What to do with **Big Old Data**?

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A key problem in the development of an open source computer algebra system is the design of mechanisms and formats for dealing with *big old data*. Here, "big old data" refers to the output of a mathematical computation that is much larger than a few lines, and is intended for storage in a repository, or for further processing by a different program. How to address this problem?

In this afternoon activity, we revisit data from prior research in algebra, geometry or combinatorics. We selected topics that are of considerable current interest. It is also meant as a fun challenge to members and friends of the OSCAR teams on *how to transfer, format and incorporate existing data*.

1. Load the ancillary files of the `arXiv` posting *Toward a Salmon Conjecture*, by Dan Bates and Luke Oeding, into `Singular` and `GAP`. How many quintics and sextics are there, and how many terms do they have? Do these polynomials really vanish on all $4 \times 4 \times 4$-tensors of rank 4?

2. Lara Bossinger *et al.* catalogued toric degenerations of flag varieties at `www.mi.uni-koeln.de/~lbossing/tropflag/tropflag5.html`. For each of her 319 rays, represent the corresponding toric ideal in `Singular` and represent the corresponding Newton-Okounkov body in `Polymake`.

3. The *binary graph model of G15* is not the same as the *binary graphical model of G15*. Find these models in Thomas Kahle's database `markov-bases.de`. Show that their toric ideals and polytopes differ.

4. Find one representative vector for each (type of) maximal cone in the tropical Grassmannian $\mathrm{Gr}(3,7)$. Compute the corresponding initial ideal and decide whether it is toric. Draw that tropical plane in $\mathbb{TP}^6$.

5. What kind of mathematical questions can be answered by using the database `www.findstat.org`? What is Christian Stump's design philosophy? Why does he like SAGE? What does he say about OSCAR?

6. Smooth reflexive lattice polytopes are cataloged in the data base `https://polymake.org/polytopes/paffenholz/www/fano.html`. Download all 3-dimensional instances and compute their toric varieties. Try the toric packages in `GAP` and `Macaulay2`. Can you find the coordinate rings of these toric varieties in `http://grdb.co.uk/`?

7. *Invariant Theory* was a hot area in the 19th century. Classical tables of invariants are still of important for today's research. How will we access them in OSCAR? Compute explicit polynomial expressions for the invariants of degree $3, 6, 9, 12$ for ternary quartics displayed by Andries Brouwer at `www.win.tue.nl/~aeb/math/ternary_quartic.html`.

8. Pick one of the 15 examples at `www.normaliz.uni-osnabrueck.de/documentation/interesting-and-challenging-examples-for-normaliz/`. Clicking on the name creates a folder with one or more ".out" files. Explore this data further. Use one of the programs in the OSCAR project.

9. Download the matroids of rank 4 available at `https://polymake.org/doku.php/data`. Compute their toric ideals and check whether *White's Conjecture* holds, i.e. whether these ideals are generated by quadrics.

10. In the paper *On permutation polytopes*, Barbara Baumeister *et al.* classify all permutations polytopes up to dimension 4 and their permutation groups. Look at the codes and data posted at `https://polymake.org/polytopes/paffenholz/www/permutations.html`. Can you construct the permutation polytopes and their groups in dimension 5?

11. A team of number theorists, including Stephan Ehlen from Köln, maintains the comprehensive database `www.lmfdb.org`. What is an L-function? ... a modular form? ... a Shimura variety? Why are these objects important? Can any of this data be transported into `Singular`?

12. Melody Chan posted the moduli spaces of tropical genus g curves for $g \leq 5$ at `www.math.brown.edu/~mtchan/torelli/`. She also offers `Mathematica` code. Can you run it and produce output that `Polymake` and `GAP` can read? How hard is it to compute this space for $g = 6$?