

ABSTRACT

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We first present partial regularity theory for the nonstationary Navier-Stokes equations in space dimensions 4. In particular, we show that there exist global-in-time partially regular weak solutions to the nonstationary Navier-Stokes equations whose singular sets have finite 2-dimensional parabolic Hausdorff measure in space dimension $n = 4$.

Secondly, in space dimensions $n \geq 3$, motivated by studying possible singularities of the Navier-Stokes equations, we prove singularity formation in a linear toy model of the axi-symmetric Navier-Stokes equations. As a by-product, we construct time-independent supercritical drifts in $L^{n-\lambda}(\mathbb{R}^n)$ with arbitrarily small $\lambda > 0$ such that the Harnack inequality and the Hölder continuity fail in both elliptic and parabolic equations associated to these drifts.