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**Entanglement property and monogamy relation of generalized mixed states**

by


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# Entanglement property and monogamy relation of generalized mixed $W$ states

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We have introduced a new class of multipartite entangled mixed states with pure state decompositions of generalized $W$ states, similar to SC states having generalized GHZ states in the pure state decomposition. The entanglement and separability properties are studied according to PPT operations. Monogamy relations related to these states are also investigated.

*Keywords*: generalized mixed $W$ state; entanglement; monogamy relation

## I. INTRODUCTION

Entanglement is a striking feature of quantum systems and responsible for many quantum tasks such as teleportation, dense coding, key distribution, error correction etc. [1], which has provided a strong motivation for the study of detection and quantification of entanglement. There have been many results related to separability criterion and entanglement measures whose effectiveness depends on detailed quantum states. For instance PPT criterion [2, 3] detects many entangled states but not bound entangled ones, while the realignment criterion does [4]. The entanglement of formation [5] and concurrence [6] are two well defined quantitative measures of quantum entanglement. However the entanglement of formation and concurrence have only explicit analytical results for some special quantum states such as Werner states and isotropic states [7].

For multipartite case there are two well known classes of pure states, the GHZ and W states. They are shown to be robust against external flux fluctuations for feasible experimental realizations [8] and the related fidelity can be determined with an effort increasing only linearly with the number of qubits [9]. Two-party and three-party quantum teleportation with GHZ state has been discussed. The W state can be also used as quantum channels for perfect two-party teleportation [10] and quantum key distribution [11]. In [12] the entanglement dynamics of GHZ state and W state have been monitored under different models of system-environment interaction and an exponential decay of entanglement as a function of time has been obtained. In [13] a protocol has been presented for distilling maximally entangled bipartite states between random pairs of parties from those sharing a tripartite

W state. Various experiments have been set up in the literature for generating three-qubit GHZ and W states by applying optical systems, nuclear magnetic resonance, cavity QED, or ion trapping techniques.

The Schmidt-correlated (SC) states are the mixtures of pure states, sharing the same Schmidt bases [14]. They are generalized to multipartite case, having the generalized GHZ states as pure state decompositions in [15]. An $N$-partite SC state $\rho_{SC} \in \mathbb{C}^M \otimes \cdots \otimes \mathbb{C}^M$ is generally of the form

$$\rho_{SC} = \sum_{m,n=0}^{M-1} \rho_{mn} |m \cdots m\rangle\langle n \cdots n|, \tag{1}$$

where $\sum_{m=0}^{M-1} \rho_{mm} = 1$. For any pure state decomposition $\rho_{SC} = \sum_k p_k |\phi_k\rangle\langle\phi_k|$, $|\phi_k\rangle$ has the from $|\phi_k\rangle = \sum_m \sqrt{\rho_{mm}} e^{i\Theta_m^{(k)}} |m \cdots m\rangle$, which is a kind of generalized $N$-partite $\mathrm{GHZ(N, M)}$ state, where

$$\mathrm{GHZ}(N, M) = \frac{1}{\sqrt{M}}(|0 \cdots 0\rangle + |1 \cdots 1\rangle + \cdots + |M-1, \cdots, M-1\rangle). \tag{2}$$

An SC state is fully separable if and only if it is PPT with respect to some subsystems [15], and it is either fully separable or genuine entangled.

In this paper we study another class of multipartite mixed states, which have the generalized W states as pure state decompositions.

## II. GENERALIZED MIXED W STATES IN MULTIQUBITS SYSTEM

First we consider multipartite qubit case. The $N$-partite $|W_N\rangle$ state reads,

$$|W_N\rangle = \frac{1}{\sqrt{N}}(|0 \cdots 01\rangle + |0 \cdots 10\rangle + \cdots + |1 \cdots 00\rangle). \tag{3}$$

The generalized $|W_N\rangle$ state is given by,

$$|W_N\rangle_g = \sum_{m=1}^{N} a_m |0 \cdots 1_m \cdots 0\rangle = a_1|0 \cdots 01\rangle + a_2|0 \cdots 10\rangle + \cdots + a_N|10 \cdots 0\rangle, \tag{4}$$

with $\sum_{m=1}^{N} |a_m|^2 = 1$.

For non zero $a_1, a_2, \cdots, a_N$, one can show that $|W_N\rangle_g$ is equivalent to $|W_N\rangle$ under stochastic local operation and classical communication (SLOCC) [17], $|W_N\rangle = A_1 \otimes A_2 \otimes \cdots \otimes A_N |W_N\rangle_g$, with

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{a_N\sqrt{N}} \end{pmatrix}, \; A_2 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{a_{N-1}\sqrt{N}} \end{pmatrix}, \; \cdots, \; A_N = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{a_1\sqrt{N}} \end{pmatrix}.$$

Therefore in this case $|W_N\rangle_g$ are all genuine $N$-partite entangled states. Similarly if the number of nonzero coefficients $a_i$ of $|W_N\rangle_g$ is $t$ $(0 < t < N)$, then the states are genuine $t$-partite entangled ones.

Let us consider mixed states with ensembles of pure state decomposition $\{p_k, |\phi_k\rangle\}$, with $|\phi_k\rangle = \sum_{m=1}^{N} \sqrt{\rho_{mm}} e^{i\theta_m^{(k)}} |0\cdots 1_m \cdots 0\rangle$, here $|0\cdots 1_m \cdots 0\rangle$ denotes a state with the $m$-th position from right one and others positions zeros, i.e. $|1_N 0\cdots 0\rangle = |10\cdots 0\rangle$, $|01_{(N-1)}\cdots 0\rangle = |01\cdots 0\rangle$ and so on. Such states are generally of the form

$$\rho = \sum_{m,n=1}^{N} \rho_{mn} |0\cdots 1_m \cdots 0\rangle\langle 0\cdots 1_n \cdots 0|, \tag{5}$$

with $\sum_{m=1}^{N} \rho_{mm} = 1$. Such mixed states $\rho$ has only ensemble realizations with pure states of the form (4).

To study the entanglement property of state (5), we consider the partial transposition, for instance, with respect to the $N$-th subsystem, which gives rise to

$$\rho^{PT} = \begin{pmatrix} 0 & 0 & 0 & \rho_{12} & 0 & \cdots & 0 & \cdots & \rho_{1N} & \cdots & 0 \\ 0 & \rho_{11} & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \rho_{22} & 0 & \rho_{23} & \cdots & \rho_{2N} & \cdots & 0 & \cdots & 0 \\ \rho_{21} & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \rho_{32} & 0 & \rho_{33} & \cdots & \rho_{3N} & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \rho_{N2} & 0 & \rho_{N3} & \cdots & \rho_{NN} & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{N1} & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \end{pmatrix}.$$

By carrying out some elementary transformations, $\rho^{PT}$ can be transformed into another matrix $(\rho^{PT})'$:

$$(\rho^{PT})' = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & \rho_{11} & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix},$$

where

$$A = \begin{pmatrix} 0 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{N1} & 0 & \cdots & 0 \end{pmatrix}, \quad C = \begin{pmatrix} \rho_{22} & \rho_{23} & \cdots & \rho_{2N} \\ \rho_{32} & \rho_{33} & \cdots & \rho_{3N} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{N2} & \rho_{N3} & \cdots & \rho_{NN} \end{pmatrix},$$

and $D$ is a zero matrix. $C$ is semi-positive as $\rho$ is a density matrix. Therefore the positivity of the matrix $(\rho^{PT})'$, and hence $(\rho^{PT})$, depends on the matrix $A$. By deduction we get that the eigenvalues of $A$ are $\pm(\sum_{j\neq 1}|\rho_{1j}|^2)^{\frac{1}{2}}$ and 0. Hence $\rho^{PT}$ is semi-positive if and only if $(\sum_{j\neq 1}|\rho_{1j}|^2)^{\frac{1}{2}} = 0$, that is, $\rho_{1j} = 0$ for $j = 2, \cdots, N$. In this case the mixed state (5) becomes

$$\rho = \rho_{11}|0\cdots 0\rangle\langle 0\cdots 0| \otimes |1\rangle\langle 1| + \sum_{m,n=2}^{N} \rho_{mn}|0\cdots 1_m\cdots 0\rangle\langle 0\cdots 1_n\cdots 0| \otimes |0\rangle\langle 0|.$$

Therefore it is a bi-separable state with respect to partition $12\cdots N-1$ and $N$ subsystems. Similar results can be obtained related to partial transpositions with respect to other subsystems. We have

**Proposition.** The mixed state (5) is a bi-separable one with respect to partition the $i$-th system and the rest systems if and only if it is semi-positive under partial transposition with respect to the $i$-th subsystem.

The above property is rather special compared with the ones of SC states. Moreover, the entanglement of the state $\rho$ is very robust against particle loss. As the state $|W_N\rangle$ remains entangled even if any $N-2$ parties lose the information, any two out of $N$ parties possess an entangled state, independent of whether the remaining $N-2$ parties decide to cooperate with them or not. Therefore if the mixed state (5) is genuine entangled, the reduced density matrix of $\rho$, for instance, $\rho_{\overline{N}} = tr_N\rho = \rho_{11}|0\cdots 0\rangle\langle 0\cdots 0| + \sum_{m,n=2}^{N}\rho_{mn}|0\cdots 1_m\cdots 0\rangle\langle 0\cdots 1_n\cdots 0|$ is still a genuine entangled state. The SC states have no such property. Any kinds of reduced density matrices of $\rho_{SC}$ states are fully separable.

## III. MONOGAMY RELATION OF THE GENERALIZED MIXED W STATES IN MULTIQUBITS SYSTEM

We now study some monogamy relations related to the generalized mixed W states in multiqubits system. Recall that the concurrence of any bipartite pure state $|\psi\rangle$ in system $\mathcal{H}_A \otimes \mathcal{H}_B$ is defined as $C(|\psi\rangle) = \sqrt{2(1 - tr\rho_A^2)}$, where $\rho_A = tr_B|\psi\rangle\langle\psi|$. The concurrence is then extended to mixed states $\rho$ by the convex roof: $C(\rho) \equiv \min_{\{p_i,|\psi_i\rangle\}}\sum_i p_i C(|\psi_i\rangle)$

for all possible ensemble realizations $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$, where $p_i \geq 0$ and $\sum_i p_i = 1$. For a pure three-qubit state $|\psi_{ABC}\rangle$, Coffman, Kundu and Wootters (CKW) [19] introduced a monogamy inequality in terms of concurrence, $C^2_{AB} + C^2_{AC} \leq C^2_{A(BC)}$, where $C_{AB}$ and $C_{AC}$ are the concurrences of the mixed states $\rho_{AB} = tr_C(|\psi_{ABC}\rangle\langle\psi_{ABC}|)$ and $\rho_{AC} = tr_B(|\psi_{ABC}\rangle\langle\psi_{ABC}|)$, respectively, and $C_{A(BC)}$ is the concurrence of $|\psi_{ABC}\rangle$ under bipartite decomposition between subsystems $A$ and $BC$. The general monogamy inequality for the case of $n$ qubits is proved in [20]. Ref. [21] provided the general monogamy relation of $|W_N\rangle_g$ state with respect to arbitrary partitions. Recently another monogamy inequality in terms of negativity is deduced in [22]. Negativity is an entanglement measure in two partite systems which can be expressed as $N(\rho) = \frac{\|\rho^{PT}\|-1}{2}$, where PT stands for partial transposition and the trace norm $\|R\|$ is given by $\|R\| = tr\sqrt{RR^\dagger}$. In fact the negativity of state $\rho$ is essentially the absolute value of the sum of negative eigenvalues of $\rho^{PT}$. For any three-qubit pure state $|\psi\rangle_{ABC}$, the following CKW-inequality-like monogamy inequality in terms of negativity holds,

$$N^2_{AB} + N^2_{AC} \leq N^2_{A(BC)}, \tag{6}$$

where $N_{AB}$ and $N_{AC}$ are the negativities of the mixed states $\rho_{AB}$ and $\rho_{AC}$ respectively. $N_{A(BC)}$ is the negativity of $|\psi_{ABC}\rangle$ for the bipartite partition of subsystems $A$ and $BC$. Similarly one has also

$$N^2_{BA} + N^2_{BC} \leq N^2_{B(AC)}, \quad N^2_{CA} + N^2_{CB} \leq N^2_{C(AB)}.$$

The general monogamy relation in terms of negativity is given by

$$N^2_{A_1A_2} + N^2_{A_1A_3} + \cdots + N^2_{A_1A_N} \leq N^2_{A_1(A_2A_3\cdots A_N)}. \tag{7}$$

Other general monogamy inequalities corresponding to different focused subsystems $A_i$ can be written down similar to the form (7). In the context of quantum cryptography, such a monogamy property is of fundamental importance because it quantifies how much information an eavesdropper could potentially obtain about the secret key extraction. The constraints on shareability of entanglement lie also at the heart of the success of many information-theoretic protocols, such as entanglement distillation [20]. In this section we prove the monogamy relation of the mixed state (5) in terms of negativity.

From the above section we have that the negativity of $\rho$ for a bipartite decomposition between subsystem $A_i$ and the rest subsystems (non $A_i$) $\overline{A_i}$ is $(\sum_{j\neq N+1-i}|\rho_{N+1-i,j}|^2)^{\frac{1}{2}}$. Therefore we get

$$N^2_{A_1(A_2\cdots A_N)} = \sum_{j\neq N}|\rho_{Nj}|^2. \tag{8}$$

As $\rho_{A_1A_2}=tr_{A_3\cdots A_N}\rho=(\rho_{11}+\cdots+\rho_{N-2,N-2})|00\rangle\langle 00|+\rho_{N-1,N-1}|01\rangle\langle 01|+\rho_{N-1,N}|01\rangle\langle 10|+\rho_{N,N-1}|10\rangle\langle 01|+\rho_{NN}|10\rangle\langle 10|$, we get

$$N_{A_1A_2}=\frac{1}{2}(\sqrt{(\rho_{11}+\cdots+\rho_{N-2,N-2})^2+4|\rho_{N-1,N}|^2}-(\rho_{11}+\cdots+\rho_{N-2,N-2})).$$

Similarly we can deduce

$$\begin{aligned}N_{A_1A_i} &= \frac{1}{2}(\sqrt{(\rho_{11}+\cdots+\overline{\rho_{N+1-i,N+1-i}}+\cdots+\rho_{N-1,N-1})^2+4|\rho_{N+1-i,N}|^2}\\ &\quad -(\rho_{11}+\cdots+\overline{\rho_{N+1-i,N+1-i}}+\cdots+\rho_{N-1,N-1})),\end{aligned}$$

where $\overline{\rho_{N+1-i,N+1-i}}$ means that the term is absent in the summation. As $\sqrt{a+b}\leq\sqrt{a}+\sqrt{b}$, we have

$$\begin{aligned}N_{A_1A_i} &= \frac{1}{2}(\sqrt{(\rho_{11}+\cdots+\overline{\rho_{N+1-i,N+1-i}}+\cdots+\rho_{N-1,N-1})^2+4|\rho_{N+1-i,N}|^2}\\ &\quad -(\rho_{11}+\cdots+\overline{\rho_{N+1-i,N+1-i}}+\cdots+\rho_{N-1,N-1}))\\ &\leq |\rho_{N+1-i,N}|.\end{aligned} \tag{9}$$

From (8) and (9) we see that (7) holds for the mixed state $\rho$. Inequality (9) becomes equality if and only if $\rho_{11}=\cdots=\overline{\rho_{N+1-i,N+1-i}}=\cdots=\rho_{N-1,N-1}=0$ or $\rho_{N+1-i,N}=0$, $i=2,\cdots,N$. Hence one arrives at that if the inequality (7) becomes equality, then $\rho$ is at least bipartite $A_i|\overline{A_i}$ separable for some $i$, $1\leq i\leq N$. In other words, monogamy inequality (7) holds strictly for genuine entangled state (5).

Similarly we have

$$N^2_{A_kA_l(A_1\cdots\overline{A_k}\cdots\overline{A_l}\cdots A_N)}=\sum_{i=N+1-k,N+1-l,\ j\neq N+1-k,N+1-l}|\rho_{ij}|^2. \tag{10}$$

For $A_1A_2|A_3\cdots A_N$ partition, the following equalities holds:

$$N^2_{A_1A_2(A_3\cdots A_N)} = \sum_{i=N-1,N,\ j\neq N-1,N}|\rho_{ij}|^2, \tag{11}$$

$$\begin{aligned}N_{A_1A_2(A_k)} &= \frac{1}{2}(((\rho_{11}+\cdots+\overline{\rho_{N+1-k,N+1-k}}+\cdots+\rho_{N-2,N-2})^2\\ &\quad +4(|\rho_{N+1-k,N-1}|^2+|\rho_{N+1-k,N}|^2))^{\frac{1}{2}}\\ &\quad -(\rho_{11}+\cdots+\overline{\rho_{N+1-k,N+1-k}}+\cdots+\rho_{N-2,N-2})).\end{aligned} \tag{12}$$

Therefore one gets

$$N^2_{A_1A_2(A_3)}+\cdots+N^2_{A_1A_2(A_N)}\leq N^2_{A_1A_2(A_3\cdots A_N)}. \tag{13}$$

If inequality (13) becomes an equality, then state $\rho$ is at least separable under some partition $A_iA_j|\overline{A_iA_j}$, $1\leq i<j\leq N$, otherwise it will be a strictly inequality. Generally for any partition $P_1=\{A_{i_1},\cdots,A_{i_k}\}$, $P_2=\{A_{i_{k+1}},\cdots,A_{i_{k+l}}\}$, $\cdots$, $P_s=\{A_{i_{k+s}},\cdots,A_{i_N}\}$, we have

$$N^2_{P_1P_2}+\cdots+N^2_{P_1P_s}\leq N^2_{P_1(P_2\cdots P_s)}. \tag{14}$$

If this inequality becomes an equality, the state is at least separable under some partition $A_{t_1}A_{t_2}\cdots A_{t_k}|\overline{A_{t_1}A_{t_2}\cdots A_{t_k}}$, $1\le t_1<t_2<\cdots<t_k\le N$. On the other hand, we can deduce another conclusion that mixed state $\rho$ (5) is biseparable if and only if it is PPT with respect to such partition.

## IV. COMMENTS AND CONCLUSIONS

The results can be generalized to $N$-partite $d$ dimensional systems. That is

$$\rho=\sum_{i,k=1}^{N}\sum_{j,l=0}^{d-1}\rho_{i(j),k(l)}|0\cdots j_i\cdots 0\rangle\langle 0\cdots l_k\cdots 0| \tag{15}$$

with $\sum_{i=1}^{N}\sum_{j=0}^{d-1}\rho_{i(j),i(j)}=1$. We can similarly prove that pure state decomposition of (15) has the form $|\psi\rangle=\sum_{i=1}^{N}\sum_{j=0}^{d-1}a_{i(j)}|0\cdots j_i\cdots 0\rangle$, which is equivalent to pure state $|\psi\rangle=\sum_{i=1}^{N}\sum_{j=0}^{d-1}|0\cdots j_i\cdots 0\rangle$ under SLOCC. Moreover state (15) is separable with respect to the subsystem $A_i$ $(1\le i\le N)$ if and only if (15) is PPT with respect to $A_i$.

The monogamy relations can be similarly studied. For example, the negative eigenvalue of $\rho^{PT}$ with respect to the first subsystem is $(\sum_{i\neq N}\sum_{j,l=0}^{d-1}|\rho_{i(j),N(l)}|^2)^{\frac{1}{2}}$. Therefore the negativity $N^2_{A_1(A_2\cdots A_N)}=\sum_{i\neq N}\sum_{j,l=0}^{d-1}|\rho_{i(j),N(l)}|^2$. By tedious calculation we can show that inequality (7) and (14) also hold for state (15). And if the inequalities become equalities, then corresponding results hold similar to qubit case. While for SC states, the equalities hold if and only if they are fully separable, as their reduced matrices are all fully separable.

In summary, similar to SC states having generalized GHZ states in the pure state decomposition, we have introduced a new class of multipartite entangled mixed states with pure state decompositions of generalized $W$ states. The entanglement and separability properties are studied according to PPT operations. It is shown that the states are bipartite separable if they are PPT corresponding such partition. Monogamy relations related to these states are also investigated. Although it is still not known if the monogamy relations in terms of negativity hold for general high dimensional mixed state, they are true for our class of states. Above all, the entanglement of these states is very robust against particle loss. If the mixed state (5) is genuine entangled, the reduced density matrix of $\rho$ is still a genuine entangled state.


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