Carnot-Carathéodory inf-convolution and limiting behavior of solutions of subelliptic heat kernels.

Abstract: It is well-known that the limiting behavior, as $\varepsilon \to 0^+$, of the solutions of

$$\begin{cases} w_t^\varepsilon - \varepsilon \Delta w^\varepsilon = 0, & x \in \mathbb{R}^n,\ t > 0, \\ w^\varepsilon(0,x) = \mathrm{e}^{-\frac{g(x)}{2\varepsilon}}, & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

is described by the Hamilton-Jacobi-Cauchy problem

$$\begin{cases} u_t + \frac{1}{2}|Du|^2 = 0, & x \in \mathbb{R}^n,\ t > 0, \\ u(0,x) = g(x), & x \in \mathbb{R}^n, \end{cases} \tag{2}$$

More precisely, if $g : \mathbb{R}^n \to \mathbb{R}$ is a bounded and continuous function, the logarithmic transform of w^ε, i.e. $u^\varepsilon = -2\varepsilon \log w^\varepsilon$, converges, locally uniformly, as $\varepsilon \to 0^+$, to the unique viscosity solution u of (2). One way of proving this is to use both the representation of w^ε as the integral convolution and the Hopf-Lax representation of the viscosity solution of (2) as the (euclidean) inf-convolution

$$g_t(x) = \inf_{y \in \mathbb{R}^n} \left[g(y) + \frac{|x-y|^2}{2t} \right],$$

and to apply the Large Deviation Principle in order to establish the validity of the limiting relation

$$\lim_{\varepsilon \to 0^+} -2\varepsilon \log w^\varepsilon = u.$$

The aim of this talk is to generalize the procedure described above in order to analyze the limiting behavior of some subelliptic diffusion equations in term of the Carnot-Carathéodory inf-convolutions, given a new proof covering as the Hörmander case as the already known euclidean case, using measure theory methods instead of the probability techniques.

At the end of the talk I'm going to give a brief sketch of my past and present researches, too.