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by

Dimitry A. Leites, and Christoph Sachse

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Dimitry Leites, Christoph Sachse

MPIMiS, Inselstr. 22, DE-04103 Leipzig, Germany;
leites, sachse@mis.mpg.de

Exactly 10 of all simple Lie superalgebras of vector fields on $1|N$-dimensional supercircles (superstrings) preserving a structure (either nothing, or a volume element, or a contact form) have nontrivial central extensions. The values of the central charges in (projective) spinor-oscillator representations of these stringy superalgebras associated with the adjoint module can be interpreted as critical dimensions of respective superstrings. Apart from the well-known values 26, 10, 2 and 0 corresponding to $N = 0$, 1, 2 and > 2, respectively, for the contact type stringy superalgebras (Neveu–Schwarz and Ramond types alike), there are two more non-zero values of critical dimension: For the general and divergence-free algebras for $N = 2$. These dimensions, found here, are -1 and -2, respectively. We also mention related problems.

Keywords: stringy Lie superalgebra, critical dimension

1. Introduction

From Ref. 7: "In various papers and books on string theories, a degenerate (in Dirac's sense) Lagrangian is considered. The Fourier harmonics of the constraints of this Lagrangian can be endowed with a Lie algebra structure; this Lie algebra is isomorphic to the Lie algebra of the group of diffeomorphisms of the circle. The complexification of this Lie algebra is a version[a] of the Witt algebra $\mathfrak{witt}$: Either $\mathfrak{der}\left(\mathbb{C}[x^{-1}, x]\right)$ or its completion. The following types of $\mathfrak{witt}$-modules (or their completions) are mainly studied:

(1) $\mathcal{F}_{\lambda,\mu} = \mathbb{C}[x^{-1}, x]x^{\mu}(dx)^{\lambda}$ (the answer to the question which of these modules are irreducible follows from Ref. 24, where the polynomial case is considered);

(2) highest (or lowest) weight modules (irreducible ones are completely

[a]Every Lie algebra of vector fields on a manifold has several "incarnations": The coefficients of the fields can be smooth functions, or polynomials, or power series, or Laurent polynomials, or (for completion) Laurent series, etc. depending on the problem.

described in Ref. 6) which are all realized as quotients of the spinor Spin(V) or oscillator Osc(V) representations constructed from modules $V = \mathcal{F}_{\lambda,\mu}$.

Since the representations of type (2) are projective, they give rise to a nontrivial central extension of $\mathfrak{witt}$, the Virasoro algebra $\mathfrak{vir}$, and actually are representations of $\mathfrak{vir}$, rather than of $\mathfrak{witt}$.

Realization of the elements of $\mathfrak{witt}$ by quadratic polynomials in creation and annihilation operators acting in the spaces of spinor and oscillator representations is interpreted in physics as quantization.

The "critical dimension" CD that appears in physical papers is the (only) dimension of the Minkowski space (in which the string under study lives) for which quantization is free of anomalies[12]. There are several, perhaps inequivalent in super setting, ways to compute the critical dimensions. The first ones are related to a study of various action functionals, see, e.g., Ref. 22, 12."

Feigin interpreted the CD as the value of the central charge of $\mathfrak{vir}$ in the spinor representation corresponding to the adjoint $\mathfrak{witt}$-module. Feigin used this CD in the computation of semi-infinite cohomology he introduced[3,7,4,5,8].

For the list of super analogues of $\mathfrak{witt}$ — simple Lie superalgebras that appear in string theories, see Ref. 10. Among them, there are precisely ten *distinguished* ones, which have a nontrivial central extension.

Superization of Polyakov's action and Nambu's action leads to interesting supersymmetric integrable systems (see, e.g., Ref. 13, 16, 21, 18, and refs. therein), super versions of minimal surfaces and constant curvature surfaces, but we can not describe critical dimensions in these terms.

Our result: We apply Feigin's picture to consider two, hopefully new, examples. (The isomorphism, up to parity, Spin$(V) \simeq$ Osc(V), *as spaces* is referred to as "Fermi–Bose correspondence" in physical papers. The values of CD in these spaces differ by a sign, and this gives us hope to be able to interpret negative CDs as reasonable dimensions.)

2. Stringy superalgebras and modules over them

2.1. Vectorial Lie superalgebras with polynomial or formal coefficients. Let $\mathcal{F} := \mathbb{C}[[y]]$ be the supercommutative superalgebra of formal power series in $y = (y_1, \dots, y_{n+m}) = (x_1, \dots, x_n, \xi_1, \dots, \xi_m)$ with parities $p(x_i) = \bar{0}$ and $p(\xi_j) = \bar{1}$ for all i, j. Denote by (y) the maximal ideal in $\mathcal{F}$ generated by the y_i. Define the topology on $\mathcal{F}$ in which the ideals $(y)^r$, where $r = 0, 1, 2, \dots$, are neighborhoods of zero. We see that $\mathcal{F}$ is complete with respect to this topology.

Let $\mathfrak{vect}(n|m)$ be the Lie superalgebra of formal vector fields, i.e., of derivations of $\mathbb{C}[[y]]$ continuous with respect to the above topology. In general questions we denote $\mathfrak{vect}(n|m)$ and its infinite dimensional subalgebras by $\mathcal{L}$. In $\mathcal{L}$, define a descending filtration $\mathcal{L} = \mathcal{L}_{-1} \supset \mathcal{L}_0 \supset \mathcal{L}_1 \supset \dots$, setting

$$\mathcal{L}_r = \{D \in \mathfrak{vect}(n|m) \mid D(\mathcal{F}) \subset (y)^{r+1}\} \tag{1}$$

Denote by $L = \oplus L_r$, where $L_r = \mathcal{L}_r/\mathcal{L}_{r+1}$, the associated graded Lie superalgebra. Let us identify L_0 with $\mathfrak{gl}(n|m)$ setting $E_{ij} \leftrightarrow y_j\partial_i$.

Remark. For some simple Lie superalgebras, the spaces of the highest and lowest weight vectors is multidimensional, cf. Ref. 11; there are also various gradings listed in Ref. 20.

2.2. Tensor fields. Let us consider a given representation ρ of $L_0 = \mathcal{L}_0/\mathcal{L}_1$ in V as a representation of $\mathcal{L}_0$. Since $\mathrm{Hom}(\mathbb{C}[\partial], \mathbb{C}) \cong \mathcal{F}$, we may set

$$T(V) := \mathrm{Hom}_{U(\mathcal{L}_0)}(U(\mathcal{L}), V).$$

Each vector field $D = \sum D_i\partial_i$ acts in $T(V)$ by means of the *Lie derivative* L_D: For any $f \in \mathcal{F}, v \in V$ and $D^{ij} = (-1)^{p(y_i)p(D_j)}\partial_i D_j$, we set

$$L_D(fv) = D(f)v + (-1)^{p(D)p(f)} f \sum D^{ij}\rho(E_{ij})(v).$$

Having fixed coordinates, we define the *divergence* as the map

$$D = \sum_i f_i \frac{\partial}{\partial u_i} + \sum_j g_j \frac{\partial}{\partial \theta_j} \longmapsto \mathrm{div} D = \sum_i \frac{\partial f_i}{\partial u_i} + \sum_j (-1)^{p(g_j)} \frac{\partial g_i}{\partial \theta_j}.$$

Set $\mathfrak{svect}(n|m) = \{D \in \mathfrak{vect}(n|m) \mid \mathrm{div} D = 0\}$.

2.3. Modules over stringy superalgebras. Simple stringy superalgebras and their nontrivial central extensions are listed in Ref. 10. These extensions only exist for contact algebras (and the corresponding CDs are known), and also for $\mathfrak{vect}^L(1|2) = \mathfrak{der}\mathbb{C}[x^{-1}, \xi_1, \xi_2][[x]]$ and

$$\mathfrak{svect}^L_\lambda(1|2) = \{D \in \mathfrak{vect}^L(1|2) \mid \mathrm{div}(x^\lambda D) = 0\}.$$

Denote by $\mathcal{T}(V)$ the $\mathfrak{vect}(1|n)$-module that differs from $T(V)$ by allowing Laurent polynomials as coefficients of its elements instead of polynomials. Clearly, $\mathcal{T}(V)$ is a $\mathfrak{vect}^L(1|n)$-module. Set $\mathcal{T}_\mu(V) := \mathcal{T}(V)x^\mu$. Some of such modules are described in Ref. 14 as an easy corollary of Ref. 1. None of these modules has a highest or lowest weight vector. The "simplest" such modules over the Lie superalgebras $\mathfrak{vect}^L(1|n)$ and $\mathfrak{svect}^L_\lambda(1|n)$ are, clearly, the rank 1 modules over $\mathcal{F}$, the algebra of functions. They are constructed as follows.

Let Vol^μ be the space of tensor fields corresponding to the "μth power" of the $\mathfrak{gl}$-module corresponding to the superdeterminant or Berezinian (infinitesimally, $X vol = \mu \mathrm{str}(X) vol$). For $\mu = 0$, this is just the space of functions, $\mathcal{F}$.

Over $\mathfrak{k}^L(N) \subset \mathfrak{vect}^L(1|N)$, the contact superalgebra which preserves the Pfaff equation with the form

$$\alpha_N = dx + \sum_{1\le i\le N} \theta_i d\theta_i = dx + \sum_{1\le i\le N/2} (\xi_i d\eta_i + \eta_i d\xi_i) + \begin{cases} & \text{for } N \text{ even} \\ \theta_N d\theta_N & \text{for } N \text{ odd,} \end{cases}$$

it is more natural to consider the modules defined in terms of α_N:

$$\mathcal{F}_\lambda = \begin{cases} \mathcal{F}\alpha_N^\lambda & \text{for } N = 0 \\ \mathcal{F}\alpha_N^{\lambda/2} & \text{for } N > 0, \end{cases} \qquad \mathcal{F}_{\lambda,\mu} = \mathcal{F}_\lambda x^\mu. \tag{2}$$

Observe that $Vol^\lambda = \mathcal{F}_{\lambda(2-n)}$ and that the Lie superalgebras of series $\mathfrak{k}^L$ do not distinguish between $\frac{\partial}{\partial x}$ and α^{-1}: their transformation rules are identical. Over $\mathfrak{k}^L(1|2)$ (and similarly over $\mathfrak{k}^M(1|3)$), there are also modules $\mathcal{F}_{\lambda,\nu;\mu} := \mathcal{F}_{\lambda;\mu}[d\xi_1]^\nu$, where $[d\xi_1] = d\xi_1 \mod \alpha_N$.

Let $\alpha_N^M = \alpha_{N-1} + x\theta d\theta$ be the "Möbius" version of the contact form α_N and let $\mathfrak{k}^M(N) \subset \mathfrak{vect}^L(1|N)$ be the Lie superalgebra that preserves the Pfaff equation $\alpha_N^M(X) = 0$ for $X \in \mathfrak{vect}^L(1|N)$. We set (clearly, $K_f \in \mathfrak{k}^L(N)$)

$$K_f = (2-E)(f)\frac{\partial}{\partial x} + (-1)^{p(f)} H_f + \frac{\partial f}{\partial x} E,$$

where $E = \sum\limits_i \theta_i \frac{\partial}{\partial \theta_i}$, and $H_f = \sum\limits_{j\le N} \frac{\partial f}{\partial \theta_j}\frac{\partial}{\partial \theta_j}$ is the hamiltonian field with Hamiltonian f that preserves $d\alpha_N$ and (clearly, $K_f^M \in \mathfrak{k}^M(N)$)

$$K_f^M = (2-E)(f)\mathcal{D} + \mathcal{D}(f)E + (-1)^{p(f)} H_f^M,$$

where

$$\mathcal{D} = \frac{\partial}{\partial x} - \frac{\theta}{2x}\frac{\partial}{\partial \theta}, \quad H_f^M = \frac{1}{x}\frac{\partial f}{\partial \theta}\frac{\partial}{\partial \theta} + \sum_{i\le N-1} \frac{\partial f}{\partial \theta_i}\frac{\partial}{\partial \theta_i}.$$

3. The spinor and oscillator representations in super setting

3.1. The Lie superalgebra $\mathfrak{hei}(2n|m)$.

Define the Heisenberg Lie superalgebra $\mathfrak{hei}(2n|m)$ as follows. Consider a $(2n+1|m)$-dimensional superspace $W = V \oplus \mathbb{C}\mathbb{1}$, where $p(\mathbb{1}) = \bar{0}$, and let $(\cdot,\cdot)$ be an even nondegenerate skew-symmetric bilinear form on V. Let W be the superspace of $\mathfrak{hei}(2n|m)$ with the bracket

$$[v, w] = (v, w)\mathbb{1}, \quad \text{for } v, w \in V; \ [\mathbb{1}, \mathfrak{hei}] = 0.$$

3.1.1. The big and small Weyl superalgebras. Being primarily interested in *irreducible* representations of $\mathfrak{hei}$, or, equivalently, of its enveloping algebra, we consider not the whole $U(\mathfrak{hei}(2n|m)$ (the "big" Weyl superalgebra) but its quotient (the "small" Weyl superalgebra)

$$U_\hbar(\mathfrak{hei}(2n|m) = U(\mathfrak{hei}(2n|m))/(\mathbb{1} - \hbar) \quad \text{for } \hbar \in \mathbb{C}.$$

The quotient $U_\hbar(\mathfrak{hei}(2n|m)$, called the *superalgebra of observables*, is, by definition, an associative superalgebra isomorphic, for $k = [\frac{m}{2}]$ and m even, to the associative superalgebra of differential operators on the supermanifold $\mathbb{C}^{n|k}$ with polynomial coefficients $\mathrm{diff}(n|k)$ = "Mat"$(\mathbb{C}[q,\xi])$ whereas, for $k = [\frac{m}{2}]$ and m odd, it is isomorphic to (for the definition of the queer analog QMat of the general matrix superalgebra, see, e.g., Ref. 15, 9)

$$\mathrm{qdiff}(n|k) = Q(\text{"Mat"}\mathbb{C}[q,\xi])) = \begin{Bmatrix} x \in \mathfrak{diff}(n|k) \mid [x,\pi] = 0 \text{ for an odd} \\ \pi \text{ such that } \pi^2 = 1. \end{Bmatrix}.$$

(For example, take $\pi = \xi_m + \frac{\partial}{\partial \xi_m}$.) For $n = 0$, the algebras diff and qdiff with super structure ignored are known under the name of *Clifford* algebras.

3.2. Spinor (Clifford–Weil–wedge–. . .) and oscillator representations. Let $\mathfrak{po}(2n|m)$ be the Poisson Lie superalgebra realized on polynomials. As is easy to see, $\mathfrak{po}(2n|m)_0 \cong \mathfrak{osp}(m|2n)$, the superspace of elements of degree 0 in the $\mathbb{Z}$-graded Lie superalgebra or, which is the same, the superspace of quadratic elements in the representation by generating functions. The complete description of deformations of $\mathfrak{po}(2n|m)$ was recently given in Ref. 19: There is only one (class of) deformations: *quantization* $\mathcal{Q}$ and we denote $\mathrm{Spin}(V)$ the module given by the through map[b]

$$\mathfrak{g} \longrightarrow \mathfrak{osp}(m|2n) \cong \mathfrak{po}(2n|m)_0 \xrightarrow{\mathcal{Q}} \begin{cases} \mathfrak{diff}(n|k) := \mathrm{diff}(n|k)_L \\ \mathfrak{qdiff}(n|k) := \mathrm{qdiff}(n|k)_L, \end{cases} \tag{3}$$

We will denote this representation $\mathrm{Spin}(V)$ and set $\mathrm{Osc}(V) := \mathrm{Spin}(\Pi(V))$, where V is the standard representation of $\mathfrak{osp}(m|2n) = \mathfrak{osp}(V)$, and Π is the change of parity. For $n = 0$, it is called the *spinor representation*; for $m = 0$ the *oscillator representation*.

Now, suppose that V is a $\mathfrak{g}$-module without any bilinear form. Then, consider the module $W = V \oplus V^*$ (in the infinite dimensional case, V^* is the *restricted* dual whose elements are sums of finitely many terms, not

[b]The subscript $_L$ makes an associative superalgebra a Lie one replacing the dot product by the supercommutator.

series) endowed with the form (for any $v_1, w_1 \in V$, $v_2, w_2 \in V^*$)

$$B((v_1, v_2), (w_1, w_2)) = v_2(w_1) \pm (-1)^{p(v_1)p(w_2)} w_2(v_1).$$

This form is symmetric for the plus sign and skew-symmetric otherwise.

In the following tables we give the results of calculations of the highest weights — the labels $(c, h; H_1, \dots, H_k)$ with respect to the central elements z; K_t and $K_{\xi_1\eta_1}, \dots, K_{\xi_k\eta_k}$ of the spinor representations $\mathrm{Spin}(\mathcal{F}_{\lambda;\mu})$ of the contact superalgebras. For the oscillator representation $\mathrm{Osc}(\mathcal{F}_{\lambda;\mu}) = \mathrm{Spin}(\Pi(\mathcal{F}_{\lambda;\mu}))$ the values of the highest weight are $(-c, h; H_1, \dots, H_k)$.

Problem. *For which projective representations of $\mathfrak{k}^L(1|N)$ and $\mathfrak{k}^M(1|N)$, where $N = 3, 4$, the values of c are nonzero?* (So far there is only a partial answer[23].)

3.3. The values c and h of the highest weight of the $\mathfrak{k}^L(1|N)$- and $\mathfrak{k}^M(1|N)$-module $\mathrm{Spin}(\mathcal{F}_{\lambda;\mu})$. The labels of the highest weight other than c, h are all 0, except for $\mathfrak{k}^L(1|2)$ and $\mathfrak{k}^M(1|3)$: the highest weight of $\mathrm{Spin}(\mathcal{F}_{\lambda,\nu;\mu})$ is $(c, h; \nu)$.

N	c	h for $\mathfrak{k}^L$	h for $\mathfrak{k}^M$
0	$12\lambda^2 - 12\lambda + 2$	$(\mu + 2\lambda)(\mu + 1)$	$-$
1	$\frac{2}{3}(3 - 12\lambda)$	$\mu + 2\lambda$	$2\mu + 3\lambda - \frac{1}{4}$
2	2	$2\mu + 2\lambda + \nu$	$2\mu + 2\lambda - \frac{1}{2}$
≥ 3	0	$2^{N-1}(\mu + \lambda) + 2^{N-3}$	$2^{N-1}(\mu + \lambda)$

Remark. For the contact superalgebras $\mathfrak{g}$, our choice of $\mathfrak{g}$-modules $V = \mathcal{F}_{\lambda;\mu}$ from which we constructed $\mathrm{Spin}(V \oplus V^*)$ is natural provided we are interested in semi-infinite cohomology of $\mathfrak{g}$. Besides, for n small, all modules of tensor fields are of this form, anyway.

For the superalgebras $\mathfrak{g}$ of series $\mathfrak{vect}$ and $\mathfrak{svect}$, the situation is the opposite one: the adjoint representation $V = \mathfrak{g}$ is of the form $V = \mathcal{T}(\mathrm{id}^*)$.

3.4. The choice of the cocycle. Now, let us fix the cocycle that determines the nontrivial central extensions of the distinguished stringy superalgebras. One choice comes from the study of semi-infinite cohomology; it is very interesting and reasonable for $\mathfrak{vir}$. Another choice is to directly calculate the bracket $[e_i, e_{-i}]$ in the spinor or oscillator representation. We get

$$[e_i, e_{-i}] = -i(2e_0 + \sum_{j\geq 1} j \cdot z) = -2ie_0 - \frac{1}{12}z. \qquad (4)$$

Nowadays such infinite sums seem meaningless (or equal to ∞) to most, but less than a century ago every student who took Calculus knew a way to compute (4): evaluate the Riemann ζ-function $\zeta(s) = \sum\limits_{n\geq 1} \frac{1}{n^s}$ at $s = -1$.

For other distinguished stringy superalgebras $\mathfrak{g}$, physicists traditionally consider the cocycles whose restriction to the subalgebra $\mathfrak{witt} \subset \mathfrak{g}$ coincides with the fixed cocycle that determines $\mathfrak{vir}$. The trouble is that there is no canonical embedding $\mathfrak{witt} \subset \mathfrak{g}$ for $\mathfrak{g} \not\cong \mathfrak{k}^L(1)$ or $\mathfrak{k}^M(1)$.

Problems. *Which cocycle to choose? In the following Theorem, can one consider* $\mathrm{Osc}(\mathfrak{g})$ *(so CDs are positive) instead of* $\mathrm{Spin}(\mathfrak{g})$*?*

3.5. Theorem. *For* $\mathfrak{vect}^L(1|2)$*, CD is equal to* -1*; for each* $\mathfrak{svect}^L_\lambda(1|2)$*, CD is equal to* -2*.*

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