

PERTURBED DIFFERENTIAL EQUATION AND REGULARIZATION BY NOISE

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Abstract:

Paths of some stochastic processes such as fractional Brownian motion have some amazing regularizing properties. It is well known that in order to have uniqueness in differential systems such as

$$dy_t = b(y_t)dt$$

b needs to be quite regular. As soon as the last equation is perturbed by a suitable stochastic process w , the oscillations of such a process will guarantee that the following system

$$y_t = x + \int_0^t b(y_r)dr + w_t \tag{1}$$

has a unique solution for really irregular b .

After recalling some basic facts we will show that the study of the following stochastic averaging operator

$$T_t^w b(x) = \int_0^t b(x + w_r)dr$$

will allow us to solve equation (1) for b on which we have some control on the growth of the Fourier transform. This will allow us to extend such equations when b are not functions but distributions.

As an application, we will show that the stochastic transport equation driven by fractional Brownian motion with $H \in (0, 1)$

$$\partial_t u + b \cdot \nabla u + \nabla u \cdot dB_t^H = 0$$

has a unique solution when $u_0 \in L^\infty$ and b is a possibly random α -Hölder continuous function for α large enough.