Intersections with random geodesics in high dimensions

Uri Grupel

Given a large subset of the sphere $A \subseteq S^{n-1}$ does the ratio of lengths between a random geodesic Γ and the intersection $\Gamma \cap A$ represent the size of A or does it tend to a zero-one law (as the dimension grows)? We will show that for any large set A we have a distribution that is not concentrated around neither zero nor one.

In contrast to the case of the sphere, for any convex body in high dimensions, we can find a subset, of half the volume, such that the ratio of lengths between the intersection of the random geodesic with the convex body and the intersection of the subset with the random geodesic will be close to zero-one law.

The analysis of the two cases has different flavors. For the sphere we analyze the singular values of the Radon transform, in order to bound the variance of the length of the random intersection. For convex bodies, we use concentration of measure phenomena.

The results on the sphere can be generalized to the discrete torus or to intersection on the sphere with higher dimensional subspaces.