On a pseudomonotone evolution equation with multiplicative noise

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SUMMARY

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete, countably generated probability space, $T > 0$, $D \subset \mathbb{R}^d$ be a bounded Lipschitz domain, $Q_T := (0,T) \times D$ and $p > 2$. Our aim is the study of the problem

$$(P) \begin{cases} du - \operatorname{div}(|\nabla u|^{p-2}\nabla u + F(u))\, dt = H(u)\, dW & \text{in } \Omega \times Q_T \\ u = 0 & \text{on } \Omega \times (0,T) \times \partial D \\ u(0,\cdot) = u_0 \in W_0^{1,p}(D) \end{cases}$$

for a cylindrical Wiener process W in $L^2(D)$ with respect to a filtration $(\mathcal{F}_t)$ satisfying the usual assumptions and $F : \mathbb{R} \to \mathbb{R}^d$ Lipschitz continuous. We consider the case of multiplicative noise with $H : L^2(D) \to HS(L^2(D))$, $HS(L^2(D))$ being the space of Hilbert-Schmidt operators, satisfying appriopriate regularity conditions. By an implicit time discretization of (P), we obtain approximate solutions. Using the theorems of Skorokhod and Prokhorov, we are able to pass to the limit and show existence of martingale solutions. Using an argument of pathwise uniqueness, we show existence and uniqueness of strong solutions.

Keywords: pseudomonotone problem, stochastic forcing, cylindrical Wiener process, martingale solution, pathwise uniqueness, strong solution

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