A Nonlinear Schrödinger Problem in Variational Formulation

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A nonlinear Schrödinger problem perturbed by multiplicative Gaussian noise will be investigated over a finite time horizon and a bounded one-dimensional domain. The appearing power-type nonlinearity has the form $f(z) = |z|^{2\sigma}z$ for $z \in \mathbb{C}$ and $\sigma \in (0, 2)$, which has many applications in mathematical physics. Being interested in the existence and uniqueness of the variational solution, a further process will be introduced which allows to transfer the stochastic Schrödinger problem into a pathwise one. Exploiting the absence of noise and using Galerkin approximations and compact embedding results, one considers first a priori estimates, existence and uniqueness of the variational solution of the pathwise Schrödinger problem. Thereafter, it is possible to extend these results to the variational solution of the nonlinear Schrödinger problem with multiplicative noise. Finally, some generalizations and a corresponding problem of stochastic optimal control will be presented.