Max-Planck-Institut
für Mathematik
in den Naturwissenschaften
Leipzig

Towards classification of simple finite dimensional modular Lie superalgebras

(revised version: January 2007)

by

Dimitry A. Leites

Preprint no.: 132 2006

Journal of Prime Research in Mathematics Volume **2**, *No. 2 (2006): 1–10*

Towards classification of simple finite dimensional modular Lie superalgebras

Dimitry Leites

Abstract

A conjectural way to construct all simple finite dimensional modular Lie (super)algebras over algebraically closed fields is offered (in characteristic 2, all graded by integers Lie (super)algebras are expected to be obtained in this way and only part of the non-graded ones). The conjecture is backed up with the latest results computationally most difficult of which are obtained with the help of Grozman's software package SuperLie.

Keywords: Cartan prolongation, Kostrikin-Shafarevich conjecture, modular Lie algebra, Lie superalgebra
Subjclass: 17B50, 70F25

Characteristic p is for the time when we retire.

Sasha Beilinson, when we all were young.

1. Introduction

The purpose of this transcript of the talk to be presented at the 3rd International Conference on 21st Century Mathematics 2007, School of Mathematical Sciences (SMS), Lahore, is to state problems, digestible to Ph.D. students (in particularly, the students at SMS) and worth (Ph.D. diplomas) to be studied, together even with ideas of their solution. In the process of formulating the problems, I'll overview the classical and latest results. To be able to squeeze the material into the prescribed 10 pages, all background is supplied by accessible references: We use standard notations of [FH, S]; for a precise definition of the Cartan prolongation and its generalizations, see [Shch]; see also [BGL1]–[BGL4]. Hereafter $\mathbb{K}$ is an (algebraically closed, unless specified) field, $\mathrm{Char}\mathbb{K} = p$.

I am thankful to P. Grozman, I. Shchepochkina, S. Bouarroudj, A. Lebedev and also to V. Serganova and Yu. Kochetkov for help; MPIMiS, Leipzig, for financial support and most creative environment.

The works of S. Lie, Killing and Cartan, now classical, completed classification over $\mathbb{C}$ of **simple Lie algebras** of a particular form:

$$\textbf{either of finite dimension or of polynomial vector fields}. \qquad (1)$$

In addition to the above two types, there are several interesting types of simple Lie algebras but they do not contribute to the solution of our problem: **classification of simple finite dimensional modular Lie (super)algebras**.

Lie algebras and Lie superalgebras over fields in characteristic $p > 0$, a.k.a. *modular* Lie (super)algebras, were distinguished in topology in the 1930s. The **simple** Lie algebras drew attention (over finite fields $\mathbb{K}$) as a step towards classification of simple finite groups, cf. [St]. Lie *super*algebras, even simple ones, did not draw much attention of mathematicians until their (outstanding) usefulness was observed by physicists in the 1970s, whereas new and new examples of simple modular Lie algebras were being discovered for decades until Kostrikin and Shafarevich ([KSh]) formulated a **conjecture** embracing all previously found examples for $p > 7$ (a **generalized KSh–conjecture**): *select a $\mathbb{Z}$-form $\mathfrak{g}_\mathbb{Z}$ of every $\mathfrak{g}$ of type*[1] (1), *take $\mathfrak{g}_\mathbb{K} := \mathfrak{g}_\mathbb{Z} \otimes_\mathbb{Z} \mathbb{K}$ and its simple subquotient $\mathfrak{si}(\mathfrak{g}_\mathbb{K})$ (there might be several). Together with deformations*[2] *of these examples we get in this way all simple finite dimensional Lie superalgebras over algebraically closed fields if $p > 5$. If $p = 5$, Melikyan's examples should be added to the examples obtained by the above method.*

After 30 years of work of several teams of researchers, Block, Wilson, Premet and Strade proved the generalized KSh conjecture for $p > 3$, see [S].

Even before the KSh conjecture was proved, its analog was offered in [KL] for $p = 2$. Although the KL conjecture was, it seems now, a bit overoptimistic, it suggested a way to get such an abundance of examples (to verify which of them are really simple is one of the tasks still open) that Strade [S] cited [KL] as an indication that the case $p = 2$ is too far out of reach by modern means[3]. Still, [KL] made two interesting points: it noted a striking similarity between modular Lie algebras and Lie *super*algebras (even over $\mathbb{C}$), and it introduced totally new characters — *Volichenko algebras* (inhomogeneous with respect to parity subalgebras of Lie superalgebras); for the classification of simple Volichenko algebras (finite dimensional and of vector fields) over $\mathbb{C}$, see [LSer].

[1] Notice that the analog of the polynomial algebra—the algebra of divided powers—*and all prolongs* (vectorial Lie algebras) acquire for $p > 0$ one more (shearing) parameter: $\underline{N}$.

[2] It is not clear, actually, if the conventional notion of deformation can always be applied if $p > 0$. This concerns both Lie algebras and Lie superalgebras (for the arguments, see [LL]); to give the correct (better say, universal) notion is an open problem, but we let it pass for the moment, besides, in some cases, the conventional definition is applicable, see [BGL4].

[3] Accordingly, **the "punch line" of this talk is: Cartan did not have the modern root technique, but got the complete list of simple Lie algebras; let's use his "old-fashioned" methods: they work!** Conjecture 2 expresses our hope in precise terms.

Recently Strade had published a monograph [S] summarizing the description of newly classified simple finite dimensional Lie algebras over the algebraically closed fields $\mathbb{K}$ of characteristic $p > 3$, and also gave an overview of the "mysterious" examples (due to Brown, Frank, Ermolaev and Skryabin) of simple finite dimensional Lie algebras for $p = 3$ with no counterparts for $p > 3$. Several researchers started afresh to work on the cases where $p = 2$ and 3, and new examples of simple Lie algebras with no counterparts for $p \neq 2, 3$ started to appear ([J1, GL4, GG, Leb1]). The "mysterious" examples of simple Lie algebras for $p = 3$ were interpreted as vectorial Lie algebras preserving certain distributions ([GL4]).

While writing [GL4] we realized, with considerable dismay, that there are reasons to put to doubt the universal applicability of the conventional definitions of the enveloping algebra $U(\mathfrak{g})$ (and its restricted version) of a given Lie algebra $\mathfrak{g}$ together with the conventional definitions of Lie algebra representations and (co)homology, cf. [LL]. But even accepting conventional definitions, there are plenty of problems to be solved before one will be able to start writing the proof of classification of simple modular Lie algebras, to wit: describe irreducible representations (as for vectorial Lie superalgebras, see [GLS]), decompose the tensor product of irreducible representations into indecomposables, cf. [Cla], and many more [Gr].

Classification of simple Lie *super*algebras for $p > 0$ and the study of their representations are of independent interest. A conjectural list of simple finite dimensional Lie superalgebras over an algebraically closed fields $\mathbb{K}$ for $p > 5$, known for some time, was recently cited in [BjL]:

Conjecture (1: Super KSh, $p > 5$). *Apply the steps of the KSh conjecture to the simple complex Lie* **super***algebras* $\mathfrak{g}$ *of types* (1)*. The examples thus obtained exhaust all simple finite dimensional Lie superalgebras over algebraically closed fields if* $p > 5$*.*

The examples obtained by this procedure will be referred to as *KSh-type Lie superalgebras*. The first step towards obtaining the list of KSh-type Lie superalgebras is classification of simple Lie superalgebras of types (1) over $\mathbb{C}$; this is done; for summaries with somewhat different emphases, see [K2, K3, LSh]. The same ideas of Block and Wilson that proved classification of simple Lie algebras for $p > 5$ will, if $p > 5$, work, I am sure, perhaps with minor changes, for Lie superalgebras also. Here I will describe the cases $p \leq 5$ where the situation is different and suggest *another* way to get simple examples.

2. How to construct simple Lie algebras and superalgebras

2.1. How to construct simple Lie algebras if $p = 0$. Let us recall how Cartan used to construct simple $\mathbb{Z}$-graded Lie algebras over $\mathbb{C}$ of polynomial growth [C] and finite depth. Now that they are classified (for examples of infinite depth, see [K]), we know that, *all of them can be endowed with a $\mathbb{Z}$-grading $\mathfrak{g} = \underset{-d\leq i}{\oplus} \mathfrak{g}_i$ of depth $d = 1$ or 2 so that $\mathfrak{g}_0$ is a simple Lie algebra $\mathfrak{s}$ or its trivial central extension $\mathfrak{cs} = \mathfrak{s} \oplus \mathfrak{c}$, where $\mathfrak{c}$ is a 1-dimensional center. Moreover, simplicity of $\mathfrak{g}$ requires $\mathfrak{g}_{-1}$ to be an irreducible $\mathfrak{g}_0$-module that generates $\mathfrak{g}_- := \underset{i<0}{\oplus} \mathfrak{g}_i$ and $[\mathfrak{g}_{-1}, \mathfrak{g}_1] = \mathfrak{g}_0$.*

Yamaguchi's theorem [Y], reproduced in [GL4, BjL], states that *for almost all simple finite dimensional Lie algebras $\mathfrak{g}$ over $\mathbb{C}$ and their $\mathbb{Z}$-gradings $\mathfrak{g} = \underset{-d\leq i}{\oplus} \mathfrak{g}_i$, the generalized Cartan prolong of $\mathfrak{g}_{\leq} = \underset{-d\leq i\leq 0}{\oplus} \mathfrak{g}_i$ is isomorphic to $\mathfrak{g}$, the rare exceptions being precisely the four series of simple vectorial algebras.*

We construct simple Lie algebras of type (1) by induction:

Depth $d = 1$. 1) we start with 1-dimensional $\mathfrak{c}$, so $\dim \mathfrak{g}_{-1} = 1$ due to irreducibility. The complete prolong is isomorphic to $\mathfrak{vect}(1)$, the partial one to $\mathfrak{sl}(2)$.

2) Take $\mathfrak{g}_0 = \mathfrak{csl}(2) = \mathfrak{gl}(2)$ and its irreducible module $\mathfrak{g}_{-1}$. The component $\mathfrak{g}_1$ of the Cartan prolong is nontrivial only if $\mathfrak{g}_{-1}$ is $R(\varphi_1)$ or $R(2\varphi_1)$, where φ_i is the ith fundamental weight.

2a) If $\mathfrak{g}_{-1}$ is $R(\varphi_1)$, the component $\mathfrak{g}_1$ consists of two irreducible submodules, say $\mathfrak{g}_1'$ or $\mathfrak{g}_1''$. We can take any one of them or both; together with $\mathfrak{g}_{-1} \oplus \mathfrak{g}_0$ this generates $\mathfrak{sl}(3)$ or $\mathfrak{svect}(2) \mathbin{\supset\!\!\!\!\!+} \mathfrak{d}$, where $\mathfrak{d}$ is spanned by an outer derivation, or $\mathfrak{vect}(2)$, respectively.

2b) If $\mathfrak{gl}(2) \simeq \mathfrak{co}(3) \simeq \mathfrak{csp}(2)$-module $\mathfrak{g}_{-1}$ is $R(2\varphi_1)$, then $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* \simeq \mathfrak{o}(5) \simeq \mathfrak{sp}(4)$.

3) Induction: Take $\mathfrak{g}_0 = \mathfrak{csl}(n) = \mathfrak{gl}(n)$ and its irreducible module $\mathfrak{g}_{-1}$. The component $\mathfrak{g}_1$ of the Cartan prolong is nontrivial only if $\mathfrak{g}_{-1}$ is $R(\varphi_1)$ or $R(2\varphi_1)$ or $R(\varphi_2)$.

3a) If $\mathfrak{g}_{-1} = R(\varphi_1)$, then $\mathfrak{g}_1$ consists of two irreducible submodules, $\mathfrak{g}_1'$ or $\mathfrak{g}_1''$. Take any of them or both; together with $\mathfrak{g}_{-1} \oplus \mathfrak{g}_0$ this generates $\mathfrak{sl}(n+1)$ or $\mathfrak{svect}(n) \mathbin{\supset\!\!\!\!\!+} \mathfrak{d}$, where $\mathfrak{d}$ is spanned by an outer derivation, or $\mathfrak{vect}(n)$, respectively.

3b) If $\mathfrak{g}_{-1} = R(2\varphi_1)$, then $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* \simeq \mathfrak{sp}(2n)$.

3c) If $\mathfrak{g}_{-1} = R(\varphi_2)$, then $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* \simeq \mathfrak{o}(2n)$.

4) The induction with $\mathfrak{g}_0 = \mathfrak{co}(2n-1)$-module $R(\varphi_1)$ returns $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* \simeq \mathfrak{o}(2n+1)$. Observe that $\mathfrak{sl}(4) \simeq \mathfrak{o}(6)$. The induction with $\mathfrak{g}_0 = \mathfrak{co}(2n)$-module $R(\varphi_1)$ returns $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* \simeq \mathfrak{o}(2n+2)$. (We have obtained $\mathfrak{o}(2n)$ twice; analogously, there many ways to obtain other simple Lie algebras as prolongs.)

5) The $\mathfrak{g}_0 = \mathfrak{sp}(2n)$-module $\mathfrak{g}_{-1} = R(\varphi_1)$ yields the Lie algebra $\mathfrak{h}(2n)$ of Hamiltonian vector fields.

$\mathfrak{e}(6)$, $\mathfrak{e}(7)$) The $\mathfrak{g}_0 = \mathfrak{co}(10)$-module $\mathfrak{g}_{-1} = R(\varphi_1)$ yields $\mathfrak{e}(6)$; the $\mathfrak{g}_0 = \mathfrak{ce}(6)$-module $\mathfrak{g}_{-1} = R(\varphi_1)$ yields $\mathfrak{e}(7)$.

Depth $d = 2$. Here we need **generalized** prolongations, see [Shch]. Again there are just a few modules $\mathfrak{g}_{-1}$ for which $\mathfrak{g}_1 \neq 0$ and $\mathfrak{g} = \oplus \mathfrak{g}_i$ is simple:

$\mathfrak{g}(2)$; $\mathfrak{f}(4)$; $\mathfrak{e}(8)$. These Lie algebras correspond to the prolongations of their non-positive part (with $\mathfrak{g}_0$ being isomorphic to $\mathfrak{gl}(2)$; $\mathfrak{o}(6)$ or $\mathfrak{sp}(6)$; $\mathfrak{e}(7)$ or $\mathfrak{o}(14)$, respectively) in the following $\mathbb{Z}$-gradings. Let us rig the nodes of the Dynkin graph with coefficients of linear dependence of the maximal root with respect to simple ones. If any end node is rigged by a 2, mark it (just one node even if several are rigged by 2's) and set the degrees of the Chevalley generators to be: $\deg X_i^\pm = \begin{cases} \pm 1 & \text{if } i\text{th node is marked} \\ 0 & \text{otherwise.} \end{cases}$

$\mathfrak{k}$. The cases where $(\mathfrak{g}_-, \mathfrak{g}_0)_*$ is simple and of infinite dimension: $\mathfrak{g}_0 = \mathfrak{csp}(2n)$ and $\mathfrak{g}_{-1} = R(\varphi_1)$. In these cases, $(\mathfrak{g}_-, \mathfrak{g}_0)_* = \mathfrak{k}(2n+1)$.

Conjecture (2: Amended KL=Super KSh, $p > 0$**).** *For* $p > 0$*, to get all* $\mathbb{Z}$**-graded** *simple finite dimensional examples of Lie algebras and Lie superalgebras, take the non-positive part of every simple (up to center) finite dimensional* $\mathbb{Z}$*-graded Lie (super)algebra (or Volichenko algebra if* $p = 2$*), consider its complete and partial prolongs and distinguish their simple subquotients. To get non-graded examples, we have to add deformations of the obtained simple algebras.*

For preliminary results, see [GL4, BjL, BGL1, BGL2, BGL3, LL, ILL, Leb3]. (For $p = 3$ and Lie algebras, this is how Grozman and me got an interpretation of all the "mysterious" exceptional simple Lie algebras known before [GL4] was published; we also found two (if not three) series of new simple algebras.) Having obtained a supply of such examples, we can sit down to d e s c r i b e t h e i r d e f o r m a t i o n s provided we will be able to understand what we are computing, cf. footnote 2); for a review of the already performed, see [KKCh, KuCh, Ch].

2.2. How to construct simple $\mathbb{Z}$**-graded Lie superalgebras of polynomial growth and finite depth if** $p = 0$**.** We start with a simple (up to center) Lie algebra or a Lie superalgebra $\mathfrak{g}_0$ and an irreducible $\mathfrak{g}_0$-module $\mathfrak{g}_{-1}$ for which we have more possibilities than for Lie algebras, but still not too many (beside finite dimensional cases, there are 34 series and 13 exceptions; as abstract (grading forgotten) algebras there are a dozen series and 5 Shchepochkina's exceptions) to obtain a simple prolong $(\mathfrak{g}_-, \mathfrak{g}_0)_*$ of depth 1 or 2, see [LSh, K2]. Observe:

1) The prolong $\mathfrak{g}_* := (\mathfrak{g}_-, \mathfrak{g}_0)_*$ might be not simple, but then the derived algebra $\mathfrak{g}_*'$ is.

2) One algebra may have several inequivalent Dynkin diagrams, in other words, it may have several inequivalent *systems of simple roots*. Only some of these systems of simple roots lead to nontrivial prolongs of their non-positive part of depth 1 or 2.

3) One vectorial algebra may have several regradings non-isomorphic as vectorial algebras; some of these regradings may be of depth 3, see [LSh, K2].

4) There are several Lie superalgebras with the same root system: there are deformations. (For an aborted discussion of the notion of the root system for Lie superalgebras, see [Se1].)

2.3. How to construct finite dimensional Lie algebras if $p > 5$**: the KSh procedure.** Observe that although there is a wider variety of pairs $(\mathfrak{g}_{-1}, \mathfrak{g}_0)$ yielding nontrivial prolongs (for the role of $\mathfrak{g}_0$ we can now take vectorial Lie algebras or their central extensions), *a posteriori* we know that we can always confine ourselves to the same pairs $(\mathfrak{g}_{-1}, \mathfrak{g}_0)$ as for $p = 0$. Melikyan's example looked as a deviation from the pattern, but Kuznetsov's observation [Ku1] elaborated in [GL4] shows that for $p \geq 5$ all is the same. Not so if $p \leq 3$:

2.4. How to construct finite dimensional Lie algebras if $p = 5$. At Kostrikin's suggestion, Melikyan took $\mathfrak{g}_0 = \mathfrak{cvect}(1;\underline{1})$ and $\mathfrak{g}_{-1} = \mathcal{O}(1;\underline{1})/\mathrm{const}$ and the generalized prolong yielded a new series of examples. Kuznetsov observed ([Ku1]) that these examples can also be obtained as $\underline{N}$-prolongs of the non-negative part of $\mathfrak{g}(2)$ (see step $\mathfrak{g}(2)$ above), with $\mathfrak{g}(2)$ obtained now as a *partial prolong*. (The same construction with $\mathfrak{g}_0 = \mathfrak{ck}(1;\underline{1}|1)$ and $\mathfrak{g}_{-1} = \mathcal{O}(1;\underline{1}|1)/\mathrm{const}$ yields another, non-isomorphic to $\mathfrak{Bj}(1;\underline{N}|7)$, super Melikyan algebra and, yet other, super versions of $\mathfrak{g}(2)$ and Brown algebras for $p = 3$, cf. [BjL] and [BGL3].)

2.5. New simple finite dimensional Lie algebras for $p = 3$. In [S], Strade listed known to him at that time examples of simple finite dimensional Lie algebras for $p = 3$. The construction of such algebras is usually subdivided into the following types and deformations of these types:

(1) algebras with Cartan matrix CM (sometimes encodable by Dynkin graphs, cf. [S, BGL1, BGL2]),

(2) algebras of vectorial type (meaning that they have more roots of one sign than of the other with respect to a partition into positive and negative roots).

Case (1) was studied in [WK] but with omissions, and the corrections in [KWK] do not correct the $p = 2$ case; still more important is the following phenomenon, never stated clearly: **there are examples of other types, distinct from (1) and (2), namely** $\mathfrak{g}(A)_{\mathbb{K}}$ suggested by the KSh-procedure: Let us be careful and d i s t i n g u i s h between $\mathfrak{g}(A_p)$ obtained from the Cartan matrix $A_p := A \bmod p$ over $\mathbb{F}_p$ or $\mathbb{K}$ (via the usual rules, compare [WK] and [BGL1]) and $\mathfrak{g}(A)_{\mathbb{K}}$ obtained via the KSh procedure. Whereas Grozman, and even his package `SuperLie` [Gr], knew how to construct $\mathfrak{g}(A_p)$ for a decade, it was only recently that we learned [BGL3, Leb2] how to construct $\mathfrak{g}(A)_{\mathbb{K}}$: it is NOT recovered from the matrix A by the usual rules and there are VARIOUS bases with respect to which the structure constants are integer, cf. [FH], pp. 345–346 and [Er]. For example, how many types of orthogonal Lie algebras are there for $p = 2$? How to present them? Do they have simple subquotients? (For the answers, see [Leb1].)

Thus, for $p < 5$, to (1)–(2) we have to add one more way to get simple Lie (super)algebras:

(3) the initial KSh construction

and deformations of (1)–(3). Conjecture 2 suggests to consider certain $\mathbb{Z}$-graded prolongs $\mathfrak{g}$. For Lie algebras and $p = 3$, Kuznetsov described various restrictions on the 0-th component of $\mathfrak{g}$ and the $\mathfrak{g}_0$-module $\mathfrak{g}_{-1}$ (for partial summary, see [GK, Ku1, Ku2], [BKK] and a correction in [GL4]). What are these restrictions for superalgebras?

2.6. Elduque's simple finite dimensional Lie superalgebras for $p = 5$ and $p = 3$. Elduque investigated which spinor modules over orthogonal algebras can serve as the odd part of a simple Lie superalgebra and discovered an exceptional simple Lie superalgebra for $p = 5$, cf. [El2, BGL2]. Elduque also superized the Freudenthal Magic Square and expressed it in a new way, and his approach yielded ten new simple (probably, exceptional) finite dimensional Lie superalgebras for $p = 3$, cf. [CE, El1, CE2]. These Lie superalgebras possess Cartan matrices (CM's) and we described all CMs and presentations of these algebras in terms of Chevalley generators, see [BGL1, BGL2]; in [BGL3], we considered their "most promising" (in terms of prolongations) $\mathbb{Z}$-gradings and discovered several new series of simple vectorial Lie superalgebras.

2.7. New simple finite dimensional Lie algebras and Lie superalgebras for $p = 2$. Lebedev [Leb1, Leb3] offered a new series of examples of simple orthogonal Lie algebras without CM. Together with Iyer, we constructed their prolongations, missed in [Lin], see [ILL]. (Lebedev also found out that the exceptional examples of Lie algebras with CM listed in [WK] as isomorphic, have different dimensions, so the $p = 2$ list of simple Lie algebras with CM in [WK] has to be corrected.)

Passing to Lie superalgebras we see that even their definition, as well as that of their prolongations, are not quite straightforward for $p = 2$, but, having defined them ([LL, Leb2]), it remains to apply the above-described procedures to get at least a supply of examples. To prove their completeness is a much more difficult task that requires serious preliminary study of the representations of the examples known and to be obtained — more topics for Ph.D. theses.

References

[BKK] Benkart, G.; Kostrikin, A. I.; Kuznetsov, M. I. The simple graded Lie algebras of characteristic three with classical reductive component L_0. Comm. Algebra 24 (1996), no. 1, 223–234

[BjL] Bouarroudj S., Leites D., Simple Lie superalgebras and non-integrable distributions in characteristic p. Zapiski nauchnyh seminarov POMI, t. 331 (2006), 15–29; Reprinted in J. Math. Sci. (NY) 2007; math.RT/0606682

[BGL1] Bouarroudj S., Grozman P., Leites D., Cartan matrices and presentations of Elduque and Cunha simple Lie superalgebras; MPIMiS preprint 124/2006

[BGL2] Bouarroudj S., Grozman P., Leites D., Cartan matrices and presentations of the exceptional simple Elduque Lie superalgebra; MPIMiS preprint 125/2006

[BGL3] Bouarroudj S., Grozman P., Leites D., New modular Lie superalgebras as generalized prolongs; MPIMiS preprint 146/2006

[BGL4] Bouarroudj S., Grozman P., Leites D., Deformations of the modular Lie superalgebras with Cartan matrices for $p = 3$; MPIMiS preprint

[C] Cartan É., Über die einfachen Transformationsgrouppen, Leipziger Berichte (1893), 395–420. Reprinted in: *Œuvres complètes.* Partie II. (French) [Complete works. Part II] Algèbre, systèmes différentiels et problèmes d'équivalence. [Algebra, differential systems and problems of equivalence] Second edition. Éditions du Centre National de la Recherche Scientifique (CNRS), Paris, 1984.

[Ch] Chebochko, N. G. Deformations of classical Lie algebras with a homogeneous root system in characteristic two. I. (Russian) Mat. Sb. 196 (2005), no. 9, 125–156; translation in Sb. Math. 196 (2005), no. 9-10, 1371–1402

[Cla] Clarke B., Decomposition of the tensor product of two irreducible $\mathfrak{sl}(2)$-modules in characteristic 3, MPIMiS preprint 145/2006

[CE] Cunha I., Elduque A., An extended Freudenthal magic square in characteristic 3; math.RA/0605379

[CE2] Cunha I., Elduque, A., The extended Freudenthal Magic Square and Jordan algebras; math.RA/0608191

[El1] Elduque, A. New simple Lie superalgebras in characteristic 3. J. Algebra 296 (2006), no. 1, 196–233

[El2] Elduque, A. Some new simple modular Lie superalgebras. math.RA/0512654

[Er] Ermolaev, Yu. B. Integral bases of classical Lie algebras. (Russian) Izv. Vyssh. Uchebn. Zaved. Mat. 2004, , no. 3, 16–25; translation in Russian Math. (Iz. VUZ) 48 (2004), no. 3, 13–22.

[FH] Fulton, W., Harris, J., *Representation theory. A first course*. Graduate Texts in Mathematics, 129. Readings in Mathematics. Springer-Verlag, New York, 1991. xvi+551 pp

[GK] Gregory, T.; Kuznetsov, M. On depth-three graded Lie algebras of characteristic three with classical reductive null component. Comm. Algebra 32 (2004), no. 9, 3339–3371

[GG] Grishkov, A.; Guerreiro, M. New simple Lie algebras over fields of characteristic 2. Resenhas 6 (2004), no. 2-3, 215–221

[Gr] Grozman P., `SuperLie`, `http://www.equaonline.com/math/SuperLie`

[GL1] Grozman P., Leites D., Defining relations for classical Lie superalgebras with Cartan matrix, Czech. J. Phys., Vol. 51, 2001, no. 1, 1–22; arXiv: `hep-th/9702073`

[GL2] Grozman P., Leites D., `SuperLie` and problems (to be) solved with it. Preprint MPIM-Bonn, 2003-39 (`http://www.mpim-bonn.mpg.de`)

[GL4] Grozman P., Leites D., Structures of $G(2)$ type and nonintegrable distributions in characteristic p. Lett. Math. Phys. 74 (2005), no. 3, 229–262; arXiv: `math.RT/0509400`

[GLS] Grozman P., Leites D., Shchepochkina I., Invariant operators on supermanifolds and standard models. In: In: M. Olshanetsky, A. Vainstein (eds.) *Multiple facets of quantization and supersymmetry. Michael Marinov Memorial Volume*, World Sci. Publishing, River Edge, NJ, 2002, 508–555. [math.RT/0202193; ESI preprint 1111 (2001)].

[ILL] Iyer U., Lebedev A., Leites, D. Prolongations of orthogonal Lie algebras in characteristic 2. IN PREPARATION

[J1] Jurman, G., A family of simple Lie algebras in characteristic two. J. Algebra 271 (2004), no. 2, 454–481.

[K] Kac V. *Infinite-dimensional Lie algebras.* Third edition. Cambridge University Press, Cambridge, 1990. xxii+400 pp.

[K2] Kac V., Lie superagebras, Adv. Math. v. 26, 1977, 8–96

[K3] Kac, V. Classification of supersymmetries. Proceedings of the International Congress of Mathematicians, Vol. I (Beijing, 2002), Higher Ed. Press, Beijing, 2002, 319–344

Cheng, Shun-Jen; Kac, V., Addendum: "Generalized Spencer cohomology and filtered deformations of $\mathbb{Z}$-graded Lie superalgebras" [Adv. Theor. Math. Phys. 2 (1998), no.

5, 1141–1182; MR1688484 (2000d:17025)]. Adv. Theor. Math. Phys. 8 (2004), no. 4, 697–709.

Cantarini, N.; Cheng, S.-J.; Kac, V. Errata to: “Structure of some $\mathbb{Z}$-graded Lie superalgebras of vector fields” [Transform. Groups 4 (1999), no. 2-3, 219–272; MR1712863 (2001b:17037)] by Cheng and Kac. Transform. Groups 9 (2004), no. 4, 399–400

[KWK] Kac, V. G. Corrections to: ”Exponentials in Lie algebras of characteristic p” [Izv. Akad. Nauk SSSR 35 (1971), no. 4, 762–788; MR0306282 (46 #5408)] by B. Yu. Veisfeiler and Kac. (Russian) Izv. Ross. Akad. Nauk Ser. Mat. 58 (1994), no. 4, 224; translation in Russian Acad. Sci. Izv. Math. 45 (1995), no. 1, 229

[KKCh] Kirillov, S. A.; Kuznetsov, M. I.; Chebochko, N. G. Deformations of a Lie algebra of type G_2 of characteristic three. (Russian) Izv. Vyssh. Uchebn. Zaved. Mat. 2000, , no. 3, 33–38; translation in Russian Math. (Iz. VUZ) 44 (2000), no. 3, 31–36

[KL] Kochetkov Yu., Leites D., Simple finite dimensional Lie algebras in characteristic 2 related to superalgebras and on a notion of finite simple group. In: L. A. Bokut, Yu. L. Ershov and A. I. Kostrikin (eds.) *Proceedings of the International Conference on Algebra. Part 1., Novosibirsk, August 1989*, Contemporary Math. 131, Part 1, 1992, AMS, 59–67

[KSh] Kostrikin, A. I., Shafarevich, I.R., Graded Lie algebras of finite characteristic, Izv. Akad. Nauk. SSSR Ser. Mat. 33 (1969) 251–322 (in Russian); transl.: Math. USSR Izv. 3 (1969) 237–304

[KuCh] Kuznetsov, M. I.; Chebochko, N. G. Deformations of classical Lie algebras. (Russian) Mat. Sb. 191 (2000), no. 8, 69–88; translation in Sb. Math. 191 (2000), no. 7-8, 1171–1190

[Ku1] Kuznetsov, M. I. The Melikyan algebras as Lie algebras of the type G_2. Comm. Algebra 19 (1991), no. 4, 1281–1312.

[Ku2] Kuznetsov, M. I. Graded Lie algebras with the almost simple component L_0. Pontryagin Conference, 8, Algebra (Moscow, 1998). J. Math. Sci. (New York) 106 (2001), no. 4, 3187–3211.

[Leb1] Lebedev A., Non-degenerate bilinear forms in characteristic 2, related contact forms, simple Lie algebras and superalgebras. arXiv: `math.AC/0601536`

[Leb2] Lebedev A., Simple Lie superalgebras preserving ortho-orthogonal forms in characteristic $p = 2$. MPIMiS preprint

[Leb3] Lebedev A., Presentations of finite dimensional classical modular Lie algebras. MPIMiS preprint

[LL] Lebedev A., Leites D., On realizations of the Steenrod algebras. J. Prime Res. Math., v. 2, 2006,

[LSer] Leites, D.; Serganova, V. Metasymmetry and Volichenko algebras. Phys. Lett. B 252 (1990), no. 1, 91–96; id, Symmetries wider than supersymmetry. In: Dupllij S., Wess J. (eds.) *Noncommutative structures in mathematics and physics* (Kiev, 2000), 13–30, NATO Sci. Ser. II Math. Phys. Chem., 22, Kluwer Acad. Publ., Dordrecht, 2001

[LSh] Leites D., Shchepochkina I., Classification of the simple Lie superalgebras of vector fields, preprint MPIM-2003-28 (`http://www.mpim-bonn.mpg.de`)

[Lvd] Leur J. van de, *Contragredient Lie superalgebras of finite growth* (Ph.D. thesis) Utrecht, 1986; a short version published in Commun. in Alg., v. 17, 1989, 1815–1841

[Lin] Lin, L., Nonalternating hamiltonian algebra $P(n, m)$ of characteristic two, Comm. Algebra 21 (2) (1993) 399–411

[RSh] Rudakov, A. N.; Shafarevich, I. R. Irreducible representations of a simple three-dimensional Lie algebra over a field of finite characteristic. (Russian) Mat. Zametki 2 (1967) 439–454 (1967, 2, 760–767)

[Se] Serganova, V., Automorphisms of simple Lie superalgebras. (Russian) Izv. Akad. Nauk SSSR Ser. Mat. 48 (1984), no. 3, 585–598

[Se1] Serganova, V., On generalizations of root systems. Comm. Algebra 24 (1996), no. 13, 4281–4299

[Ser] Sergeev A., Orthogonal polynomials and Lie superalgebras, arXiv: `math.RT/9810110`.

[Shch] Shchepochkina I., How to realize Lie algebras by vector fields. Theor. Mat. Fiz. 147 (2006) no. 3, 821–838; arXiv: `math.RT/0509472`

[Sh14] Shchepochkina I., Five exceptional simple Lie superalgebras of vector fields and their fourteen regradings. Representation Theory (electronic journal of AMS), v. 3, 1999, 3 (1999), 373–415; arXiv: `hep-th/9702121`

[Sh] Shen, Guang Yu, Variations of the classical Lie algebra G_2 in low characteristics. Nova J. Algebra Geom. 2 (1993), no. 3, 217–243.

[Sk] Skryabin, S. M. New series of simple Lie algebras of characteristic 3. (Russian. Russian summary) Mat. Sb. 183 (1992), no. 8, 3–22; translation in Russian Acad. Sci. Sb. Math. 76 (1993), no. 2, 389–406

[St] Steinberg, R. *Lectures on Chevalley groups.* Notes prepared by John Faulkner and Robert Wilson. Yale University, New Haven, Conn., 1968. iii+277 pp.

[S] Strade, H. *Simple Lie algebras over fields of positive characteristic. I. Structure theory.* de Gruyter Expositions in Mathematics, 38. Walter de Gruyter & Co., Berlin, 2004. viii+540 pp.

[WK] Weisfeiler, B. Ju.; Kac, V. G. Exponentials in Lie algebras of characteristic p. (Russian) Izv. Akad. Nauk SSSR Ser. Mat. 35 (1971), 762–788.

[Y] Yamaguchi K., Differential systems associated with simple graded Lie algebras. Progress in differential geometry, Adv. Stud. Pure Math., 22, Math. Soc. Japan, Tokyo, 1993, 413–494

MPIMiS, Inselstr. 22, DE-04103 Leipzig, Germany
on leave from Department of Mathematics, University of Stockholm, Roslagsv. 101, Kräftriket hus 6, SE-106 91 Stockholm, Sweden;
mleites@math.su.se, leites@mis.mpg.de