Regularization by noise for linear SPDEs

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Abstract

We say that a regularization by noise phenomenon occurs for a possibly ill-posed differential equation if this equation becomes well-posed under addition of noise. In this talk we show such a regularization for stochastic linear transport-like equations, namely

$$\partial_t v + b \cdot \nabla v + hv + \nabla v \circ \dot{W} = 0,$$

where $b = b(t, x)$, $h = h(t, x)$ are given deterministic, possibly irregular vector fields, W is a d-dimensional Brownian motion, $\circ$ denotes Stratonovich integration and $v = v(t, x, \omega)$ is the solution.

We show, under a certain integrability assumption on b and h (the Ladyzhenskaya-Prodi-Serrin integrability condition), that this equation admits a unique distributional solution (in a suitable integrability class), which is also Sobolev regular for regular initial condition. The result is false in general without noise.

The existence of a (Sobolev) regular solution is obtained by a priori estimates: using the renormalization property of the transport equation, we get a system of transport-like SPDEs for the powers of the derivative of the solution v; then we use parabolic estimates to bound the expectation of such powers.

The uniqueness we get is of pathwise type (actually even stronger than pathwise) and is obtained through a duality method, using the regular solution to the dual SPDE.

These results can be applied to get well-posedness for the associated SDE

$$\mathrm{d}X = b(X)\mathrm{d}t + \mathrm{d}W,$$

again for b non-smooth.

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