

Regularity results and new perspectives for degenerate Kolmogorov equations

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We are concerned with the regularity theory of strongly degenerate Kolmogorov equations and we also study a relativistic generalization of such equations. We divide this presentation into three parts.

In the first part, we present some results which lie in the classical regularity theory of Kolmogorov-type operators with regular coefficients. In particular, we here discuss some Schauder estimates for classical solutions to Kolmogorov equations in non-divergence form with Dini-continuous coefficients contained in [4]. Furthermore, we show new pointwise regularity results and a Taylor-type expansion up to second order with estimate of the rest in L^p norm (see [3]).

The second part focuses on the weak regularity theory of degenerate Kolmogorov equations with discontinuous coefficients, which is nowadays the main focus of the research community. More precisely, we present a Harnack inequality and the Hölder continuity for weak solutions to the Kolmogorov equation with measurable coefficients, integrable lower order terms and nonzero source term, following the work [1].

Finally, in the last part of this presentation, we address a possible generalization of the kinetic Kolmogorov-Fokker-Planck equation, which is in accordance with the theory of special relativity and was studied in [2]. In particular, we explain why the operator proposed is the suitable relativistic generalization of the Fokker-Planck operator and we describe it as a Hörmander operator which is invariant with respect to Lorentz transformations.

References

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