New Dependencies of Hierarchies in Polynomial Optimization

In science and engineering we regularly face hard, nonlinear polynomial optimization problems. Solving these problems is essentially equivalent to certifying nonnegativity of multivariate, real polynomials – a key problem in real algebraic geometry since the 19th century. In the first part of this talk, we discuss how to tackle such problems both from the perspective of algebra and optimization.

In the second part, we compare four key hierarchies for solving Constrained Polynomial Optimization Problems (CPOP): *Sum of Squares (SOS)*, *Sum of Diagonally Dominant (SDSOS)*, *Sum of Nonnegative Circuits (SONC)*, and the *Sherali Adams (SA)* hierarchies. We prove a collection of dependencies among these hierarchies both for general CPOPs and for optimization problems on the Boolean Hypercube. Key results include for the general case that the SONC and the SOS hierarchy are polynomially incomparable, while SDSOS is contained in SONC. On the Boolean Hypercube, we show that Schmüdgen-like versions SDSOS*, SONC*, and SA* of the hierarchies are polynomially equivalent. A direct consequence is the non-existence of a Putinar-like Positivstellensatz for SDSOS. This part is joint work with Adam Kurpisz.