f} = \text{Tr}(\rho^{-1} A^2).$$

To evaluate a QFI one has just to know what happens for $(T_\rho \mathcal{D}_n^1)^o$ whose typical element has the form

$$i[\rho, A] \qquad\qquad A \text{ s.a.}$$

Regular and non-regular QFI

$$\mathcal{F}_{op} := \{f \text{ op. mon.} | f(1) = 1, \quad t f(t^{-1}) = f(t)\}$$

$$\mathcal{F}_{op}^{r} := \{f \in \mathcal{F}_{op} | f(0) := \lim_{t\to 0} f(t) > 0\}$$

$$\mathcal{F}_{op}^{n} := \{f \in \mathcal{F}_{op} | f(0) = 0\}$$

$$\mathcal{F}_{op} = \mathcal{F}_{op}^{r} \dot\cup \mathcal{F}_{op}^{n}$$

Why is this decomposition relevant?

Riemannian metrics on the sphere

$$B_3 := \{(x,y,z) \in R^3 | x^2 + y^2 + z^2 \leq 1\}$$

$$S_2 := \overline{B}_3 \qquad \underline{0} := (0,0,0)$$

$$\mathcal{M} := B_3/(S_2 \cup \{\underline{0}\})$$

$\mathcal{M}$ is a fiber bundle over S_2 with projection

$$\pi : \mathcal{M} \to S_2$$

$$\pi(x,y,z) := \frac{1}{\sqrt{x^2+y^2+z^2}}(x,y,z)$$

Riem. metrics on the sphere II

$\mathcal{M} \ni D_n \to \rho \in S_2$ *radially* iff
$\pi(D_n) = \rho \quad \forall n$ and $\lim D_n = \rho$
Differential

$$T\pi : T\mathcal{M} \to TS_2$$

Horizontal-Vertical decomposition

$$T_D\mathcal{M} = \mathrm{Ker}(T_D\pi) \oplus H_D$$

$H_D =$ horizontal tangent vectors at the point D

Riem. metrics on the sphere III

Restriction

$$T_D\pi = H_D \to T_\rho S_2$$

is a linear isomorphism between H_D and $T_\rho S_2$ (where $\rho = T(D)$).

We may “lift" tangent vectors $u, v \in T_\rho S_2$ to $u_D, v_D \in T_D\mathcal{M}$.

Radial extensions

Suppose we have:
i) a Riemannian metric $g(\cdot, \cdot)$ on $\mathcal{M}$;
ii) a Riemannian metric $h(\cdot, \cdot)$ on S_2.

h is the *radial extension* of g if

$$D_n \to \rho \quad \text{radially}$$

$$\Downarrow$$

$$g(u_{D_n}, v_{D_n}) \to k(u, v)$$

The Bloch sphere

2×2 matrices, I identity, $\sigma_1, \sigma_2, \sigma_3$ Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

Stokes parametrization of qubits

$$\rho = \frac{1}{2}\left(I + \langle (x, y, z), (\sigma_1, \sigma_2, \sigma_3) \rangle\right)$$

$$x^2 + y^2 + z^2 \leq 1$$

Petz-Sudar theorem

Pure states $\to x^2+y^2+z^2=1$ (the sphere S_2)
Faithful mixed states $\to x^2+y^2+z^2<1$
(manifold $\mathcal{M}$ plus the origin)

Theorem

If $\langle\cdot,\cdot\rangle_{FS}$ denotes the standard Riemannian metric on the sphere S_2 (pure states), then a QFI $\langle\cdot,\cdot\rangle_{\rho,f}$ has a radial extension iff it is regular. The extension is given by

$$\frac{1}{2f(0)}\langle\cdot,\cdot\rangle_{FS}$$

General P-S theorem

Remark

True in general using the *Fubini–Study* metric on the projective space CP^n.

More delicate because for $n > 2$:
extreme boundary (pure states) $\neq$ topological boundary ($\det\rho = 0$).

The function $\tilde{f}$

$$\tilde{f}(x) := \frac{1}{2}\left[(x+1) - (x-1)^2 \frac{f(0)}{f(x)}\right]$$

Theorem

$f \in \mathcal{F}_{op}^{r}$ (f is a regular n. s. o. m. function)

$$\Downarrow$$

$\tilde{f} \in \mathcal{F}_{op}^{n}$ ($\tilde{f}$ is a non-regular n. s. o. m. function)

Moreover $f \to \tilde{f}$ is bijection.
Gibilisco-Imparato-Isola-Hansen (a different proof also from Petz-Szabo)

Regular and non-regular means

$$f \;\to\; \tilde{f}$$

$$m_f \;\to\; m_{\tilde{f}}$$

Examples

$$\frac{x+y}{2} \;\to\; \frac{2}{\frac{1}{x}+\frac{1}{y}}$$

$$\left(\frac{\sqrt{x}+\sqrt{y}}{2}\right)^2 \;\to\; \sqrt{xy}$$

Classical and quantum covariance

Classical covariance

$$\mathrm{Cov}_p(X,Y) := \mathbb{E}_p(XY) - \mathbb{E}_p(X)\mathbb{E}_p(Y).$$

Quantum covariance ($A_0 := A - \mathrm{Tr}(\rho A)\cdot I$)

$$\mathrm{Cov}_\rho(A,B) := \frac{1}{2}\mathrm{Tr}(\rho(AB+BA)) - \mathrm{Tr}(\rho A)\cdot\mathrm{Tr}(\rho B) =$$

$$= \mathrm{Tr}\left[\left(\frac{L_\rho + R_\rho}{2}\right)(A_0)B_0\right].$$

g-Covariance

To each operator monotone $g \in \mathcal{F}_{op}$ one associate the means $m_g(\cdot,\cdot)$.

Define the g-covariance as

$$\mathrm{Cov}_\rho^g(A,B) := \mathrm{Tr}(m_g(L_\rho,R_\rho)(A_0)B_0)$$

Remark: in a commuting setting all the g-covariances coincide with the classical covariance.

Fundamental formula

Theorem

If f is regular then

$$\frac{f(0)}{2}\langle i[\rho, A], i[\rho, B]\rangle_{\rho,f} = \mathrm{Cov}_\rho(A,B) - \mathrm{Cov}_\rho^{\tilde{f}}(A,B).$$

Gibilisco-Imparato-Isola

1. The dynamical UP

Let $A_1 \ldots, A_N \in \mathcal{M}_{n,sa}(\mathbb{C})$.

$$\det\{\mathrm{Cov}_\rho(A_h, A_j)\} \geq \det\left\{\frac{f(0)}{2}\langle i[\rho, A_h], i[\rho, A_j]\rangle_{\rho,f}\right\}$$

for $h, j = 1, \ldots, N$,
for all $f \in \mathcal{F}_{op}$.

Nontrivial bound also if N is odd!

$N = 1$
$\mathrm{Var}_\rho(A) \geq I_\rho^f(A) := \frac{f(0)}{2}||i[\rho, A]||_{\rho,f}^2$

Why “dynamical"?

Let $\rho > 0$ be a state and $H, K \in M_{n,sa}$. Suppose that $\rho = \rho_H(0) = \rho_K(0)$. Then, for any $f \in \mathcal{F}_{op}$, one has (taking the square root of both sides of the DUP)

$$\mathrm{Area}_\rho^{\mathrm{Cov}}(H, K) \geq \frac{f(0)}{2} \cdot \mathrm{Area}_\rho^f(\dot{\rho}_H(0), \dot{\rho}_K(0)).$$

The bound on the right side of the inequality can be seen as a measure of the difference between the dynamics generated by H and K.

2. The DUP on von Neumann alg.

While the first papers on the DUP where forced to use eigenvalues (a Kosaki remark) now one has to generalize something like the mean of the operators L_ρ and R_ρ. These are commuting operator therefore

$$m_{\tilde{f}}(L_\rho, R_\rho) = L_\rho \tilde{f}(R_\rho L_\rho^{-1}) = L_\rho \tilde{f}(\Delta_\rho)$$

So we are dealing with the modular operator and this construction makes sense in the general setting of von Neumann algebras.
Gibilisco-Isola
Petz-Szabo

3. How the bound depend on f

Define for $f \in \mathcal{F}_{op}^{r}$

$$S(f) := \det \left\{ \frac{f(0)}{2} \langle i[\rho, A_h], i[\rho, A_j] \rangle_{\rho, f} \right\}$$

$$\tilde{f}(x) := \frac{1}{2} \left[(x+1) - (x-1)^2 \frac{f(0)}{f(x)} \right].$$

Then, for any $f, g \in \mathcal{F}_{op}^{r}$

$$\tilde{f} \leq \tilde{g} \quad \Longrightarrow \quad S(f) \geq S(g).$$

The optimal bound

Let $f_{SLD}(x) := \frac{1+x}{2}$. Since for any $f \in \mathcal{F}_{op}^{r}$

$$\frac{2x}{1+x} = \tilde{f}_{SLD} \leq \tilde{f}$$

then

$$S(f_{SLD}) \geq S(f)$$

namely the optimal bound is given by Bures-Uhlmann metric.

Relation with standard UP - I

Let $f \in \mathcal{F}_{op}^{r}$. The inequality

$$\det\left\{\frac{f(0)}{2}\langle i[\rho, A_h], i[\rho, A_j]\rangle_{\rho,f}\right\} \geq \det\left\{-\frac{i}{2}\mathrm{Tr}(\rho[A_h, A_j])\right\}$$

is (in general) false for any $N = 2m$. The proof is a consequence of Hadamard inequality:

$$\det(H) \leq \prod_{j=1}^{N} h_{jj}$$

for any $H \in M_{N,sa}$.

Relation with standard UP - II

Let $f \in \mathcal{F}_{op}^{r}$. The inequality

$$\det\left\{\frac{f(0)}{2}\langle i[\rho, A_h], i[\rho, A_j]\rangle_{\rho,f}\right\} \leq \det\left\{-\frac{i}{2}\mathrm{Tr}(\rho[A_h, A_j])\right\}$$

is (in general) false for any $N = 2m$.

WYD-information

Wigner-Yanase-Dyson information

A s.a. matrix (observable in QM)
ρ density matrix (state in QM)

$$I_\rho^\beta(A) := -\frac{1}{2}\mathrm{Tr}([\rho^\beta, A][\rho^{1-\beta}, A]) \qquad \beta \in (0, \frac{1}{2}]$$

plays a role in ....

WYD II

- strong subadditivity of entropy (Lieb-Ruskai,1973)
- homogeneity of the state space of factors of type III_1 (Connes-Stormer,1978);
- measures for quantum entanglement (Chen,2005; Klyachko-Oztop-Shumovsky,2006);
- uncertainty relations ;
- quantum hypothesis testing (Calsamiglia et al., 2008)

Explanation

Indeed WYD information is a quantum Fisher information. To prove it one has to prove that the function

$$f_\beta(x) = \beta(1-\beta)\frac{(x-1)^2}{(x^\beta - 1)(x^{1-\beta} - 1)} \qquad 0 < \beta < 1,$$

is operator monotone.
Original proof: Hasegawa-Petz.

The inversion formula

Theorem

For $g \in \mathcal{F}_{op}^{n}$ and $x \neq 1$ set

$$\check{g}(x) = g''(1) \cdot \frac{(x-1)^2}{2g(x) - (x+1)}$$

Define $g(1) = 1$.

Then

$$\check{\check{f}} = f$$

Gibilisco-Hansen-Isola

4. WYD as QFI: a new proof

The function $f_\beta \in \mathcal{F}_{op}^r$ for $0 < \beta < 1$.

Proof

The function

$$g_\beta(x) = \frac{x^\beta + x^{1-\beta}}{2} \qquad 0 < \beta < 1$$

is operator monotone. It easily follows that $g_\beta \in \mathcal{F}_{op}$ and that g_β is non-regular. Since $\tilde{f}_\beta = g_\beta$ we get the desired conclusion.

3.bis A result by Kosaki

Note that if $x > 0$ is fixed the function

$$\left(0, \frac{1}{2}\right] \ni \beta \to \frac{x^\beta + x^{1-\beta}}{2} = \tilde{f}_\beta(x)$$

is decreasing and therefore

Theorem (Kosaki)
The function $\beta \to S(f_\beta)$ is increasing in $\left(0, \frac{1}{2}\right]$.

5. Skew information from entropy

$$\frac{\partial^2}{\partial t \partial s} S_{\tilde{f}}(\rho + i[\rho, A]t, \rho + i[\rho, A]s)|_{t=s=0} =$$

$$= f(0)\langle i[\rho, A], i[\rho, A]\rangle_{\rho, f}$$

where $S_F(\cdot, \cdot)$ is quantum version of Csiszar F-entropy.

Petz-Szabo

End

Some more applications for the $f - \tilde{f}$ correspondence?

Maybe at IGAIA4 ...

Thank you!