An abstract perturbation principle for semigroups and weak well-posedness of some critical singular SPDEs and other singular systems

Building on the approach from [GP20] we show weak-wellposedness for a stochastic surface quasi-geostrophic equation and the critical 1-D fractional stochastic Burgers equation, both of which are scaling-critical singular stochastic PDEs, by constructing unique Markovian L2-semigroups with respect to a nice reference measure, the law of white noise. The approach also yields uniqueness of energy solutions to these equations.

The main technical tool is a perturbation-type result for semigroups formulated in an abstract Hilbert space setting which allows the perturbation to be singular in a certain sense if certain structural properties are fulfilled.

If time permits, we outline applications to other singular systems, like SDEs with singular drift, infinite-dimensional SDEs with singular interaction terms or a partially alternative approach to Cannizzaro-Erhard-Toninelli-type scaling results [CET21].

Joint work with Nicolas Perkowski.

[GP20] M. Gubinelli, N. Perkowski. The infinitesimal generator of the Stochastic Burgers equation. Probability Theory and Related Fields (178), 1067–1124, 2020.

[CET21] G. Cannizzaro, D. Erhard, F. Toninelli. Weak coupling limit of the Anisotropic KPZ equation, 2021.