Max-Planck-Institut
für Mathematik
in den Naturwissenschaften
Leipzig

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by

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Preprint no.: 89 2008

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## Abstract

We study quantum states for which the PPT criterion is both sufficient and necessary for separability. We present a class of $3 \times 3$ bipartite mixed states and show that these states are separable if and only if they are PPT.

Keywords: PPT, Entanglement, Separability

PACS numbers: 03.65.Bz, 89.70.+c

Quantum entanglement has been recently the subject of much study as a potential resource for communication and information processing [1]. Thus characterization and quantification of entanglement become an important issue. Entanglement of formation (EOF) [2] and concurrence [3] are two well defined quantitative measures of quantum entanglement. For two-quibt systems it has been proved that EOF is a monotonically increasing function of the concurrence and an elegant formula for the concurrence was derived analytically by Wootters [4]. However with the increasing dimensions of the subsystems the computation of EOF and concurrence becomes formidably difficult. A few explicit analytic formulae for EOF and concurrence have been found only for some special symmetric states [5].

In fact if one only wants to know wether a state is separable or not, it is not necessary to compute the exact values of the measures for quantum entanglement. The estimation of lower bounds of entanglement measures can be just used to judge the separability of a quantum state [6]. There are also many separability criteria, e.g., PPT (positive partial transposition) criterion [7], realignment [8] and generalized realignment criteria [9], as well as some necessary and sufficient operational criteria for low rank density matrices [10]. Further more, separability criteria based on local uncertainty relation [11] and the correlation matrix [12] of the Bloch representation for a quantum state have been derived, which are strictly stronger than or independent of the PPT and realignment criteria.

The PPT criterion is generally a necessary condition for separability. It becomes sufficient only for the cases $2 \times 2$ and $2 \times 3$ bipartite systems [13]. Other states of such property are the Schmidt-correlated (SC) states [14], which are the mixtures of pure states sharing the same Schmidt bases and naturally appear in a bipartite system dynamics with additive integrals of motion [15]. In this paper we consider another special class of $3 \times 3$ quantum mixed states. We show that for the states in this class, PPT is both necessary and sufficient for separability.

We consider $3 \times 3$ quantum mixed states given by

$$\rho = \lambda|X >< X| + \lambda'|X' >< X'| + \lambda''|X'' >< X''|, \tag{1}$$

where $\lambda + \lambda' + \lambda'' = 1$, $0 < \lambda, \lambda', \lambda'' < 1$, $|X >, |X' >, |X'' >$ are orthonormal vectors,

$$\begin{array}{l} |X >= (\alpha, 0, 0, 0, \beta, 0, 0, 0, \gamma)^t, \\ |X' >= (0, \alpha', 0, 0, 0, \beta', \gamma', 0, 0)^t, \\ |X'' >= (0, 0, \alpha'', \beta'', 0, 0, 0, \gamma'', 0)^t, \end{array} \tag{2}$$

where $\alpha, \beta, \gamma, \alpha', \beta', \gamma'\alpha'', \beta'', \gamma'' \in \mathbb{C}$, $t$ stands for transposition.

Theorem: State $\rho$ is separable if and only if it is PPT.

To prove the theorem we first note that after partial transposition $\rho$ has the form

$$
\rho^{pt} =
\left(\begin{array}{ccccccccc}
\lambda\alpha\alpha^* & 0 & 0 & 0 & 0 & \lambda''\alpha''\beta''^* & 0 & \lambda'\alpha'\gamma'^* & 0 \\
0 & \lambda'\alpha'\alpha'^* & 0 & \lambda\alpha\beta^* & 0 & 0 & 0 & 0 & \lambda''\alpha''\gamma''^* \\
0 & 0 & \lambda''\alpha''\alpha''^* & 0 & \lambda'\alpha'\beta'^* & 0 & \lambda\alpha\gamma^* & 0 & 0 \\
0 & \lambda\alpha^*\beta & 0 & \lambda''\beta''\beta''^* & 0 & 0 & 0 & 0 & \lambda'\beta'\gamma'^* \\
0 & 0 & \lambda'\alpha'^*\beta' & 0 & \lambda\beta\beta^* & 0 & \lambda''\beta''\gamma''^* & 0 & 0 \\
\lambda''\alpha''^*\beta'' & 0 & 0 & 0 & 0 & \lambda'\beta'\beta'^* & 0 & \lambda\beta\gamma^* & 0 \\
0 & 0 & \lambda\alpha^*\gamma & 0 & \lambda''\beta''^*\gamma'' & 0 & \lambda'\gamma'\gamma'^* & 0 & 0 \\
\lambda'\alpha'^*\gamma' & 0 & 0 & 0 & 0 & \lambda\beta^*\gamma & 0 & \lambda''\gamma''\gamma''^* & 0 \\
0 & \lambda''\alpha''^*\gamma'' & 0 & \lambda'\beta'^*\gamma' & 0 & 0 & 0 & 0 & \lambda\gamma\gamma^*
\end{array}\right).
$$

$\rho^{pt}$ is hermitian. The non-negativity of $\rho^{pt}$, $\rho^{pt} \geq 0$, implies that $<\psi|\rho^{pt}|\psi> \geq 0$ for all vector $|\psi> \in H \otimes H$, which is equivalent to the non-negativity of the following three $3 \times 3$ matrices:

$$
A_1 = \left(\begin{array}{ccc}
\lambda\alpha\alpha^* & \lambda''\alpha''\beta''^* & \lambda'\alpha'\gamma'^* \\
\lambda''\alpha''^*\beta'' & \lambda'\beta'\beta'^* & \lambda\beta\gamma^* \\
\lambda'\alpha'^*\gamma' & \lambda\beta^*\gamma & \lambda''\gamma''\gamma''^*
\end{array}\right), \tag{3}
$$

$$
A_2 = \left(\begin{array}{ccc}
\lambda'\alpha'\alpha'^* & \lambda\alpha\beta^* & \lambda''\alpha''\gamma''^* \\
\lambda\alpha^*\beta & \lambda''\beta''\beta''^* & \lambda'\beta'\gamma'^* \\
\lambda''\alpha''^*\gamma'' & \lambda'\beta'^*\gamma' & \lambda\gamma\gamma^*
\end{array}\right), \tag{4}
$$

and

$$
A_3 = \left(\begin{array}{ccc}
\lambda''\alpha''\alpha''^* & \lambda'\alpha'\beta'^* & \lambda\alpha\gamma^* \\
\lambda'\alpha'^*\beta' & \lambda\beta\beta^* & \lambda''\beta''\gamma''^* \\
\lambda\alpha^*\gamma & \lambda''\beta''^*\gamma'' & \lambda'\gamma'\gamma'^*
\end{array}\right). \tag{5}
$$

The non-negativity of $A_1$ is equivalent to the following inequalities:

$$
\lambda\lambda'|\alpha|^2|\beta'|^2 \geq \lambda''^2|\alpha''|^2|\beta''|^2, \tag{6}
$$

$$
\lambda\lambda''|\alpha|^2|\gamma'''|^2 \geq \lambda'^2|\alpha'|^2|\gamma'|^2, \tag{7}
$$

$$
\lambda'\lambda''|\beta'|^2|\gamma''|^2 \geq \lambda^2|\gamma|^2|\beta|^2 \tag{8}
$$

and

$$
\begin{array}{l}
\lambda\lambda'\lambda''|\alpha|^2|\beta'|^2|\gamma''|^2 + 2\lambda\lambda'\lambda'' Re\alpha'\alpha''^*\beta''\beta^*\gamma\gamma'^* - \lambda^3|\alpha|^2|\beta|^2|\gamma|^2 \\
-\lambda'^3|\alpha'|^2|\beta'|^2|\gamma'|^2 - \lambda''^3|\alpha''|^2|\beta''|^2|\gamma''|^2 \geq 0.
\end{array} \tag{9}
$$

Similarly the non-negativity of $A_2$ and $A_3$ give rise to

$$\lambda'\lambda''|\alpha'|^2|\beta''|^2 \geq \lambda^2|\alpha|^2|\beta|^2, \tag{10}$$

$$\lambda\lambda'|\alpha'|^2|\gamma|^2 \geq \lambda''^2|\alpha''|^2|\gamma''|^2, \tag{11}$$

$$\lambda\lambda''|\gamma|^2|\beta''|^2 \geq \lambda'^2|\gamma'|^2|\beta'|^2, \tag{12}$$

$$\begin{aligned}&\lambda\lambda'\lambda''|\alpha'|^2|\beta''|^2|\gamma|^2 + 2\lambda\lambda'\lambda'' Re\alpha\alpha''^*\beta'\beta^*\gamma''\gamma'^* - \lambda^3|\alpha|^2|\beta|^2|\gamma|^2\\ &-\lambda'^3|\alpha'|^2|\beta'|^2|\gamma'|^2 - \lambda''^3|\alpha''|^2|\beta''|^2|\gamma''|^2 \geq 0,\end{aligned} \tag{13}$$

and

$$\lambda\lambda''|\alpha''|^2|\beta|^2 \geq \lambda'^2|\alpha'|^2||\beta'|^2, \tag{14}$$

$$\lambda'\lambda''|\alpha''|^2|\gamma'|^2 \geq \lambda^2|\alpha|^2|\gamma|^2, \tag{15}$$

$$\lambda\lambda'|\beta|^2|\gamma'|^2 \geq \lambda''^2|\beta''|^2|\gamma''|^2, \tag{16}$$

$$\begin{aligned}&\lambda\lambda'\lambda''|\alpha''|^2|\beta|^2|\gamma'|^2 + 2\lambda\lambda'\lambda'' Re\alpha'\alpha^*\beta'^*\beta''\gamma''^*\gamma - \lambda^3|\alpha|^2|\beta|^2|\gamma|^2\\ &-\lambda'^3|\alpha'|^2|\beta'|^2|\gamma'|^2 - \lambda''^3|\alpha''|^2|\beta''|^2|\gamma''|^2 \geq 0.\end{aligned} \tag{17}$$

We can show that the inequalities (6)-(8), (10)-(12), (14)-(16) are equalities. In fact if (6) is an inequality, $\lambda\lambda'|\alpha|^2|\beta'|^2 > \lambda''^2|\alpha''|^2|\beta''|^2$, then from (10) and (14), one would have, by multiplying $\lambda'|\alpha'|^2$ on both sides, $\lambda'|\alpha'|^2\lambda''^2|\alpha''|^2|\beta''|^2 < \lambda\lambda'^2|\alpha|^2|\alpha'|^2|\beta'|^2 \leq \lambda\lambda''|\alpha''||\beta|^2\lambda|\alpha|^2 \leq \lambda'|\alpha'|^2\lambda''^2|\alpha''|^2|\beta''|^2$, which contradicts. Therefore (6)-(8), (10)-(12), (14)-(16) become

$$\sqrt{\lambda\lambda'}\alpha\beta' = \lambda''\alpha''\beta''e^{i\theta_1''},\ \sqrt{\lambda\lambda''}\alpha\gamma'' = \lambda'\alpha'\gamma'e^{i\theta_2'},\ \sqrt{\lambda''\lambda'}\gamma''\beta' = \lambda\gamma\beta e^{i\theta_3} \tag{18}$$

$$\sqrt{\lambda''\lambda'}\alpha'\beta'' = \lambda\alpha\beta e^{i\theta_1},\ \sqrt{\lambda\lambda'}\alpha'\gamma = \lambda''\alpha''\gamma''e^{i\theta_2''},\ \sqrt{\lambda\lambda''}\gamma\beta'' = \lambda'\gamma'\beta'e^{i\theta_3'} \tag{19}$$

$$\sqrt{\lambda\lambda''}\alpha''\beta = \lambda'\alpha'\beta'e^{i\theta_1'},\ \sqrt{\lambda''\lambda'}\alpha''\gamma' = \lambda\alpha\gamma e^{i\theta_2},\ \sqrt{\lambda\lambda'}\gamma'\beta = \lambda''\gamma''\beta''e^{i\theta_3''}, \tag{20}$$

where all $\theta \in [-\pi, \pi]$. From (18)-(20), (9), (13) and (17) become

$$Re(\alpha'\alpha''^*\beta''\beta^*\gamma\gamma'^*) \geq |\alpha|^2|\beta'|^2|\gamma''|^2, \tag{21}$$

$$Re(\alpha\alpha''^*\beta'\beta^*\gamma''\gamma'^*) \geq |\alpha'|^2|\beta''|^2|\gamma|^2, \tag{22}$$

$$Re(\alpha'\alpha^*\beta'^*\beta''\gamma''^*\gamma) \geq |\alpha''|^2|\beta|^2|\gamma'|^2. \tag{23}$$

Applying these 9 equalities one obtains also

$$\begin{aligned}|\alpha'\alpha''^*\beta''\beta^*\gamma\gamma'^*| &\leq |\alpha'||\alpha''||\beta''||\beta||\gamma||\gamma'|\\ = \frac{\lambda^2}{\lambda'\lambda''}|\alpha||\beta||\alpha||\gamma||\beta||\gamma| &= |\alpha|^2|\beta'|^2|\gamma''|^2.\end{aligned} \tag{24}$$

Similarly one has

$$|\alpha\alpha''^*\beta'\beta^*\gamma''\gamma'^*| \leq |\alpha'|^2|\beta''|^2|\gamma|^2, \quad |\alpha'\alpha^*\beta'^*\beta''\gamma''^*\gamma| \leq |\alpha''|^2|\beta|^2|\gamma'|^2. \tag{25}$$

By using (18)-(20), we get

$$Re(\alpha'\alpha''^*\beta''\beta^*\gamma\gamma'^*) = \alpha'\alpha''^*\beta''\beta^*\gamma\gamma'^* = |\alpha|^2|\beta'|^2|\gamma''|^2, \tag{26}$$

$$Re(\alpha\alpha''^*\beta'\beta^*\gamma''\gamma'^*) = \alpha\alpha''^*\beta'\beta^*\gamma''\gamma'^* = |\alpha'|^2|\beta''|^2|\gamma|^2, \tag{27}$$

$$Re(\alpha'\alpha^*\beta'^*\beta''\gamma''^*\gamma) = \alpha'\alpha^*\beta'^*\beta''\gamma''^*\gamma = |\alpha''|^2|\beta|^2|\gamma'|^2. \tag{28}$$

From (26) and (27), we have

$$\frac{\alpha'\beta''\gamma}{\alpha\beta'\gamma''} = \frac{|\alpha|^2|\beta'|^2|\gamma''|^2}{|\alpha'|^2|\beta''|^2|\gamma|^2} = \frac{\lambda^2|\alpha|^2 \cdot \frac{1}{\lambda'\lambda''}|\beta|^2|\gamma|^2}{|\alpha'|^2|\beta''|^2|\gamma|^2} = 1.$$

While from (26) and (28), we have

$$(\frac{\alpha''\beta\gamma'}{\alpha\beta'\gamma''})^* = 1.$$

Therefore

$$\alpha'\beta''\gamma = \alpha\beta'\gamma'' = \alpha''\beta\gamma'$$

and, from (18)-(20),

$$\theta_1 = \theta_2 = \theta_3 \equiv \theta \ , \theta_1' = \theta_2' = \theta_3' \equiv \theta' \ , \theta_1'' = \theta_2'' = \theta_3'' \equiv \theta''.$$

Now by using these PPT conditions of $\rho$ we prove that $\rho$ has a pure state decomposition $\rho = \sum_{l=1}^{l=3} |\psi_l >< \psi_l|$ such that all states $|\psi_l >$, $l = 1, 2, 3$, are separable. $|\psi_l >$ can be generally expressed as $|\psi_l >= \sum_m^3 U_{ml}|X_m >= \sum_{ij}^3 a_{ij}^l|ij >$ for some $a_{ij}^l \in \mathbb{C}$ under some basis $|ij >$, where $U_{ml}$ are the entries of a $3 \times 3$ unitary matrix $U$, $|X_1 >= \sqrt{\lambda}|X >$, $|X_2 >= \sqrt{\lambda'}|X' >$, $|X_3 >= \sqrt{\lambda''}|X'' >$. We denote $A_l$ the $3 \times 3$ matrix with entries $a_{ij}^l$. Suppose the matrix $U$ has the following form

$$U = \begin{pmatrix} u_1 & u_2 & u_3 \\ u_1' & u_2' & u_3' \\ u_1'' & u_2'' & u_3'' \end{pmatrix} = \begin{pmatrix} c_1e^{i\theta} & c_2e^{i\theta} & c_3e^{i\theta} \\ c_1'e^{i\theta'} & c_2'e^{i\theta'} & c_3'e^{i\theta'} \\ c_1''e^{i\theta''} & c_2''e^{i\theta''} & c_3''e^{i\theta''} \end{pmatrix}, \tag{29}$$

where according to the unitary condition of $U$,

$$\sum_{l=1}^{l=3} c_lc_l^* = \sum_{l=1}^{l=3} c_l'c_l'^* = \sum_{l=1}^{l=3} c_l''c_l''^* = 1, \ \sum_{l=1}^{l=3} c_lc_l'^* = 0, \ \sum_{l=1}^{l=3} c_lc_l''^* = 0, \ \sum_{l=1}^{l=3} c_l'c_l''^* = 0. \tag{30}$$

Then $A_l$, $l=1,2,3$, has the following form

$$A_l = \begin{pmatrix} \sqrt{\lambda}\alpha u_l & \sqrt{\lambda'}\alpha' u_l' & \sqrt{\lambda''}\alpha'' u_l'' \\ \sqrt{\lambda''}\beta'' u_l'' & \sqrt{\lambda}\beta u_l & \sqrt{\lambda'}\beta' u_l' \\ \sqrt{\lambda'}\gamma' u_l' & \sqrt{\lambda''}\gamma'' u_l'' & \sqrt{\lambda}\gamma u_l \end{pmatrix}. \tag{31}$$

It is straightforward to verify that the matrix $A_l$ has rank one if

$$c_l^2 e^{i2\theta} = c_l' c_l''. \tag{32}$$

As $0 < rank(A_l A_l^+) \leq rank(A_l) rank(A_l^+)$, if the rank of $A_l$ is one, matrix $A_l A_l^+$ has also rank one and $|\psi_l>$ is separable. Therefore if we can find $c_l$, $c_l'$, $c_l''$, $l=1,2,3$, satisfying the unitary condition (30) and the rank one condition (32), then $\rho = \lambda|X><X| + \lambda'|X'><X'| + \lambda''|X''><X''|$ has separable pure state decomposition, $\rho = \sum_{l=1}^{l=3} |\psi_l><\psi_l|$ and $\rho$ is then separable if it is PPT. We show now that there exit such coefficients $c_i$, $c_i'$, $c_i''$, $i=1,2,3$, satisfying both the unitary condition and the rank-one condition. Set $c_l = \frac{1}{\sqrt{3}} e^{i\varphi_l}$, $c_l' = \frac{1}{\sqrt{3}} e^{i\varphi_l'}$, $c_l'' = \frac{1}{\sqrt{3}} e^{i\varphi_l''}$, $l=1,2,3$, with $\varphi_l$, $\varphi_l'$, $\varphi_l''$, $l=1,2,3$, satisfying

$$\varphi_1 - \varphi_1' = \xi',\ \varphi_2 - \varphi_2' = \xi' + \frac{2\pi}{3},\ \varphi_3 - \varphi_3' = \xi' - \frac{2\pi}{3},$$

$$\varphi_1 - \varphi_1'' = \xi'' + \frac{2\pi}{3},\ \varphi_2 - \varphi_2'' = \xi'',\ \varphi_3 - \varphi_3'' = \xi'' - \frac{2\pi}{3}.$$

Then the unitary conditions (30) are satisfied.

The rank-one conditions require that $-2\varphi_l + \varphi_l' + \varphi_l'' = 2\theta$, $-2\varphi_l' + \varphi_l + \varphi_l'' = 2\theta'$, $-2\varphi_l'' + \varphi_l + \varphi_l' = 2\theta''$, which can be realized by simply choosing $\xi' = \frac{2}{3}\theta' - \frac{2}{3}\theta$, $\xi'' = -\frac{2}{3}\theta' - \frac{4}{3}\theta - \frac{2\pi}{3}$. Therefore if state $\rho$ is PPT, then it is separable. In fact as there are still free parameters $\varphi_l$, $\varphi_l'$, $\varphi_l''$, $l=1,2,3$, therefore there exit many separable pure state decompositions.

We have studied a special kind of bipartite quantum mixed states. For this class of states, the PPT criterion is both sufficient and necessary for separability. Here the states we concerned are rank three ones on $3 \times 3$ bipartite systems. It has been shown that any bipartite entangled states of rank three are distillable [16], that is, there is no rank three bipartite bound entangled states. Therefore if the state $\rho$ is not PPT, i.e. conditions

$$\frac{\lambda}{\sqrt{\lambda'\lambda''}}|\alpha\beta\gamma| = \frac{\lambda'}{\sqrt{\lambda\lambda''}}|\alpha'\beta'\gamma'| = \frac{\lambda''}{\sqrt{\lambda\lambda'}}|\alpha''\beta''\gamma''|$$

are not satisfied, $\rho$ must be not only entangled, but also distillable. This gives an example that a separable state could directly become a distillable state when some parameters varies continuously. There could be no bound entangled states between separable states and distillable states. Above all, with a similar construction of states (2), rank $2k+1$ states on $(2k+1)\times(2k+1)$, $k \in \mathbb{N}$, bipartite system can be obtained. Analogous investigations could be applied to get similar results.

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