Regularity Theory for Viscosity Solutions of Fully Nonlinear Parabolic Equations

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Abstract.

We introduce the notion of *viscosity solutions* of second order fully nonlinear parabolic equations. This type of solution is apriori assumed to be merely continuous and satisfies the equation only in a weak sense which, roughly speaking, says that smooth functions that touches the solution from below/above at a point must be super/sub-solutions of the equation in the classical sense. Therefore it is reasonable to focus on whether such a solution can be proven to be more regular satisfying delicate Hölder estimates.

We start by giving a brief overview of the theory that have been developed the last decades concerning interior and boundary regularity properties of viscosity solutions of elliptic/parabolic fully nonlinear equations. Then we concentrate on oblique derivative problems of the form

$$\begin{cases} F(D^2u) - u_t = f, & \text{in } \ B_1^+ \times (-1,0] \\ \beta \cdot Du = g, & \text{on } \ (B_1^+ \cap \{x_n = 0\}) \times (-1,0] \end{cases} \tag{0.1}$$

where $B_1^+ = B_1 \cap \{x_n > 0\} \subset \mathbb{R}^n$ is the unit half ball. We discuss how, under suitable assumptions on the given data, we can obtain Hölder estimates for the first and second derivatives of viscosity solutions of (0.1) up to the flat boundary. We will close by presenting very briefly a free boundary problem of obstacle type where the regularity properties of (0.1) can be applied.

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