FAMILIES OF BLOWUPS OF THE REAL AFFINE PLANE AND THEIR VISUALIZATION

We present ongoing work with Peter Schenzel. Let $Z \subset \mathbb{R}^2$ be finite and let $U \subset \mathbb{R}^2$ be open bounded and star-shaped such that $Z \subset U$. Consider a pair of polynomials $\underline{f} := (f_0, f_1) \in \mathbb{R}[\mathbf{x}, \mathbf{y}]^2$ such that $\mathrm{Z}_{\overline{U}}(\underline{f}) := \{p \in \overline{U} \mid f_0(p) = f_1(p) = 0\} = Z$. The *embedded blowup* $\mathrm{Bl}_U(\underline{f})$ of U with respect to the pair $\underline{f}$ is defined as the real Zariski closure of the graph of the map

$$\varepsilon_{U,\underline{f}} : U \setminus Z \longrightarrow \mathbb{P}^1, \text{ given by } p \mapsto [\underline{f}(p)] = (f_0(p) : f_1(p)).$$

We write $\mathfrak{Bl}_U(Z)$ for the set $\{\mathrm{Bl}_U(\underline{f}) \mid \underline{f} \in \mathbb{R}[\mathbf{x}, \mathbf{y}]^2 \text{ with } Z_{\overline{U}}(\underline{f}) = Z\}$ of all these blowups. An embedded blowup $\mathrm{Bl}_U(\underline{f}) \in \mathfrak{Bl}_U(Z)$ is *regular* if the Jacobian $\partial \underline{f}$ is of rank 2 at each point $p \in Z$. We write $\mathfrak{Bl}_U^{\mathrm{reg}}(Z)$ for the set of all regular blowups $B \in \mathfrak{Bl}_U(Z)$. Two embedded blowups $B, C \in \mathfrak{Bl}_U(Z)$ are said to be *isomorphic* if there is an U-automorphism $\varphi : U \times \mathbb{P}^1 \xrightarrow{\cong} U \times \mathbb{P}^1$ such that $\varphi(B) = C$. We prove:

(1) **Deformation Theorem.** If two embedded blowups $B, C \in \mathfrak{Bl}_U(Z)$ are isomorphic, they are connected via an isotopy within the class $\mathfrak{Bl}_U(Z)$.

(2) **Classification Theorem.** Two regular embedded blowups $\mathrm{Bl}_U(\underline{f}), \mathrm{Bl}_U(\underline{g}) \in \mathfrak{Bl}_U^{\mathrm{reg}}(Z)$ are isomorphic if and only if $\mathrm{sgn}\big(\det(\partial \underline{f})(p)\big) = \mathrm{sgn}\big(\det(\partial \underline{g})(p)\big)$ for all $p \in Z$.

On use of the Computer Graphic Program developed in [1] we visualize various families $\big(B^{(t)}\big)_{0 \le t \le 1}$ of blowups $B^{(t)} \in \mathfrak{Bl}_U(Z)$ coming from an isotopy as mentioned in (1).

References

1. SCHENZEL, P. and STUSSAK, C.: *Interactive Visualizations of Blowups of the Plane.* IEEE Transactions on Visualization and Computer Graphics 19 (2013) 978-990.

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