GETTING STARTED WITH $\mathbb{Q}_p$

We list some references here for participants unfamiliar with $\mathbb{Q}_p$ with a quick introduction. A familiarity with the following topics would be helpful for this conference:

(1) The structure of $\mathbb{Q}_p$ and $\mathbb{Z}_p$ as metric measure spaces.
(2) Hensel's lemma.
(3) The basics of $\mathbb{Q}_p$-Banach spaces.
(4) Generalizations of the above to general non-archimedean fields, non-archimedean local fields.

These topics are covered excellently in a number of different sources.

(1) Those new to the non-archimedean world may find the lecture series by Liang Xiao interesting. Video recordings of these lectures are available publicly at

- Lecture 1: https://www.youtube.com/watch?v=3uEp4Eao5mE&list=PLYpVTXjEi1ocTbHYF DYulYxJ1Sey_Jqob&index=2

  Notes: https://ctnt-summer.math.uconn.edu/wp-content/uploads/sites/1632/2020/ 06/CTNT-2020-Xiao-Lecture-1.pdf

- Lecture 2: https://www.youtube.com/watch?v=Mq1Ap4KfBR8&list=PLYpVTXjEi1ocTbHYF DYulYxJ1Sey_Jqob&index=3

  Notes: https://ctnt-summer.math.uconn.edu/wp-content/uploads/sites/1632/2020/ 06/CTNT-2020-Xiao-Lecture-2.pdf

- Lecture 3: https://www.youtube.com/watch?v=-gJP6Z14mF8&list=PLYpVTXjEi1ocTbHYF DYulYxJ1Sey_Jqob&index=4

(2) A nice exposition on how to visualize the tree structure of $\mathbb{Q}_p$ is given by Holly in [5].

(3) For a more thorough introduction to the topic, we recommend the following books.
- *p*-adic Numbers: An Introduction, Fernando Q. Gouvêa [4]
  Link: https://www.springer.com/gp/book/9783030472948
- *p*-adic Analysis Compared with Real, Svetlana Katok [6]
  Link: https://bookstore.ams.org/stml-37
- A Course in *p*-adic Analysis, Alain M. Robert [9]
  Link: https://www.springer.com/gp/book/9780387986692
- *p*-adic Numbers, *p*-adic Analysis, and Zeta-Functions, Neal Koblitz [7]
  Link: https://www.springer.com/gp/book/9780387960173

Computing with p-adic fields. Folks who like computer algebra systems will be pleased to know that p-adic rings are implemented in `Magma`, `OSCAR`, and `Sage`, as well as other computer algebra systems. The following references discuss the subtleties of finite precision implementation:

- Computations with p-adic numbers, Caruso [2]
- Sage documentation for p-adic precision [10]

References for the mini courses. Below is a list references that the mini-course instructors will be following.

- (Fieker) User manual for OSCAR [8]
- (Schneider) p-adic Lie Groups [11]
- (Ngoc) Local Gaussians [3, 12, 13]

Fieker will be using the `OSCAR` computer algebra system. A full version can be downloaded at `https://oscar.computeralgebra.de/install/`

T-shirts. The society of p-addicts has T-shirts available [1].

References

[1] *The society of p-addicts.* `http://math.mit.edu/~roed/#other`.

[2] Xavier Caruso, *Computations with p-adic numbers*, Les cours du CIRM **5** (2017), no. 1, 1–75.

[3] Steven N. Evans, *Local fields, Gaussian measures, and Brownian motions*, Topics in probability and Lie groups: boundary theory, 2001, pp. 11–50. MR1832433

[4] Fernando Q. Gouvêa, *p-adic numbers*, Universitext, Springer, Cham, [2020] ©2020. An introduction, Third edition of [1251959]. MR4175370

[5] Jan E. Holly, *Pictures of ultrametric spaces, the p-adic numbers, and valued fields*, The American Mathematical Monthly **108** (2001), no. 8, 721–728, available at `https://doi.org/10.1080/00029890.2001.11919803`.

[6] Svetlana Katok, *p-adic analysis compared with real*, Student Mathematical Library, vol. 37, American Mathematical Society, Providence, RI; Mathematics Advanced Study Semesters, University Park, PA, 2007. MR2298943

[7] Neal Koblitz, *p-adic numbers, p-adic analysis, and zeta-functions*, Second, Graduate Texts in Mathematics, vol. 58, Springer-Verlag, New York, 1984. MR754003

[8] OSCAR developers, *Oscar user manual.* `https://oscar-system.github.io/Oscar.jl/dev/`.

[9] Alain M Robert, *A course in p-adic analysis*, Vol. 198, Springer Science & Business Media, 2013.

[10] SAGE developers, *Sage user manual for the padics.* `https://doc.sagemath.org/pdf/en/reference/padics/padics.pdf`.

[11] Peter Schneider, *p-adic Lie groups*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 344, Springer, Heidelberg, 2011. MR2810332

[12] Ngoc Tran and Yassine El Maazouz, *Statistics of gaussians on local fields and their tropicalizations*, 2019. `https://arxiv.org/abs/1909.00559`.

[13] Ngoc Mai Tran, *Tropical Gaussians: a brief survey*, Algebr. Stat. **11** (2020), no. 2, 155–168. MR4229208