Max-Planck-Institut
für Mathematik
in den Naturwissenschaften
Leipzig

Random perturbations of spiking activity in a pair of coupled neurons

by

Boris Gutkin, Jürgen Jost, and Henry Tuckwell

Preprint no.: 49 2007

Random perturbations of spiking activity in a pair of coupled neurons

Boris Gutkin*, Jürgen Jost and Henry C. Tuckwell†

May 14, 2007

Abstract

We examine the effects of stochastic input currents on the firing behaviour of two coupled Type 1 or Type 2 neurons. In Hodgkin-Huxley model neurons with standard parameters, which are Type 2, in the bistable regime, synaptic transmission can initiate oscillatory joint spiking, but white noise can terminate it. In Type 1 cells (models), typified by a quadratic integrate and fire model, synaptic coupling can cause oscillatory behaviour in excitatory cells, but Gaussian white noise can again terminate it. We locally determine an approximate basin of attraction, $\mathcal{A}$, of the periodic orbit and explain the firing behaviour in terms of the effects of noise on the probability of escape of trajectories from $\mathcal{A}$.

1 Introduction

Hodgkin (1948) found that various squid axon preparations responded in qualitatively different ways to applied currents. Some preparations gave a frequency of firing which rose smoothly from zero as the current increased whereas others manifested the sudden appearance of a train of spikes at a particular input current. Cells that responded in the first manner were called Class 1 (which we refer to as Type 1) whereas cells with a discontinuous frequency-current curve were called Class 2 (Type 2). Mathematical explanations for the two types are found in the bifurcation which accompanies the transition from rest state to a periodic firing mode. For Type 1 behaviour, a resting potential vanishes via a saddle-node bifurcation whereas for Type 2 behaviour the instability of the rest point is due to an Andronov-Hopf bifurcation, see Rinzel and Ermentrout (1989).

Stochastic effects in the firing behaviour of neurons have been widely reported, discussed and analyzed since their discovery in the 1940's. One of the first reports for the central nervous system was by Frank and Fuortes (1955) for cat

*Group for Neural Theory, Departement des Etudes Cognitives, Ecole Normale Supérieure, 3 rue d'Ulm, 75005 Paris, France

†Max Planck Institute for Mathematics in the Sciences, Inselstr. 22, 04103 Leipzig, Germany

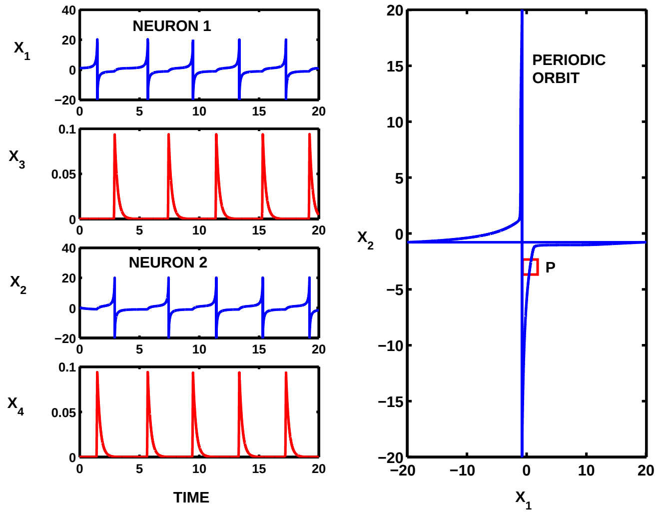

Figure 1: On the left are shown the solutions of (1)-(4) for two coupled QIF model neurons with the standard parameters. X_1 and X_2 are the potential variables of neurons 1 and 2 and X_3 and X_4 are the inputs to neurons 1 and 2, respectively. On the right is shown the periodic orbit in the (x_1, x_2)-plane. The square marked P was explored in detail in reference to the extent of the basin of attraction of the periodic orbit.

spinal neurons. Although there have been many single neuron studies, the effect of noise on systems of coupled neurons have not been extensively investigated. Some preliminary studies are those of Gutkin, Hely and Jost (2004) and Casado and Baltanás (2003).

2 The quadratic integrate and fire model

A relatively simple neural model which exhibits Type 1 firing behaviour is the quadratic integrate and fire (QIF) model. We couple two model neurons in the following manner (Gutkin, Hely and Jost, 2004).
Let $\{X_1(t), X_2(t), t \geq 0\}$ be the depolarizations of neurons 1 and 2, where t is the time index. Then the model equations are, for subthreshold states of two identical neurons,

$$dX_1 = [(X_1 - x_R)^2 + \beta + g_s X_3]dt + \sigma dW_1 \quad (1)$$

$$dX_2 = [(X_2 - x_R)^2 + \beta + g_s X_4]dt + \sigma dW_2 \quad (2)$$

$$dX_3 = -\frac{X_3}{\tau} + F(X_2) \quad (3)$$

$$dX_4 = -\frac{X_4}{\tau} + F(X_1) \quad (4)$$

where X_3 is the synaptic input to neuron 1 from neuron 2 and X_4 is the synaptic input to neuron 2 from neuron 1. The quantity x_R is a resting value. g_s is the coupling strength. β is the mean background input. W_1 and W_2 are independent standard Wiener processes which enter with strength σ. This term may model variations in nonspecific inputs to the circuit as well as possibly intrinsic membrane and channel noise. By construction, we take this term to be much weaker than the mutual coupling between the cells in our circuit. The function F is given by

$$F(x) = 1 + \tanh(\alpha(x - \theta))$$

where θ characterizes the threshold effect of synaptic activation. Since when a QIF neuron is excited and it receives no inhibition, its potential reaches an infinite value in a finite time, for numerical simulations a cutoff value x_{max} is introduced so that the above model equations for the potential apply only if X_1 or X_2 are below x_{max}. To complete a "spike" in any neuron, taken as occurring when its potential reaches x_{max}, its potential is instantaneously reset to some value x_{reset} which may be taken as $-x_{max}$. At the bifurcation point $g_s = g_s^*$, two heteroclinic orbits between unstable rest points turn into a periodic orbit of antiphase oscillations.

3 Results and theory

In the numerical work, the following constants are employed throughout. $x_R = 0$, $x_{max} = 20$, $\theta = 10$, $\alpha = 1$, $\beta = -1$, $g_s = 100$ and $\tau = 0.25$. The initial values of the neural potentials are $X_1(0) = 1.1$, $X_2(0) = 0$ and the initial values of the synaptic variables are $X_3(0) = X_4(0) = 0$.

When there is no noise, $\sigma = 0$, the results of Figure 1 are obtained. The spike trains of the two coupled neurons and their synaptic inputs are shown on the left. The firing settles down to be quite regular and the periodic orbit, S, is shown on the right. The patch marked P is the location of the region explored in detail below.

The effects of a small amount of noise are shown in Figure 2. The neural excitation variables are shown on the left and the corresponding trajectories in the (x_1, x_2)- plane are shown on the right. In the top portion an example of the trajectory for $\sigma = 0.1$ is shown. Here three spikes arise in neuron 1 and two in neuron 2, but the time between spikes increases and eventually the orbit collapses away from the periodic orbit. In the example (lower part) for $\sigma = 0.2$ there are no spikes in either neuron. In 10 trials, the average numbers of spikes obtained for the pair of neurons were $(2.5, 2.2)$ for $\sigma = 0.1$, $(1.4, 1.1)$ for $\sigma = 0.2$ and $(1.3, 0.9)$ for $\sigma = 0.3$; these may be compared with $(5, 5)$ for zero noise.

3.1 Exit-time and orbit stability

If a basin of attraction for a periodic orbit can be found, then the probability that the process with noise escapes from the region of attraction gives the probability, in the present context, that spiking will cease. Since the system

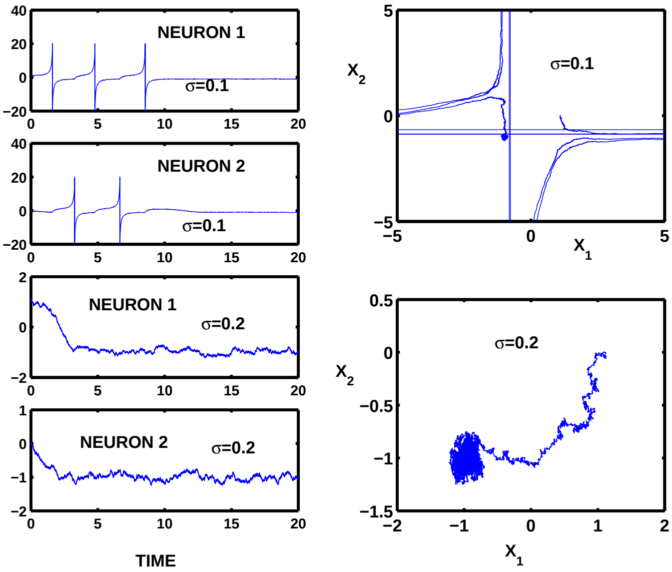

Figure 2: On the left are shown examples of the neuronal potentials for neurons 1 and 2 (QIF model) for two values of the noise, $\sigma = 0.1$ and $\sigma = 0.2$. On the right are shown the trajectories corresponding to the results on the left, showing how noise pushes or keeps the trajectories out of the basin of attraction of the periodic orbit.

(1)-(4) is Markovian, we may apply standard first-exit time theory (Tuckwell, 1989). Letting A be a set in R^4 and letting $\mathbf{x} = (x_1, x_2, x_3, x_4) \in A$ be a values of $X_1, X_2, X_3, X_4)$ at some given time, the probability $p(x_1, x_2, x_3, x_4)$ that the process ever escapes from A is given by

$$\begin{aligned} \mathcal{L}p \quad \equiv \quad & \frac{\sigma^2}{2}\frac{\partial^2 p}{\partial x_1^2} + \frac{\sigma^2}{2}\frac{\partial^2 p}{\partial x_2^2} \\ + \quad & [(x_1 - x_R)^2 + \beta + g_s x_3]\frac{\partial p}{\partial x_1} + [(x_2 - x_R)^2 + \beta + g_s x_4]\frac{\partial p}{\partial x_2} \\ + \quad & \big(F(x_2) - \frac{x_3}{\tau}\big)\frac{\partial p}{\partial x_3} + \big(F(x_1) - \frac{x_4}{\tau}\big)\frac{\partial p}{\partial x_4} = 0, \qquad \mathbf{x} \in \mathcal{A} \end{aligned} \tag{5}$$

with boundary condition that $p = 1$ on the boundary of $\mathcal{A}$ (since the process is continuous). If one also adds an arbitrarily small amount of noise for X_3 and X_4 (or considers those solutions of (5) that arise from the limit of vanishing noise for X_3, X_4), the solution of the linear elliptic partial differential equation (5) is unique and $\equiv 1$, that is, the process will eventually excape from $\mathcal{A}$ with probability 1. Hence, the expected time $f(\mathbf{x})$ of exit of the process from $\mathcal{A}$ satisfies $\mathcal{L}f = -1, \quad \mathbf{x} \in \mathcal{A}$ with boundary condition $f = 0$ on the boundary of $\mathcal{A}$. In fact, for small noise, the logarithm of the expected exit time from $\mathcal{A}$, that

is, the time at which firing stops, behaves like the inverse of the square of the noise amplitude (Freidlin and Wentzell, 1998).
These linear partial differential equations can be solved numerically, for example by Monte-Carlo techniques. The basin of attraction $\mathcal{A}$ must be found in order to identify the domain of (5). We have done this approximately for the square P in Figure 1. The effects of perturbations of the periodic orbit S within P on the spiking activity were found by solving (1)-(4) with various initial conditions in the absence of noise. The values of x_1 were from -0.43 to 1.57 in steps of 0.2 and the values of x_2 were from -4 to 2 also in steps of 0.2. For this particular region, as expected from geometrical considerations, the system responded sensitively to to variations in x_1 but not x_2. For example, to the left of S there tended to be no spiking activity whereas just to the right there was a full complement of spikes and further to the right (but still inside P) one spike.

4 Coupled Hodgkin-Huxley neurons

As an example of a Type 2 neuron, we use the standard Hodgkin-Huxley (HH) model augmented with synaptic input variables as in the model for coupled QIF neurons given by equations (3) and (4), but with different parameter values. It has been long known that additive noise has a facilitative effect on single HH neurons (Yu and Lewis, 1989). Coupled pairs of HH neurons have been employed with a different approach using conductance noise in order to analyze synchronization properties (e.g. Casado and Baltánas, 2003). For the present approach, with X_1 and X_2 as the depolarizations of the two cells, we put

$$dX_1 = \frac{1}{C}[\beta+\overline{g}_K n^4(V_K-X_1)+\overline{g}_{Na}m^3h(V_{Na}-X_1)+g_l(V_l-X_1)]dt+g_sX_3dt+\sigma dW_1$$

with a similar equation for X_2. The equations for the auxiliary variables are standard and all parameters associated with the Hodgkin-Huxley equations were the standard values.
When each cell is in the oscillatory regime, synaptic connections can modulate the spike rate, and noise can speed it up. In the bistable regime, when each cell has a stable rest point as well as a stable periodic spiking orbit, synapses can initiate or augment joint oscillatory spiking, but noise can stop it. This can be seen in simulations and deduced from the qualitative theory of dynamical systems.

5 Discussion

We have studied the effect of noise in systems of two coupled neurons of Type 1 and Type 2. In Type 1, we found a generic effect, because quadratic integrate and fire neurons represent a generic model. That effect is that while coupling can cause asynchronous oscillatory activity in excitable neurons, noise can terminate that sustained spiking (near to the bifurcation point where the asynchronous periodic orbit emerges). This circuit model is a stochastic analogue of the deterministic case previously studied by Gutkin et al. (2001) where it was found that transient synchronization can terminate sustained activity. For Type 2 neurons, we have investigated coupled Hodgkin-Huxley neurons and found that in the bistable regime, noise can again terminate sustained spiking activity initiated by synaptic connections. We have investigated a minimal circuit model of sustained neural activity. Such sustained activity in the prefrontal cortex has been proposed as a neural correlate of working memory (Fuster and Alexander, 1973).

References

Casado,J.M., Baltánas,J.P. (2003). Phase switching in a system of two noisy Hodgkin-Huxley neurons coupled by a diffusive interaction. *Phys. Rev. E* **68**, 061917,

Frank,K., Fuortes, M.G. (1955). Potentials recorded from the spinal cord with microelectrodes, *J.Physiol.* **130**, 625-654.

Freidlin,M.I., Wentzell,A.D. (1998), *Random Perturbations of Dynamical Systems*, 2nd ed., Springer, New York Fuster, J.M. and Alexander, G.E. (1971), Neuron activity related to short-term memory. *Science* **652-654**

Gutkin,B., Ermentrout,G.B. (1998). Dynamics of membrane excitability determine interval variability: a link between spike generation mechanisms and cortical spike train statistics. *Neural Comp.* **10**, 1047-1065.

Gutkin,B.S. et al. (2001) Turning on and off with excitation. *J. Comp. Neurosc.* **11:2**, 121-134

Gutkin,B., Hely,T., Jost,J. (2004). Noise delays onset of sustained firing in a minimal model of persistent activity. *Neurocomputing* **58-60**, 753-760.

Hodgkin,A.L. (1948). The local changes associated with repetitive action in a non-medullated axon. *J. Physiol.* **107**, 165-181.

Rinzel,J., Ermentrout,G. B. (1989). Analysis of neural excitability and oscillations; in: Koch C. & Segev I., eds. MIT Press.

Tateno,T., Harsch,A., Robinson,H.P.C. (2004). Threshold firing frequency-current relationships of neurons in rat somatosensory cortex: Type 1 and Type 2 dynamics. *J. Neurophysiol.* **92**, 2283-2294.

Tuckwell,H.C. (1989). *Stochastic Processes in the Neurosciences.* SIAM, Philadelphia.

Yu,X., Lewis,E.R. (1989). Studies with spike initiators: linearization by noise allows continuous signal modulation in neural networks. *IEEE Trans. Biomed. Eng.* **36**, 36-43.