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Bosonization of the two-dimensional electron gas in the lowest Landau level

by

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Abstract

We develop a bosonization scheme for the collective dynamics of a spinless two-dimensional electron gas (2DEG) in the lowest Landau level. The system is treated as a continuous elastic medium, and quantum commutation relations are imposed between orthogonal components of the elastic displacement field. This theory provides a unified description of bulk and edge excitations of compressible and incompressible phases, and explains the results of recent tunneling experiments at the edge of the 2DEG.

73.40.Hm, 73.20.Mf, 71.10.Pm

Bosonization [1] is a powerful mathematical technique to describe the long-wavelength dynamics of correlated electronic systems, in which the motion of the electrons is highly collectivized. This technique has proved very useful in studies of the behavior of interacting electrons in one-dimensional metals. Higher-dimensional generalizations have also been proposed [2]. More recently, it has been realized that the *edges* of a two-dimensional electron gas (2DEG) in a strong magnetic field behave as a "chiral Luttinger liquid" [3] and are therefore amenable to bosonization [3,4]. A collective description of the *uniform* incompressible 2DEG in a strong magnetic field such that all the electrons reside within the lowest Landau level (LLL) [5] was pioneered by Girvin *et al.* [6], and an effective bosonic theory was developed shortly afterwards [7]. The similarity between the local structure of the 2DEG in the LLL and that of a classical Wigner crystal was exploited by Johansson and Kinaret [8] to set up an "independent boson model" for the electronic spectral function; an equivalent model was derived by Haussmann [9] from diagrammatic theory. Finally, several authors have used bosonization schemes to describe the dynamics of a smooth compressible edge in a quantum Hall liquid [10–12], or of the electron gas in a weak magnetic field [13].

In this Letter we formulate a bosonic theory of the linear dynamics of a *general* uniform or nonuniform distribution of electrons in the LLL. Unlike previous theories [7–13], the present one treats bulk and edge excitations of compressible and incompressible states on an equal footing. One of our main results is a simple explanation for recent experimental observations of one-electron tunneling characteristics in the edge [14] of the 2DEG.

The basic idea of our approach is to treat the electrons in the LLL as a continuous elastic medium characterized by an equilibrium density $\rho_0(\vec{r})$, and by a local displacement field $\vec{u}(\vec{r},t)$, such that the time-dependent density is (to first order in $\vec{u}$) $\rho(\vec{r},t) = \rho_0(\vec{r}) - \vec{\nabla}\cdot[\rho_0(\vec{r})\vec{u}(\vec{r},t)] \equiv \rho_0(\vec{r}) + \delta\rho(\vec{r},t)$. We ignore the spin in this paper. Under the assumption that both the equilibrium density and the displacement are *slowly varying* on the scale of the magnetic length $l = (\hbar c/eB)^{1/2}$ (where B is the magnetic field) the effective long-wavelength Hamiltonian is

$$H = \frac{1}{2}\int d\vec{r}\int d\vec{r'}\delta\rho(\vec{r})\left(\frac{e^2}{|\vec{r}-\vec{r'}|} - \tilde{\chi}^{-1}(\vec{r},\vec{r'})\right)\delta\rho(\vec{r'}) + \int d\vec{r}\mu\left[\rho_0(\vec{r})\right]\sum_{\alpha,\beta}\left[s_{\alpha\beta}(\vec{r}) - \frac{1}{2}\delta_{\alpha\beta}\vec{\nabla}\cdot\vec{u}(\vec{r})\right]^2 , \quad (1)$$

where $\tilde{\chi}^{-1}(\vec{r},\vec{r'})$ is the inverse of the *proper* static density-density response function [15], μ is the shear modulus (discussed below), $s_{\alpha\beta} \equiv [\partial u_\alpha(\vec{r})/\partial r_\beta + \partial u_\beta(\vec{r})/\partial r_\alpha]/2$ is the strain tensor, and α, β are cartesian indices. Eq. (1) generalizes the Hamiltonian of classical elasticity theory [16] to take into account the long range of the Coulomb interaction, and of $\tilde{\chi}(\vec{r},\vec{r'})$ in the incompressible states of the 2DEG [6].

The second term of Eq. (1) is the crucial new feature of our theory. Ordinarily, the shear modulus of a liquid is taken to be zero, but this is only true for shear deformations which occur slowly compared to the thermalization time τ of the system. At frequencies higher than $1/\tau$ the correlated electron liquid behaves as a Wigner glass, and therefore exhibits a nonvanishing shear modulus. The essential assumption is that, in this system, the relaxation frequency scale $1/\tau$ is much smaller than the characteristic frequency of the collective dynamics.

The algebra of the displacement operators u_x, u_y, in the LLL, is deduced from the canonical quantization condition for the hydrodynamical momentum and displacement fields $[p_\alpha(\vec{r}), u_\beta(\vec{r'})] = -i\delta(\vec{r}-\vec{r'})\delta_{\alpha\beta}$, by projecting out the higher Landau levels, i.e. averaging over the fast cyclotron motion. This leads to

$$[u_\alpha(\vec{r}), u_\beta(\vec{r'})] = -i\epsilon_{\alpha\beta}\delta(\vec{r}-\vec{r'})\, l^2/\rho_0(\vec{r}) , \quad (2)$$

where $\epsilon_{\alpha\beta}$ is the two-dimensional Levi-Civita tensor. This is consistent with the commutation relation between projected density fluctuations [6] *in the long wavelength limit*, namely $[\rho(\vec{r}), \rho(\vec{r'})] = -i\vec{\nabla}_r \times \vec{\nabla}_{r'}\rho_0(\vec{r})l^2\delta(\vec{r}-\vec{r'})$.

Bosonization of (1) is accomplished by

$$u_\alpha(\vec{r}) = \frac{l}{\rho_0^{1/2}(\vec{r})}\sum_{n>0}\left[b_n g^*_{n\alpha}(\vec{r}) + b_n^\dagger g_{n\alpha}(\vec{r})\right] , \quad (3)$$

where b_n and $b_n^\dagger$ are boson operators, satisfying the usual algebra $[b_n, b_{n'}^\dagger] = \delta_{nn'}$ etc.. The functions $g_{n\alpha}(\vec{r})$ ($g_{n\alpha}^*(\vec{r})$), with $n > 0$, are the positive (negative) frequency solutions of the eigenvalue problem

$$\int H_{\alpha\beta}(\vec{r}, \vec{r'}) g_{n\beta}(\vec{r'}) d\vec{r'} = \omega_n g_{n\alpha}(\vec{r}) \tag{4}$$

which is equivalent to the classical equation of motion. (We adopt the convention of summing over repeated cartesian indices.) The operator $H_{\alpha\beta}$ is given by

$$\begin{aligned} H_{\alpha\beta}(\vec{r}, \vec{r'}) = & -il^2 \rho_0^{1/2}(\vec{r}) \epsilon_{\alpha\gamma} \left[\partial_\gamma \partial'_\beta \chi^{-1}(\vec{r}, \vec{r'})\right] \rho_0^{1/2}(\vec{r'}) \\ & + i \frac{l^2}{\rho_0(\vec{r})} \epsilon_{\alpha\gamma} \left[\partial_\beta \partial'_\gamma - \partial_\gamma \partial'_\beta + \delta_{\gamma\beta} \partial_\eta \partial'_\eta\right] \left[\delta(\vec{r} - \vec{r'}) \mu\right], \end{aligned} \tag{5}$$

where $\chi^{-1}(\vec{r}, \vec{r'}) \equiv \tilde{\chi}^{-1}(\vec{r}, \vec{r'}) - e^2/|\vec{r} - \vec{r'}|$, and ∂_α and ∂'_β denote derivatives with respect to r_α and r'_β respectively. It is hermitian with respect to the scalar product $(f, g) \equiv \int f_\alpha^*(\vec{r}) i\epsilon_{\alpha\beta} g_\beta(\vec{r}) d\vec{r}$, and its eigenvalues are therefore real. For each eigenfunction g_n with positive eigenvalue ω_n, there is a complex conjugate one $g_n^* \equiv g_{-n}$ with negative eigenvalue $-\omega_n$. These eigenfunctions can be chosen to satisfy orthonormality relations $(g_n, g_m) = sgn(n)\delta_{nm}$ (n and m can now have either sign) [17]. The "completeness relation" has the form

$$\sum_{n>0} [g_{n\alpha}(\vec{r}) g_{n\beta}^*(\vec{r'}) - g_{n\alpha}^*(\vec{r}) g_{n\beta}(\vec{r'})] = i\epsilon_{\alpha\beta} \delta(\vec{r} - \vec{r'}). \tag{6}$$

These equations guarantee that the commutation relation (2) is satisfied. Then, substitution of Eq. (3) into Eq. (1) yields the Hamiltonian in the desired form $H = \sum_{n>0} \left(b_n^\dagger b_n + 1/2\right) \omega_n$.

Our next step is to construct the electron tunneling operator $\psi(\vec{R})$ associated with the LLL coherent state orbital centered at $\vec{R}$. This operator must satisfy the commutation relation

$$[\psi(\vec{R}), \rho(\vec{r})] = \psi(\vec{R}) \exp[-|\vec{r} - \vec{R}|^2/2l^2]/2\pi l^2 \,, \tag{7}$$

i.e., destroy a gaussian density near $\vec{R}$. Eq. (7) alone does not uniquely determine $\psi(\vec{R})$. In order to do this we must also specify its commutator with the vorticity $\rho_v(\vec{r}) \equiv$

$-\epsilon_{\alpha\beta}\partial_\alpha[\rho_0(\vec{r})u_\beta(\vec{r})]$. For the purpose of studying tunneling, we choose $[\psi(\vec{R}),\rho_v(\vec{r})]=0$, because the incoming electron is not expected to change the vorticity at the initial time. The solution has the form

$$\psi(\vec{R})=\exp\left[-\sum_{n>0}\left(\frac{M_n^*(\vec{R})}{\omega_n}b_n-\frac{M_n(\vec{R})}{\omega_n}b_n^\dagger\right)\right] \tag{8}$$

where the "electron-phonon" matrix elements $M_n(\vec{R})$ are written as $M_n(\vec{R})=\int\exp[-|\vec{r}-\vec{R}|^2/2l^2]\tilde{M}_n(\vec{r})d\vec{r}/2\pi l^2$, and

$$\frac{\tilde{M}_n(\vec{r})}{\omega_n}=\frac{i}{2\pi}\int\epsilon_{\alpha\beta}\frac{g_{n\alpha}(\vec{r'})}{\rho_0^{1/2}(\vec{r'})l}\partial_\beta\log|\vec{r}-\vec{r'}|d\vec{r'}\,. \tag{9}$$

The calculation of the local Green's function $G(\vec{R},t)=-i\langle T\psi(\vec{R},t)\psi^\dagger(\vec{R},0)\rangle$ can be carried out by standard techniques [18,8]. Here we simply report the zero temperature result for the integral equation connecting the electronic spectral function $A(\vec{R},\omega)$ (the Fourier transform of $G(\vec{R},t)$) to the collective excitation spectrum:

$$\omega A(\vec{R},\omega)=\int_0^\omega g(\vec{R},\Omega)\,A(\vec{R},\omega-\Omega)d\Omega \tag{10}$$

where $\omega>0$ and

$$g(\vec{R},\Omega)=\frac{1}{\Omega}\sum_{n>0}|M_n(\vec{R})|^2\delta(\Omega-\omega_n) \tag{11}$$

is closely related to the local dynamical structure factor of the liquid. The above equations constitute a complete scheme for the calculation of single particle and collective properties of a general distribution of electrons in the LLL. We now discuss some specific examples.

(1) *The uniform 2DEG.* Let ρ_0 be the uniform density. The normalized eigenfunctions of Eq. (4) have the form

$$g_{\vec{q}L}(\vec{r})=iql\left(\frac{\mu}{2\omega_q\rho_0}\right)^{1/2}e^{i\vec{q}\cdot\vec{r}}, \tag{12}$$

and

$$g_{\vec{q}T}(\vec{r})=\frac{1}{ql}\left(\frac{\omega_q\rho_0}{2\mu}\right)^{1/2}e^{i\vec{q}\cdot\vec{r}}, \tag{13}$$

where

$$\omega_q = \mu^{1/2} \left(v(q) + \frac{K(q)+\mu}{\rho_0^2} \right)^{1/2} (ql)^2 \tag{14}$$

is the frequency of the mode at wavevector $\vec{q}$, $v(q)$ is the Fourier transform of the electron-electron interaction, $K(q) = -\rho_0^2 \tilde{\chi}^{-1}(q)$ is the q-dependent bulk modulus, and the labels L (longitudinal) and T (transverse) refer to the components parallel and perpendicular to $\vec{q}$. The electron-phonon matrix element of Eq. (9) has the form

$$M_q(\vec{r}) = \left[v(q) + \frac{K_{\vec{r}}(q)+\mu}{\rho_0^2} \right] e^{-q^2 l^2/2} \rho_q(\vec{r}) \,, \tag{15}$$

where $\rho_q \equiv -iql\rho_0^{1/2} g_{qL}(\vec{r})$ and $K_{\vec{r}}(q) \equiv -\rho_0^2 \int \tilde{\chi}^{-1}(\vec{r}-\vec{r'}) \exp[i\vec{q}\cdot(\vec{r'}-\vec{r})] d\vec{r'}$. (The reason for the apparently unnecessary subscript $\vec{r}$ will become clear below).

We now distinguish two cases: *(i) Compressible case.* $\lim_{q\to 0} K(q) = K$ is finite. The values of K and μ can be approximated by those of a classical Wigner crystal, namely, $\mu \sim \mu_c = 0.09775\rho_0(e^2/l)\nu^{1/2}$ and $K \sim K_c = 6\mu_c$ [19]. The long wavelength modes have frequencies $\omega_q \propto q^{3/2}$ for Coulomb-like interaction ($v(q) = 2\pi e^2/q$) and $\omega_q \propto q^2$ for short-range interaction ($v(q) = e^2 d$). We have now all the necessary input for calculating the spectral function from Eqs. (10,11). The low-frequency behavior (where "zero" frequency is the chemical potential) is found to be $A(\omega) \propto \omega^{-5/4} e^{-\gamma/\omega^{1/2}}$ for Coulomb interaction and $A(\omega) \propto \omega^{(7+(2.7\nu)^{1/2} d/l)^{1/2}/4\nu - 1}$ for short-range interaction. Complete numerical results are shown in Fig. 1. They are in good agreement with previous calculations [8,9,20] as well as with experimental data [21].

(ii) Incompressible case. Within the single-pole approximation $K(q \to 0) \simeq \Delta\rho_0/2\alpha q^4$, where Δ is the collective excitation gap at $q = 0$, and $\alpha = (1-\nu)/8\nu$ is derived from the Laughlin wavefunction at filling factors $\nu \equiv 2\pi l^2 \rho_0 = 1/\textit{odd integer}$ [6]. Using Eq. (14) we find at long wavelength $\omega_q = (\Delta\mu/2\alpha\rho_0)^{1/2}$ independent of q. This formula can be used to deduce the value of μ if $\Delta = \lim_{q\to 0} \omega_q$ is known. Alternatively, one can substitute $\mu = \mu_c$ from the classical Wigner crystal and obtain $\Delta_\nu = 0.391\nu^{3/2}(1-\nu)^{-1} e^2/l$, which gives $\Delta_{1/3} = 0.11 e^2/l$, $\Delta_{1/5} = 0.044 e^2/l$ and $\Delta_{1/7} = 0.025 e^2/l$. These results are in good

agreement with variational estimates [6,22], with the exception of the first one which is almost 30% lower than the variational one, but compares favorably with exact diagonalization studies [23].

The calculation of the spectral function is more subtle. Straightforward application of the linear response formulas (10-15) is incorrect, because the addition (or removal) of charge at point $\vec{R}$ creates a compressible region in the middle of the liquid, changing the *topology* of the incompressible region from simply to doubly connected. The change in topology can be taken into account in the following manner. We stipulate that the bulk modulus kernel $-\rho_0^2\tilde{\chi}^{-1}(\vec{r},\vec{r'})$ has the form characteristic of an incompressible liquid (namely, $|\vec{r}-\vec{r'}|^2\log|\vec{r}-\vec{r'}|$) when both $\vec{r}$ and $\vec{r'}$ are within the incompressible region, but it is given by the local form $K_c\delta(\vec{r}-\vec{r'})$ when either $\vec{r}$ or $\vec{r'}$ are within the compressible "core" of the excitation. Because the size of the core region is microscopic, its presence does not affect significantly the frequencies of the long wavelength modes. On the other hand, in the "electron-phonon" matrix element, given by Eq. (15), we must use $K_{\vec{r}}(q) \simeq K_c$, because $\vec{r}$ is inside the core region. Within this scheme, the calculation of the spectral function can be straightforwardly carried out, without adjustable parameters. In Fig. 1, we plot the results for $\nu = 1/3$. We have used q-dependent K and μ in order to fit accurately the collective mode dispersion and structure factor [6] at finite q. The essential difference between this and the compressible case is the appearance of a δ-function peak at $\omega = 0$, which now corresponds to μ_+ (recall that in the incompressible liquid the chemical potentials for addition or removal of charge, μ_+ and μ_-, are different [24]). The strength of the δ-function is $Z = \exp[-\sum_q |M_q|^2/\omega_q^2]$, which would have vanished in the compressible case, and *does not vanish here because of the gap*. The peak at μ_+ reflects the ability of the incompressible liquid to accommodate the incoming electron in the ground state as topological defects of the initial incompressible state, without creating collective excitations. The incoherent part of the spectral function, at higher frequencies, corresponds to the creation of additional collective excitations [25].

(2) *Edge dynamics.* The case of a smooth compressible edge has been treated in previous

publications [11]. Here we focus on the case of a *sharp* edge, which is directly relevant to the interpretation of recent *lateral* tunneling experiments by Chang *et al.* and Grayson *et al.* [14]. Let us consider a straight edge along the y axis, and let $eE\hat{x}$ be the gradient of the confinement potential at the edge. The density ρ_0 is assumed to be uniform for $x < 0$, and zero for $x > 0$. The presence of the edge electric field breaks the rotational symmetry of the Hamiltonian (1), and must be taken into account with the additional term

$$H^{\rm edge} = e \int \rho(\vec{r}) \vec{u}(\vec{r}) \cdot \vec{E}\, d\vec{r}. \tag{16}$$

With this term included into the eigenvalue problem (4), we obtain a new set of solutions, which satisfy the conditions $\vec{\nabla} \cdot \vec{g} = 0$ and $\vec{\nabla} \times \vec{g} = 0$ for $x < 0$. Neglecting the logarithmic correction arising from the long-range Coulomb interaction, they obey the equation of motion

$$-i\omega g_\alpha(\vec{r}) = v\epsilon_{\alpha\beta}\partial_\beta g_x(\vec{r})\,, \tag{17}$$

where $v = cE/B$ is the classical drift velocity. The orthonormal solutions are (for $x < 0$)

$$\vec{g}_q(\vec{r}) = q^{1/2} e^{qx - iqy}(\hat{x} - i\hat{y})\,, \tag{18}$$

where $q > 0$ is a one-dimensional vector along the edge, and the eigenvalues are $\omega_q = vq$. These are analogous to gravity waves on the surface of a liquid. Because there is neither density change nor shear strain in the interior of the system, these solutions do not depend on the values of the bulk elastic constants. The use of elasticity theory is justified at small q since the displacement field $\vec{u}$ is slowly varying.

The effective edge dynamics can be derived from the full dynamics by projecting the latter onto the subspace spanned by the edge-wave solutions of Eq. (18). Within this subspace, we can define an "edge density" operator $\rho^{\rm edge}(y) \equiv \int \delta\bar{\rho}(x,y)dx = \rho_0 \bar{u}_x(0,y)$, where the bar denotes projection onto the edge-wave subspace, i.e., for instance, $\bar{u}_x(0,y) \equiv (l/\rho_0^{1/2}) \sum_{q>0} q^{1/2}[b_q e^{iqy} + b_q^\dagger e^{-iqy}]$. It is easy to verify that the edge density satisfies the standard Kac-Moody algebra [3] $[\rho_q^{\rm edge}, \rho_{-q'}^{\rm edge}] = (\nu q/2\pi)\delta_{qq'}$ where ν is the bulk filling factor. Thus, we have deduced the dynamics of the chiral Luttinger liquid at the edge from a

projection of the canonical dynamics of displacement fields in the bulk. The edge tunneling operator, obtained by imposing $[\psi(y), \rho^{\rm edge}(y')] = \psi(y)\delta(y-y')$ in the edge-wave subspace, is still given by Eq. (8), with the sum running over the edge modes, and the matrix element $M_q(y) = v(2\pi q/\nu)^{1/2} e^{-iqy}$. Use of Eqs. (10) and (11) then leads to the conclusion that the tunneling current must vanish, at low bias V, as $V^{1/\nu}$. This result is in good agreement with the experimental findings. The present derivation explains why the tunneling exponent is found to depend *only* on the bulk *density*, and not on whether the bulk is compressible or not.

In summary, we have developed a magneto-elastic bosonization scheme for the long wavelength dynamics of the 2DEG in the LLL. Our results show that this scheme can provide a unified description of different physical effects in the bulk and at the edge of the system.

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REFERENCES

[1] S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950); J. M. Luttinger, J. Math. Phys. **4**, 1154 (1963); D. Mattis and E. Lieb, J. Math. Phys. **6**, 304 (1965); A. Luther, Phys. Rev. B **14**, 2153 (1975); F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).

[2] A. Luther, Phys. Rev. B **19**, 320 (1979); F. D. M. Haldane, Varenna Lectures (1992) and Helv. Phys. Acta **65**, 152 (1992); A. Houghton and J. B. Marston, Phys. Rev. B **48**, 7790 (1993); A. H. Castro Neto and E. Fradkin, Phys. Rev. B **49**, 10877 (1994).

[3] X. G. Wen, Phys. Rev. B **41**, 12838 (1990); *ibidem* **44**, 5708 (1991); Int. Journ. Mod. Phys. B **6**, 1711 (1992).

[4] U. Zülicke and A. H. MacDonald, Phys. Rev. B **54**, 8349 (1996); Dror Orgad, Phys. Rev. Lett. **79**, 475 (1997).

[5] R. B. Laughlin, Phys. Rev. B **23**, 5632 (1981).

[6] S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B **33**, 2481 (1986).

[7] S.-C. Zhang, H. Hansson and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989); N. Read, *ibidem* **62**, 86 (1989); D. H. Lee and M. P. A. Fisher, *ibidem* **63**, 903 (1989).

[8] P. Johansson and J. M. Kinaret, Phys. Rev. B **50**, 4671 (1994).

[9] R. Haussmann, Phys. Rev. B **53**, 7357 (1996).

[10] I. L. Aleiner and L. I. Glazman, Phys. Rev. Lett. **72**, 2935 (1994).

[11] S. Conti and G. Vignale, Phys. Rev. B **54**, 14309 (1996); and Phys. E **1**, 101 (1998), cond-mat/9709055.

[12] J. H. Han and D. J. Thouless, Phys. Rev. B **55**, 1926 (1997); J. H. Han, *ibidem* **56**, 15806 (1997).

[13] I. L. Aleiner, H. U. Baranger, and L. I. Glazman, Phys. Rev. Lett. **74**, 3435 (1995); H.

Westfahl, A. H. Castro Neto, and A. O. Caldeira, Phys. Rev. B **55**, 7347 (1997).

[14] A. M. Chang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **77**, 2538 (1996) and Grayson *et al.*, Phys. Rev. Lett. **80**, 1062 (1998).

[15] P. Nozières, *The theory of interacting Fermi systems* (W. A. Benjamin, New York, 1964).

[16] L. D. Landau and E. Lifshitz, *Theory of Elasticity*, Vol. 7 of *Course of theoretical Physics*, 3rd ed. (Pergamon Press, Oxford, 1986).

[17] The extra sign is caused by the property of the scalar product $(f, f) = -(f^*, f^*)$ together with the definition $g_{-n} = g_n^*$. The fact that the positive frequency eigenfunctions ($n > 0$) are the ones with positive norm follows from the stability of the ground-state.

[18] G. D. Mahan, *Many-particle Physics* (Plenum Press, New York, 1990).

[19] L. Bonsall and A. A. Maradudin, Phys. Rev. B **15**, 1959 (1977).

[20] Y. Hatsugai, P.-A. Bares, and X. G. Wen, Phys. Rev. Lett. **71**, 424 (1993); Song He, P. M. Platzman, and B. I. Halperin, *ibidem* **71**, 777 (1993).

[21] J. P. Eisenstein, L. N. Pfeiffer, K. W. West, Phys. Rev. Lett. **69**, 3804 (1992).

[22] J. K. Jain and R. K. Kamilla, Int. J. Mod. Phys. **11**, 2621 (1997) and references therein.

[23] D. Yoshioka, J. Phys. Soc. Jap. **55**, 885 (1986).

[24] R. Morf and B. I. Halperin, Phys. Rev. B **33**, 2221 (1986).

[25] Neither the peak at $\omega = \mu_+$ nor the oscillatory behavior at higher frequency were seen in the experiment of Ref. [21]. A possible explanation is that tunneling took place primarily between compressible regions. This would be consistent with the observed linear dependence of the density on bias voltage.

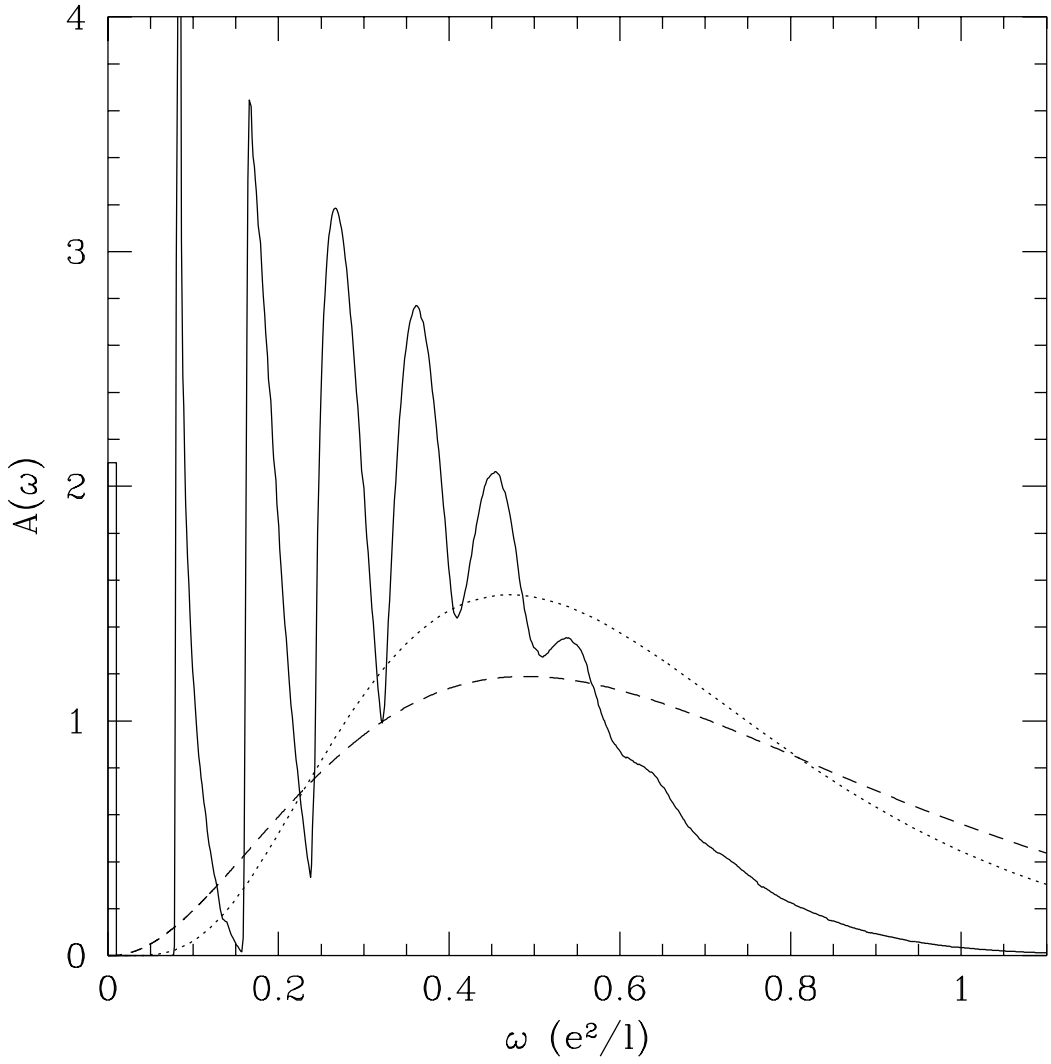

FIG. 1. Spectral function for a compressible liquid at $\nu = 0.3$ with short range interactions ($d = 10l$, dashed curve), with long-range interactions (dotted curve) and for an incompressible liquid at $\nu = 1/3$ (full curve, the δ-function peak at $\omega = 0$ contains around 2% of the spectral strength).