Energy release rate for cracks in finite–strain elasticity

Dorothee Knees
Weierstrass Institute for Applied Analysis and Stochastics
Berlin, Germany
kneesde@wias-berlin.de

Griffith's fracture criterion is an energetic criterion, which describes in a quasistatic setting whether or not a preexisting crack in an elastic body is stationary for given external forces. It is usually formulated in terms of the energy release rate (ERR), which is the derivative of the potential deformation energy of the body with respect to a virtual crack extension. In the simplest case, the fracture criterion has the following form: if the ERR is smaller than a specific constant, then the crack is stationary, otherwise it will grow. Formulas for the calculation of the ERR are needed.

One can find several formulas for the energy release rate in the literature on fracture mechanics (Griffith–formula, J–integral, formulas based on stress intensity factors). In the linear elastic case, these formulas are proved rigorously using in an essential way regularity results for weak solutions of linear elliptic systems. In the case of finite–strain elasticity, similar formulas are derived under additional assumptions on the smoothness or the asymptotic behavior of the stress and deformation fields near the crack tip. But in general, such regularity results are not proved yet.

In this talk we consider geometrically nonlinear elastic models with polyconvex energy densities W which may take the value $+\infty$ in case of nonpositive deformation gradients. The main assumption besides polyconvexity are growth conditions on the derivatives of W:

$$|A^\top DW(A)| \leq \kappa(W(A) + 1) \quad \text{for every } A \in \mathbf{M}_+^{d\times d}, \tag{1}$$

and similar for the second derivative. These assumptions allow us to derive rigorously the above mentioned formulas for the ERR without further assumptions on the smoothness of the deformation fields. The main tool is a weak convergence theorem for Eshelby tensors $W(\nabla u)\mathbf{1} - \nabla u^\top DW(\nabla u)$, which we prove on the basis of (1).

References

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