Max-Planck-Institut
für Mathematik
in den Naturwissenschaften
Leipzig

On the emergence of complex systems on the basis of the coordination of complex behaviors of their elements

by

Fatihcan M. Atay and Jürgen Jost

Preprint no.: 102 2003

On the emergence of complex systems on the basis of the coordination of complex behaviors of their elements

Fatihcan M. Atay*, Jürgen Jost*†

November 5, 2003

Abstract

We argue that the coordination of the activities of individual complex agents enables a system to develop and sustain complexity at a higher level. We exemplify relevant mechanisms through computer simulations of a toy system, a coupled map lattice with transmission delays. The coordination here is achieved through the synchronization of the chaotic operations of the individual elements, and on the basis of this, regular behavior at a longer temporal scale emerges that is inaccessible to the uncoupled individual dynamics.

The purpose of this article is to challenge the view, often expressed and perhaps prevalent in most discussions, that the essence of complex systems lies in the emergence of complex structures from the non-linear interaction of many simple elements that obey simple rules. Typically, these rules consist only of 0-1 alternatives selected in response to the input received, as in many prototypes like cellular automata, Boolean networks, spin systems, etc. We do not intend to deny that quite intricate patterns and structures can occur in such systems. However, these are toy systems, and the systems occurring in reality rather consist of elements that individually are quite complex themselves.[1] This brings in a new aspect that seems essential and indispensable to the emergence

*Max Planck Institute for Mathematics in the Sciences, Inselstr. 22-26, 04103 Leipzig, Germany, {fatay, jost}@mis.mpg.de

†Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA, jost@santafe.edu

[1]Throughout this essay, we employ the term "complex" only in some vague and metaphorical manner, without any attempt at quantifying it. This should not obscure the general thesis presented here.

and functioning of complex systems, namely the coordination of individual agents or elements that themselves are complex at their own scale of operation. This coordination dramatically reduces the degrees of freedom of those participating agents. Understanding the mechanisms responsible for achieving and maintaining this coordination seems the key to understanding, for example, the major transitions in evolution [11]. Even the constituents of molecules, the atoms, are rather complicated conglomerations of subatomic particles, perhaps ultimately excitation patterns of superstrings. Genes, the elementary biochemical coding units, are complicated macromolecular strings, as are the metabolic units, the proteins. Neurons, the basic elements of cognitive networks, themselves are cells. While their activity follows an apparently simple pattern of firing vs. resting, this depends on a slower learning dynamics tuning the strengths of the synaptic connections between them according to the history of temporal correlations between pre- and postsynaptic activities. At an even higher level of aggregation, an economic system consists of the interaction of humans, obviously highly complex agents. Nevertheless, standard economic theory is rather successful even though it assumes that these agents follow quite simple rules as laid down in utility functions and optimization patterns.

In any of these examples, it is by no means evident that the interactions of the elements or agents leads to a coherent structure at a higher level. If you bring a heterogenous group of people together, they will not automatically build a smoothly functioning economic system. It is rather that a functioning economic system has some subtle means to suppress the individual and disruptive behavior of its members and coerce them to operate in a manner that to a sufficiently large degree is predictable for the others. The rules and institutions that guarantee the functioning of the economic system are either directly imposed like the legal framework of contracts and the monetary system and then adapted by the economic system according to its internal exigencies, or acquired by the participants through processes of socialization, education, and experience. It is not our purpose here to enter the ongoing debate to what extent the rationality assumptions underlying standard economic theory are justified when contrasted with empirical investigations of the behavior of individual economic agents. We rather wish to make the point that economic agents behave rationally to whatever degree they do so because and to the extent to which they are participants in an economic system. Turning to another one of our examples, the behavior of neurons in vivo is different from the one in vitro, the former one exhibiting more regularities that are not intrinsic to the operation of the individual neuron itself, but rather imposed by the neural system in which the neuron is participating. In other words, in this and other complex systems, it is an important feature that the potential complexity of the behavior of the individual agents gets dramatically simplified through the global interactions within the system. The individual degrees of freedom are drastically reduced, or, in a more formal terminology, the factual state space of the system is much smaller than the product of the state spaces of the individual elements. This is one key aspect. The other one is that on this basis, that is utilizing the coordination between the activities of its members, the system then becomes able to develop and express a coherent structure at a higher level, that is, an emergent behavior that transcends what each element is individually capable of. Our thesis then is that the essence of a theory of complex systems should rest in analyzing and understanding the interplay of those two aspects. The reduction of the individual possibilities opens new possibilities at a higher level.

For a deeper conceptual analysis, one should then consider the elements or agents not as part of the system, but rather as constituting an inner or interior environment for the system, as in [10], so as to focus on the principally irreducible context of the system level. Here, however, rather than pursueing these conceptual aspects (see [4, 5] in that direction), we wish to elucidate this through a formal model system. As argued, such a system should not consist of simple agents, but rather ones that already by themselves possess a certain degree of complexity. We choose a discrete time chaotic dynamical system, namely the iteration of the logistic map

$$f(x) = \rho x(1-x) \tag{1}$$

with a parameter $1 \leq \rho \leq 4$. The iteration proceeds via

$$x(n+1) = f(x(n)) \tag{2}$$

for some starting value $x(0)$ ($n \in \mathbb{N}$). $f = f_\rho$ maps the unit interval $[0, 1]$ to itself. As we let ρ increase towards 4, periodic orbits appear through successive period doubling bifurcations until the behavior eventually becomes fully chaotic. Since the iteration of f_ρ for ρ sufficiently close to 4 amplifies small differences of the starting values, the future of an iteration cannot be predicted unless one makes the unrealistic assumption that the starting value is known with infinite precision. See [1] or a similar textbook for an introduction.

Our system couples such individual chaotic dynamical systems. We assume that we have some graph Γ. Vertices x, y connected by an edge are called neighbors, symbolically denoted by $x \sim y$. The number of neighbors of x is denoted by n_x. For a parameter ϵ, the coupling leads to the system

$$x(n+1) = f(x(n)) + \frac{\epsilon}{n_x} \sum_{y \sim x} (f(y(n)) - f(x(n))). \tag{3}$$

Thus, x now adjusts its state not only the basis of its own present state, but also takes the state differences from its neighbors into account. The coefficients on the right hand side are chosen in such a manner that the total weight of all the contributions is 1, that is, the same as in (2). This a coupled map lattice as introduced and studied for fully connected graphs by Kaneko [7, 8]. In particular, he discovered the phenomenon of synchronization of chaos, that is for certain values of ϵ and certain graphs, the individual chaotic iterations operate synchronously, that is

$$x(n) = y(n) =: \Xi(n) \tag{4}$$

for all vertices x, y of the graph and for sufficiently large n, regardless of the different starting values for the iterations at the individual nodes. In other cases, one may also observe intermittent behavior, that is, synchrony goes on and off. Mathematically, the stability of the synchronized solution can be studied through perturbations by eigenfunctions of the graph Laplacian

$$\Delta\phi(x) := \frac{1}{n_x}\sum_{y\sim x}(\phi(y) - \phi(x)), \tag{5}$$

and this is the reason why we prefer to write the system as in (3) instead of in the apparently simpler form

$$x(n+1) = (1-\epsilon)f(x(n)) + \frac{\epsilon}{n_x}\sum_{y\sim x} f(y(n)). \tag{6}$$

See for example [6] for an analysis. Whether synchronization occurs depends essentially on the spectral gap of the graph Γ, that is on the value of the first non-trivial eigenvalue of $\Delta = \Delta_\Gamma$ (which in turn reflects the topology, that is, the connection structure of the underlying graph Γ), and, of course, on the coupling parameter ϵ.

Synchronization is perhaps the most basic mechanism for the coordination of the behavior of individual elements or agents whose intrinsic dynamics are coupled. See [12] for a general introduction. Synchronization dramatically reduces the degrees of freedom for the dynamics of the coupled system when compared to the uncoupled dynamics of the individual agents, inasmuch as the synchronized dynamics is fully characterized by the dynamics of a single element.

The situation described so far is one where the synchronized collective dynamics coincides with the individual dynamics of an element in the uncoupled state, and so can be predicted by the latter. In particular, this only corresponds to the first one of the two key aspects for the emergence of complex behavior that we identified above. In order to obtain a new type of collective dynamics, we need to introduce an additional feature. Following [2], the feature we choose is a temporal delay in the transmission of the activities between vertices. In formal terms, we consider the coupled system

$$x(n+1) = f(x(n)) + \frac{\epsilon}{n_x}\sum_{y\sim x}(f(y(n-d_{yx})) - f(x(n))) \tag{7}$$

where $d_{yx} \in \mathbb{N}$ is the delay[2] from vertex y to x. This leads to several new dynamical features that we shall describe and explore in more detail and utilize to support the paradigm developed above. On one hand, we can generate synchronized – chaotic or

[2]In the sequel, we shall only consider constant transmission delays, $d_{yx} \equiv d$.

regular – behavior that is different from the chaotic dynamics (1), (2) of an uncoupled element. On the other hand, we can also generate regularities on a longer time scale that transcend the capabilities of isolated elements. On a technical level, we can even sustain period 3 oscillations stably over some parameter range, in contrast to the fact that for an isolated dynamical iteration, this is the penultimate state before chaos sets in [13, 9].

The first observation relevant here is that the uncoupled dynamics (2) is no longer a solution of (7), in contrast to the system (3) without delays. In the simplest possible case of global synchronization, the collective behavior can be obtained through a temporal averaging from individual dynamics, but in more interesting cases, it is fully irreducible. Thus, even the dynamics of a collective synchronized behavior cannot be reduced to the individual dynamics anymore, but rather reflects a new collective system dynamics.

We now describe some of the simulation results in more detail; the mathematical treatment will be given elsewhere. We consider a scale-free graph with 10,000 nodes.The results for random graphs are qualitatively similar, whereas for regular graphs with nearest-neighbor coupling, synchronization is typically not observed, see [6] for the case without delays where this behavior finds an explanation from the properties of the spectrum of the graph Laplacian.

We start with the case that is maximally chaotic in the uncoupled case, namely $\rho = 4$. Figure 1 indicates through gray values for which values of the coupling parameter ϵ and constant delay d the dynamic synchronize. In particular, the system synchronizes more readily, that is, starting at lower values of ϵ in the presence of delays than without. Also, there is a critical region roughly between $\epsilon = .1$ and $.2$ where synchronization occurs for odd, but not for even delays.

We now explore some of these effects in more detail. We consider constant delay $d = 1$ for the transmission between vertices, and we display the behavior of the coupled and delayed dynamics as depending on the coupling parameter ϵ. Figure 2 is a bifurcation diagram, exhibited in the range $\epsilon > 0.6$ for which synchronization is observed. As ϵ varies, we see chaos intermittent with periodic behavior; period 5 is quite stable in the range of ϵ between .8 and .9, while we see period 3 around $\epsilon = .94$. Periodic solutions become rarer for larger d.

We next fix ϵ at .8 which is within the synchronization region, take $d = 1$ (the results for other odd d are similar), and let ρ increase from 2.8 to 4. We find a constant solution up to $\rho \approx 3.2$ and then a direct transition to high period solution without intermediate successive period doublings (Figure 3). Thus, the route to chaos is different here from the standard period doubling paradigm. Another important difference is that we now get two positive Lyapunov exponents instead of one in the undelayed case, that is, we see a higher level of complexity at the system level than could be sustained by the individual dynamics. For $d = 2$, we see one period doubling at $\rho \approx 3$ which is also the

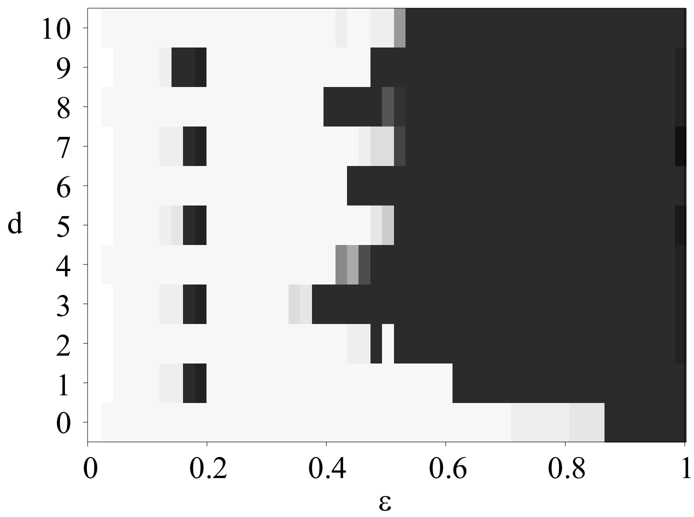

Figure 1: Synchronization of a scale-free network. The gray scale shows the degree of synchronization, with black corresponding to full synchronization.

value near which period doubling occurs for the standard uncoupled logistic map. In contrast to the latter, however, we do not observe further period doublings, but rather a Neimark-Sacker bifurcation at $\rho \approx 3.45$. That means that we get a pair of complex conjugate eigenvalues crossing the unit circle, but in contrast to the standard Hopf bifurcation for continuous time dynamics, here high period solutions bifurcate from a fixed point. For $d = 4$, the behavior is similar, but we see two consecutive period doublings before the Neimark-Sacker bifurcation.

While all this is technically, but perhaps not so much qualitatively differently from the chaotic behavior of the uncoupled logistic dynamics, in the region between the ϵ-values .1 and .2, already mentioned above as yielding different behavior depending on the parity of d, we find a qualitatively different behavior for larger even values of d, namely, a non-synchronized region with an enveloping curve of the dynamics that shows long time periodic behavior (Figure 5). This behavior is only seen in the collective dynamics, but not the individual one, and it occurs on a longer time scale than accessible to the latter.

Obviously, one can find emergent collective dynamics in other coupled systems, liking networks of spiking neurons, see for example [3]. In most such cases, however, this type of behavior is caused by underlying stochastic effects. The framework exhibited here offers some possibilities for a direct analytic approach to understanding such phenomena (see [2] in this direction). When compared with the conceptual setting described in the beginning, one deficit of the present model is perhaps that the coupling parameter ϵ has to be set by hand, instead of self-emerging from the intrinsic dynamics of the system. Also, the setting here is completely unrealistic in the sense that no

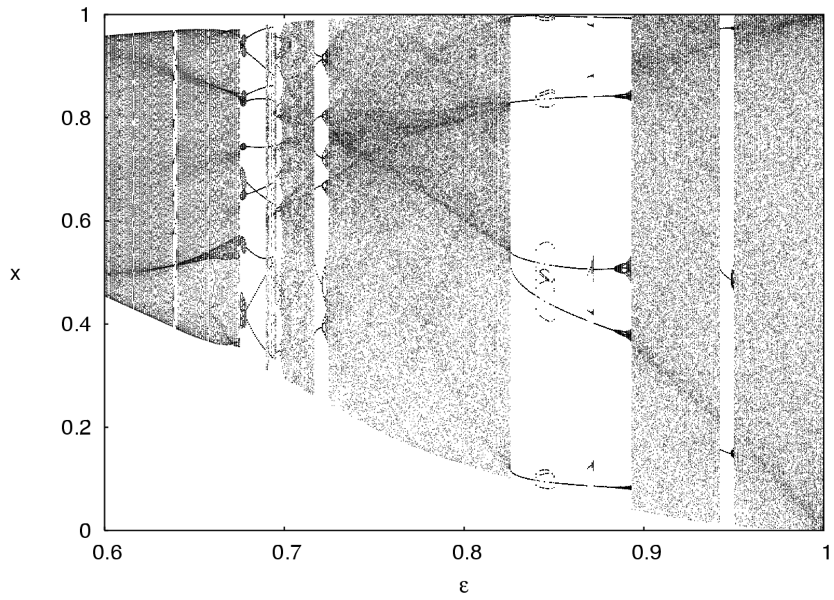

Figure 2: The dependence of the synchronized solution on the coupling parameter ϵ.

individual variations are allowed, that is, all elements operate with the same parameter values,[3] with the only exception that the underlying graph structure is not uniform or regular, but random. Nevertheless, looking at our simulation results, hopefully some understanding can be gained for our thesis that higher level complex behavior depends on the coordination of the activities of the participating agents which are complex themselves. As long as these operate in an uncoordinated manner, no higher scale is available for the encompassing system.

In any case, our formal model on which the simulations are based is obviously woefully inadequate to reflect the richness of the examples of higher level complex systems quoted. It shares this deficit with the formal models mentioned in the beginning of this essay. We think, however, that the present model captures one important aspect that is not represented in those ones. We also hope that a deeper formal analysis of that aspect will yield further insights into the mechanisms leading to the emergence of higher level complex systems.

Acknowledgement: The computer code for the simulations displayed here was written by Andreas Wende.

[3]It is not principally difficult, however, to extend the model and the simulations to cases where individual variations of the parameters like ρ are allowed.

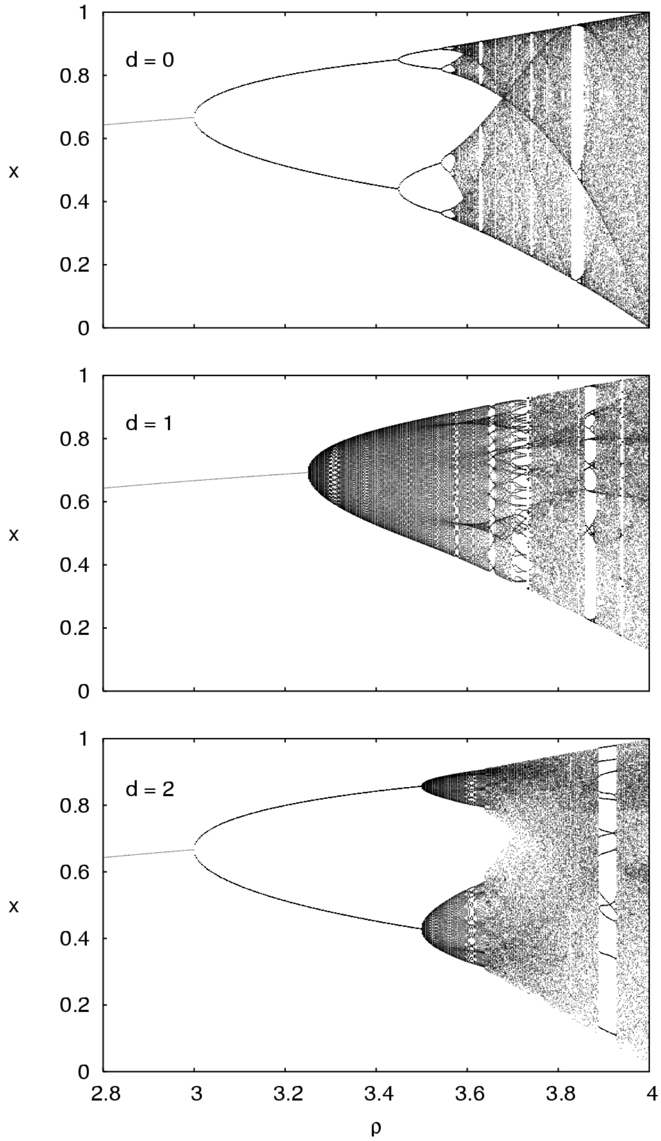

Figure 3: The dependence of the synchronized solution on the parameter ρ.

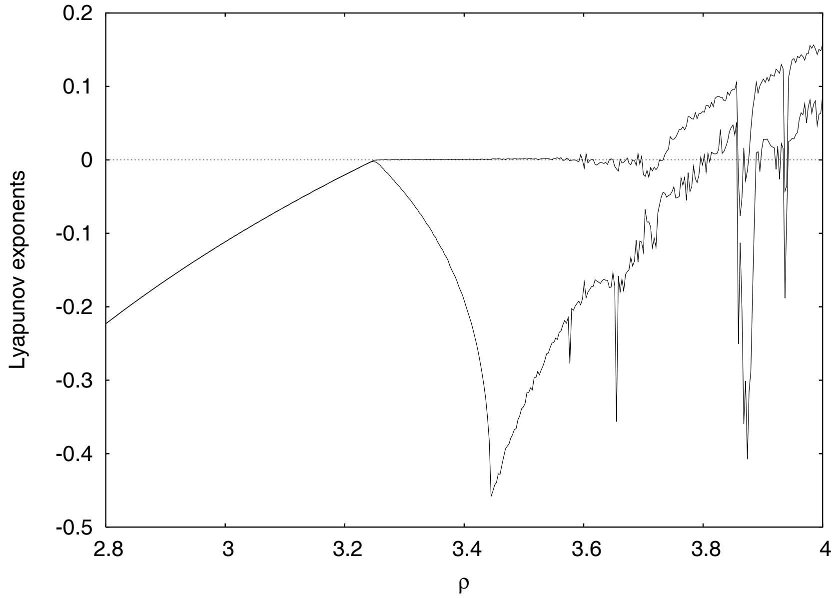

Figure 4: The Lyapunov exponents of the synchronized solution calculated for $d = 1$ and $\epsilon = 0.8$.

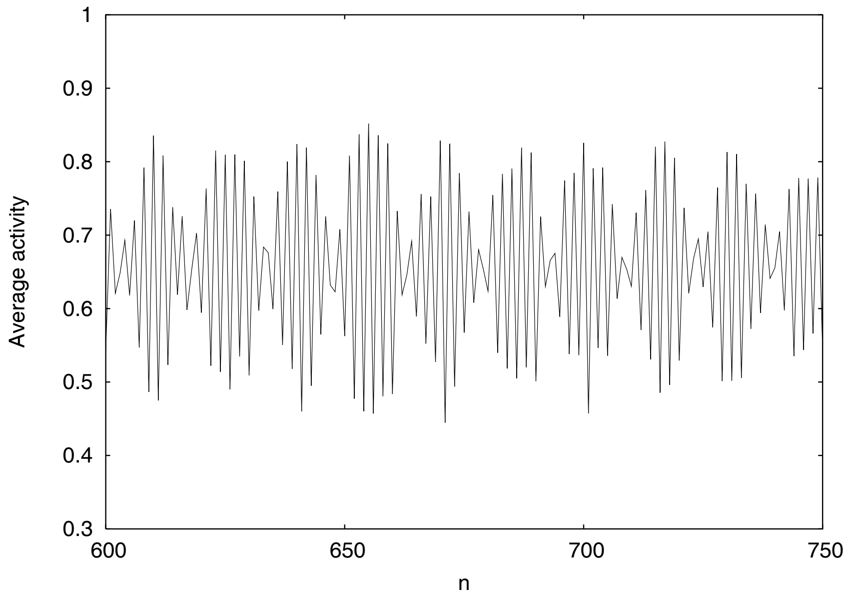

Figure 5: The average activity of the network for a non-synchronized solution, obtained for $d = 8$, $\rho = 4$, and $\epsilon = .135$.

References

[1] K. Alligood, T. Sauer, J. Yorke, Chaos. An introduction to dynamical systems, Springer, 1996.

[2] F. M. Atay, J. Jost, A. Wende, Delays, connection topology, and synchronization of coupled chaotic maps, to appear.

[3] W. Gerstner, W. Kistler, Spiking neuron models, Cambr. Univ. Press, 2002.

[4] J. Jost, External and internal complexity of complex adaptive systems, Theory Biosciences, to appear.

[5] J. Jost, Complex systems and cognitive structures, Monograph, to appear.

[6] J. Jost, M. P. Joy, Spectral properties and synchronization in coupled map lattices, Phys. Rev. E 65, 016201, 2002.

[7] K. Kaneko, Period-doubling of kink-antikink patterns, quasi-periodicity in antiferro-like structures and spatial intermittency in coupled map lattices – toward a prelude to a field theory of chaos, Prog. Theor. Phys. 72, 480–486, 1984.

[8] K. Kaneko (ed.), Theory and applications of coupled map lattices, Wiley, New York, 1993.

[9] T. Y. Li, J. Yorke, Period three implies chaos, Amer. Math. Monthly 82, 985–992, 1975.

[10] N. Luhmann, Soziale Systeme, Suhrkamp, Frankfurt/M., 1984, 71999.

[11] J. Maynard Smith, E. Szathmáry, The major transitions in evolution, Oxford Univ. Press, 1995.

[12] A. Pikovsky, M. Rosenblum, J. Kurths, Synchronization. A universal concept in nonlinear sciences, Cambr. Univ. Press, 2001.

[13] A. N. Sharkovsky, Coexistence of cycles of a continuous map of the line into itself, Ukrainskii Matematicheskii Zhurnal 16, 61–71, 1964 (in Russian).