PARABOLIC ANDERSON MODEL DRIVEN BY FRACTIONAL WHITE GUASSIAN NOISE

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We consider the 'parabolic Anderson model' which is the following PDE:

$$\begin{cases} \dfrac{\partial}{\partial t} u(t,x) = \kappa \mathbf{\Delta} u(t,x) + \xi(t,x)\, u(t,x) \ : \quad x \in \mathbb{Z}^d (\text{or } \mathbb{R}^d)\ , t \geq 0 \\ u(0,x) = u_o(t) \ : \ x \in \mathbb{Z}^d (\text{or } \mathbb{R}^d), \end{cases}$$

where $\kappa > 0$ is the diffusion constant, and $\mathbf{\Delta}$ denotes the Laplacian operator. We are interested in the case where for every $x \in \mathbb{Z}^d$ (or $x \in \mathbb{R}^d$)), $\xi(t,x)$ is a fractional white Guassian noise in the 'time' variable t. In other words, for every x, let $\Xi(t,x)$ be a fractional Brownian motion in t; then for any given site x, $\xi(t,x)$ is the distributional time derivative of $\Xi(t,x)$, i.e. the derivative taken with respect to t in the distributional sense. We will discuss the concept of solving this equation and the existence of a Feynman-Kac representation for the solution. We will also have a glance at some of our recent results on the asymptotic behavior of the solution when time goes to infinity ($t \to \infty$), in particular the Lyapunov exponent of the solution.