Complete quadrics and algebraic statistics.

Tim Seynnaeve

May 25, 2020

Let L be a general d-dimensional linear space of symmetric $n \times n$ matrices over $\mathbb{C}$. What is the degree $\phi(n,d)$ of the variety L^{-1} obtained by inverting all matrices in L? This is an interesting geometric question in its own right, but is also interesting from the point of view of algebraic statistics: $\phi(n,d)$ is the maximum likelihood degree of the generic linear concentration model. In 2010, Sturmfels and Uhler computed $\phi(n,d)$ for $d \leq 5$, and conjectured that for any fixed d, $\phi(n,d)$ is a polynomial of degree $d-1$. Using Schubert calculus and intersection theory on the space of complete quadrics, we obtain a formula for $\phi(n,d)$ in terms of the coefficients arising in the Schur expansion of certain symmetric polynomials. Our formula allows us to prove the Sturmfels-Uhler polynomiality conjecture, and to compute the polynomials $\phi(n,d)$ for $d \leq 47$. This talk is based on joint work in progress with Laurent Manivel, Mateusz Michałek, Leonid Monin, Martin Vodicka, Andrzej Weber, and Jarosław Wiśniewski.