OPTIMAL CONTROL OF A RATE-INDEPENDENT SYSTEM CONSTRAINED TO BALANCED VISCOSITY SOLUTIONS

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Abstract

We analyze an optimal control problem which is constrained to *balanced viscosity (BV) solutions* of a *rate-independent system*. This system is given in terms of a *state variable* $z : [0, T] \to \mathcal{Z}$, a time-dependent *external load* ℓ, a *stored energy functional* $\mathcal{E}$ depending on ℓ and z, and a *dissipation potential* $\mathcal{R} : \mathcal{Z} \to [0, \infty)$, which captures the dissipation due to internal friction. The evolution of z can now be described by the following differential inclusion:

$$0 \in \partial\mathcal{R}(\dot{z}(t)) + D_z\mathcal{E}(\ell(t), z(t)) \text{ for almost all } t \in [0, T], \quad z(0) = z_0,$$

where $\mathrm{D}_z\mathcal{E}$ is the Gâteaux derivative of $\mathcal{E}$ w.r.t. z and $\partial\mathcal{R} : \mathcal{Z} \rightrightarrows \mathcal{Z}^*$ denotes the convex subdifferential of $\mathcal{R}$. In rate-independent applications (e.g., dry friction, plasticity, fracture), the dissipative force does not depend on the velocity of the process, giving rise to a *positively 1-homogeneous* dissipation potential. We are further dealing with a *semi-linear*, but *non-convex* stored energy functional. In this case, we cannot expect continuous global energetic solutions, even if ℓ is smooth. Therefore, we instead consider so-called BV solutions, which can be obtained via an approximation with viscously regularized systems, see, e.g., [1].

We then show existence of solutions of an optimal control problem which is governed by the rate-independent system, where the external load ℓ serves as control variable. Since we constrain the problem to BV solutions, the focus is on the proof of compactness of the corresponding solution sets. In order to obtain the necessary a priori estimates, we introduce a reparametrization in such a way that the transformed solutions satisfy an autonomous rate-independent system on $\mathbb{R}_+$. We then obtain the essential estimates for solutions of this autonomous system by ODE-arguments and transfer them back to BV solutions.

References

[1] D. Knees, S. Thomas. *Optimal control of a rate-independent system constrained to parametrized balanced viscosity solutions.* arXiv preprint 1810.12572

[2] A. Mielke, R. Rossi, G. Savaré. *Balanced viscosity solutions to infinite-dimensional rate-independent systems.* J. Eur. Math. Soc. (JEMS), 18 (2016) pp. 2107-2165.

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