**Title.** An approximation to the ratio of prior normalizing constants for the $G$-Wishart

**Abstract.**
Graphical Gaussian model selection in a Bayesian framework is done through the pairwise comparison of neighbouring models differing by one edge, using the Bayes factor. When the prior distribution for the precision matrix is the $G$-Wishart, computing the Bayes factor involves computing the ratio of prior normalizing constants of the $G$-Wishart

$$I_{G^{-e}}(\delta, I)/I_G(\delta, I)$$

where the graphs $G$ and $G^{-e}$ underlying the two models being compared differ by the edge $e$ and, $\delta$ and the identity matrix $I$ are respectively the shape and scale parameters of the $G$-Wishart. This computation has been a stumbling block in model selection in moderate to high dimensional problems since it can only be computed through Monte Carlo methods.

In this work, we give an explicit analytic approximation to the ratio of prior normalizing constants. This approximation is expressed in terms of the ratio of two Gamma functions evaluated at $(\delta + d)/2$ and $(\delta + d + 1)/2$ where $d$ is the number of paths of length 2 between the two end points of $e$ in $G$. Though we can only give its accuracy when certain assumptions are satisfied, numerical experiments show that it holds quite widely. It allows us to avoid Monte Carlo methods, is computationally inexpensive and is scalable to high-dimensions. We also note that this approximation is exact for some particular graphs.