# An $\varepsilon$-regularity theorem for Griffith minimizers in $\mathbf{R}^N$ under a separating condition

Camille Labourie (University of FAU Erlangen-Nürnberg),
joint work with Antoine Lemenant (University of Lorraine)

The Griffith energy is a functional introduced by Francfort and Marigo to model the equilibrium state of a fracture in a brittle fracture in linear elasticity. The formula $\mathcal{G}$ given below is a bit simplified.

Let $\Omega$ be a bounded open set of $\mathbf{R}^N$, which stands for the reference configuration (without crack) of a homogeneous, isotropic, brittle body. We apply a deformation (constant over time) at the boundary of the solid and we assume that the solid only undergoes infinitesimal transformations. Francfort and Marigo formulate the problem as the minimization of

$$\mathcal{G}(u, K) := \int_{\Omega \setminus K} |e(u)|^2 \, dx + \mathcal{H}^{N-1}(K),$$

among pairs $(u, K)$ such that $K$ is a $(N-1)$-dimensional subset of $\Omega$, $u\colon \Omega \setminus K \to \mathbf{R}^N$ is a smooth function which satisfies a Dirichlet condition at the boundary $\partial\Omega$, and the matrix $e(u) := (Du + Du^T)/2$ is the symmetric part of the gradient of $u$. We interpret $u$ as a displacement field (the deformation of the solid is $x \mapsto x + u(x)$), the matrix $e(u)$ as its linear strain tensor (it describes the local deformation of $\Omega$), $K$ as a crack. The energy $\mathcal{G}(u, K)$ puts in competition the elastic energy stored outside of the crack and the surface energy required to create the crack.

This formulation does not say anything about the topology and the regularity of the crack a priori and as of yet, we know very few regularity results about minimizers. Although it looks like the Mumford-Shah energy, the Griffith energy provides in fact a lot of suprising new difficulties as one works with the symmetrized gradient instead of the full gradient. The goal of the talk is to present a new $\varepsilon$-regularity theorem for Griffith minimizers in all dimensions $N$. This is a joint work with Antoine Lemenant.