

Regularity of optimal sets for spectral functionals

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We consider the variational shape optimization problem of the minimization of the sum of the first k Dirichlet eigenvalues of a variable set Ω under the volume constraint $|\Omega| = 1$,

$$\min \left\{ \sum_{j=1}^k \lambda_j(\Omega) : \Omega \subset \mathbb{R}^d, |\Omega| = 1 \right\}.$$

We will prove that the free boundary of the optimal set is $C^{1,\alpha}$ regular up to a set of zero $(d-1)$ -Hausdorff measure. Since the optimal set is a solution of a free boundary problem of Alt-Caffarelli type involving vector valued functions, we will dedicate most of our attention to the study of the free boundary $\partial\{|U| > 0\}$ of the local minimizers $U : \mathbb{R}^d \rightarrow \mathbb{R}^k$ of the functional

$$H_{loc}^1(\mathbb{R}^d; \mathbb{R}^k) \ni U \mapsto \int |\nabla U|^2 dx + |\{|U| > 0\}|.$$

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