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by

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Self-Similarity and Network Perspective of the Chinese Fund Market

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Abstract

By testing 88 different funds of the Chinese fund market (CFM), we find fractal behavior and long-range correlations in the return series, which are insensitive to the funds kind. Meanwhile, a power-law relationship between the deviation D of prices and the Hurst exponent H has been obtained, which may be useful for predicting the price time series. In addition, with funds being viewed as nodes, and the connections among the funds being determined by the cross-correlation coefficients, using a winner-take-all approach, we investigate the topological properties of the fund network. Our analysis reveals that, during different time periods, the cumulative degree distributions of the fund network all obey the double power-law format. Moreover, the small-world property is also found for the fund network.

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Key words: Self-similarity; scaling; network; double power-law; topology.

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1 Introduction

To understand the universal properties of the financial market, such as the scaling behaviors [1–3], the cross-correlations [4,5], etc, physicists are searching for some unified framework [6–8] which is applicable to a wide class of financial systems. Particularly in the past few years, the empirical evidence of fractal behavior [9–13], long-range correlations [14–18] in the financial time series has received widespread attention, since such self-similarity characteristics are analogous to those displayed by the complex physical systems [19–22].

In the recent progress of research on complex networks, much attention has been given to analyze the formation principles [23] and the topological properties [24–26] of financial networks. For instance, using the minimal spanning tree (MST) method, Mantegna [23] proposed a stock network to investigate the interactions among the stocks. Schweitzer and Stanley [27,28] anticipate a challenging research agenda in economic networks, built upon a methodology that strives to capture the rich process resulting from the interplay between agents' behavior and the dynamic interactions among them.

The fund market plays an important role in the world economy [29,30]. As an indispensable part of the Chinese financial system, the Chinese fund market (CFM) receives an increasing attention [31]. Especially, with the rapid development of the Chinese economy, there will be huge room for the development of the CFM. Therefore, to find some commonly shared characteristics of different funds, and to understand the cross-correlations among different funds, will make great significance [32,33].

It has been found that there exist fractal behavior and long-range correlations in the return series of CFM [34]. In order to check whether these properties are valid for the majority of funds, we test on more examples and show further the self-consistency of the two self-similarity characteristics. One of the main goals of time series analysis is to forecast future values of certain variables. We find the power-law relationship between the deviation D of the price series and the Hurst exponent H, which may be used to predict the change of prices.

Fluctuations of the fund prices are not independent, since the funds with similar investment perspectives are strongly correlated with each other and tend to be grouped into communities, which influences all other funds' transactions. Thus to analyze the cross-correlations of the CFM is crucial to understand the interactions among the funds, which we can investigate from the complex network perspective [35].

The whole text is organized as follows: The fractal behavior and long-range correlations of the CFM are depicted in Subsection 2.1. We present the self-consistency of the fractal behavior and long-range correlations in Subsection

2.2. The power-law relationship between the deviation D and the Hurst exponent H is shown in Subsection 2.3. We construct the Chinese fund network in Subsection 3.1. The cumulative degree distribution is exhibited in Subsection 3.2, and the basic parameters of the network are calculated in Subsection 3.3. The conclusion is given in the last part, Section 4.

2 Self-similarity of the time series

Self-similarity [36] is ubiquitous in nature and human society, for instance, the financial markets [37], the telecommunications [38], the hydrology [39] and the geophysics [40] etc. While for financial markets, it means the time series have similar statistical features in different time scales, and exhibit the characteristics of long-range correlations.

2.1 Fractal behavior and long-range correlations of the time series

The Hurst exponent H [41] has been widely used to analyze the fractal behavior in time series, one can classify the time series into three categories: $H \approx 0$ indicates a random walk, $0<H<0.5$ means the anti persistent behavior, and $0.5<H<1.0$ implies the persistent behavior.

We can create a new time series by calculating the standard deviation of a certain time series. And we can proceed further to calculate the standard deviation of the new time series as follows,

$$Std(t) \sim t^H, \tag{1}$$

where $Std(t)$ is the standard deviation, t is the time scale, and H, the Hurst exponent.

The method of detrended fluctuation analysis (DFA) [11] is a scaling analysis method which has been employed to characterize long-range correlations in time series:

$$F(S) \propto S^{\alpha}, \tag{2}$$

where $F(S)$ is the root mean square fluctuation of the integrated and detrended time series, S is the size of time scale, and α is used to classify the time series which appear to be long-memory process or $1/f$ noise [11].

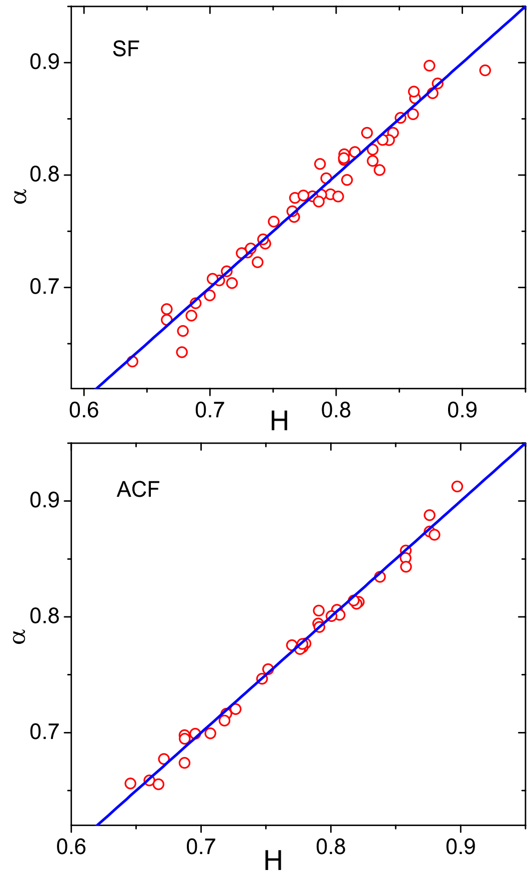

Fig. 1. The linear relationship between the Hurst exponent H and long-range correlations scaling exponent α, with slope 1.2. The top panel corresponds to 52 stock funds (SF), while the bottom one, 36 active configuration funds (ACF).

2.2 Self-consistency of fractal behavior and long-range correlations

Based on the investment objective and the degree of investment risk, the funds can be classified into stock fund (SF), and active configuration fund (ACF), etc. For example, a stock fund is one that invests mainly in stocks, and a bond fund adopts a collective investment scheme that invests mostly in bonds and other debt securities, etc. Due to limited financial data, here we choose a 4-year research period ranging from June 2005 to October 2009, and collected the daily closed prices of 52 stock funds and 36 active configuration funds from the web site of the Chinese fund market [42].

Applying the methods in the above subsection, we test 88 different funds. Fig. 1 presents the relationship between the Hurst exponent H and long-range correlations scaling exponent α. As seen, on the one hand, the values of the scaling exponents H and α of the 88 different funds are all greater than 0.5, which suggests the time series possess characteristics of the fractal behavior and persistent long-range power law correlations.

On the other hand, one can see that the relationship between H and α is linear with slope 1.2, which reflects the self-consistency of the two characteristics. The self-similar time series may have some similar statistical properties in different time scales, and the probability distributions of the series still retain the same profile even if the time scale changes.

2.3 A way to predict the fluctuation of prices

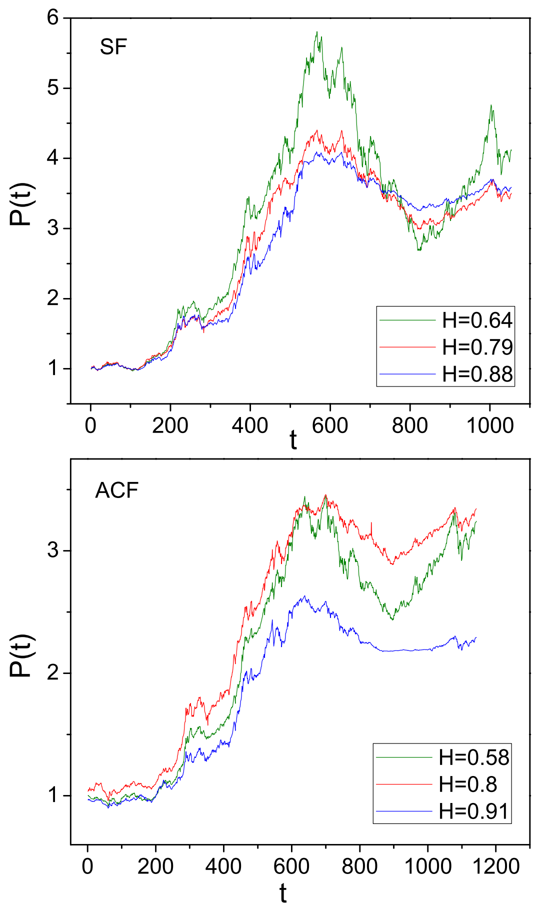

Fig. 2. Time series of prices of the stock fund (SF) and the active configuration fund (ACF), with small, medium and large Hurst exponents H.

The price time series of the funds with large, medium and small Hurst exponent are presented in Fig. 2, as one can observe that the larger the Hurst exponent H, the stronger persistent behavior of its price series, and the less risk of the fund. Therefore the question is, whether we can predict the risk of the fund by its Hurst exponent, or whether we can find a quantity based on the price series, which can be related to the Hurst exponent. We formulated as follows:

(1) Time series sample N of the price is divided into n bins, the length of each bin being T, with $T = N/n$.

(2) With p_i being the price of a fund at time i, the standard deviation $S(j)$ is calculated in all non-overlapping bins of length T, which denotes the risk distributed in every bin.

$$S(j) = \sqrt{\frac{1}{T-1}\sum_{i=1}^{T}(p_i - \bar{p})^2}, j = 1, 2...n. \tag{3}$$

$$\bar{p} = \frac{1}{T}\sum_{i=1}^{T} p_i. \tag{4}$$

(3) The average value S_T of the standard deviation $S(j)$ is calculated within n bins.

$$S_T = \frac{1}{n}\sum_{j=1}^{n} S(j). \tag{5}$$

(4) Repeat steps (1) – (3) over different lengths of T ranging from 3 to 20, one can then define the deviation D as the average value of S_T.

$$D = \frac{1}{18}\sum_{T=3}^{20} S_T. \tag{6}$$

The relationship between the deviation D and the Hurst exponent H is shown in Fig. 3, which has been tested by using a KS type goodness-of-fit test for power law distribution hypothesis [43]. Firstly, the power-law exponent has been calculated by applying the maximum likelihood estimation (MLE), then the KS test has been employed to calculate the maximum distance between the hypothesized cumulative distribution and the empirical one. It is found that $K = 0.0512$ and 0.0526, is below, the 0.9 quantile and 100 samples, therefore, with the observed significance level greater than 10%, there is insufficient evidence to reject the hypothesis that the relationship is a power-law. $D \sim H^{-\beta}$ means the larger the Hurst exponent of a fund, the smaller its corresponding deviation. Therefore, if the Hurst exponent H of a fund in a certain period is obtained, then the deviation D can be estimated, which reflects the fluctuations of the price. This feature might be used to make predictions.

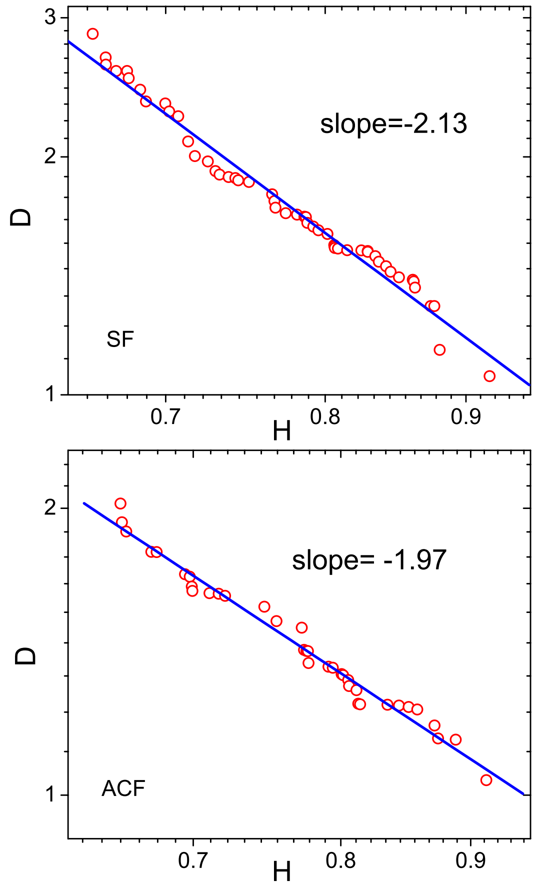

Fig. 3. The power-law relationship between the Hurst exponent H and the deviation D of the time series with a simple definition.

3 Network perspective of the Chinese fund market

Based on the cross-correlation matrix, the method of minimal spanning tree (MST) [23] has been introduced to study the correlations of prices in the financial network, in which the nodes are stocks or funds, and the distances between them are obtained from the corresponding cross-correlation coefficients. While in this paper we employ the winner-take-all approach [35] to establish edges of the network, which makes binary decision on connecting two fund prices according to the absolute values of their cross-correlation coefficients being greater than a threshold value.

3.1 Construction of the Chinese Fund Network

Let $p_i(t)$ be the price of fund i at moment t, then the return of the fund i after a time interval Δt is defined as

$$r_i(t) = \ln p_i(t + \Delta t) - \ln p_i(t), \tag{7}$$

where the return $r_i(t)$ denotes the logarithm fluctuation of the price during the time interval Δt, which can be taken to be 1 day, 1 week, or 1 month.

To reduce the influence of different investment portfolio, we define the relative return

$$R_i(t) = r_i(t) - \frac{1}{N}\sum_j r_j(t), \tag{8}$$

where $R_i(t)$ denotes the relative return of the fund i at time t, which refers to the return $r_i(t)$ as compared to average returns of the whole 88 funds at time t.

Then the cross-correlation coefficient C_{ij} is defined as

$$C_{ij} = \frac{\langle R_i R_j\rangle - \langle R_i\rangle\langle R_j\rangle}{\sqrt{(\langle R_i^2\rangle - \langle R_i\rangle^2)(\langle R_j^2\rangle - \langle R_j\rangle^2)}}, \tag{9}$$

where the values of C_{ij} range from -1 to 1: $C_{ij} = 1$ means the prices of funds i and j have perfect similar variation profiles, while $C_{ij} = -1$ shows the perfect inverse variation features and $C_{ij} = 0$ predicts no correlations.

Our aim is to analyze the interactions among the funds in a certain period, therefore, if the absolute value of the cross-correlation coefficient between two funds is greater than a threshold (e.g. 0.8), then an edge is assumed to connect them. Supposing the threshold is λ, thus the connection criterion between funds i and j is

$$|C_{ij}| > \lambda. \tag{10}$$

To reflect the strong interactions among the funds, we start with the relatively large values of λ to construct the Chinese fund network, with $\lambda = 0.7$, 0.75 and 0.8, respectively.

3.2 Cumulative degree distribution

The cumulative degree distributions of the Chinese fund network from June 2005 to May 2009 are shown in Fig. 4. It was indicated in the literature that many similar degree distributions have been found in the transport networks, such as the airport network of China [44], the US flight network [45], the worldwide air transportation network [46], the urban road network of Le Mans in France [47], etc. This kind of degree distributions have been very well fitted by the following function [48]:

$$P(K > k) = \frac{1}{ak^{r_1} + bk^{r_2}}, \tag{11}$$

where a, b are two fitting parameters, r_1, r_2 are the scaling exponents in the two regions of power-law.

From this formula, the two regimes of power-law are separated by a critical degree k_c, which can be determined through

$$ak_c^{r_1} = bk_c^{r_2}. \tag{12}$$

When $0 < k < k_c$, there will be $ak_c^{r_1} \gg bk_c^{r_2}$, thus, $p(K > k) \sim k^{-r_1}$. When $k > k_c$, then $bk_c^{r_2} \gg ak_c^{r_1}$, and hence, $p(K > k) \sim k^{-r_2}$.

We fitted our empirical data with Eq. (11), the results are shown in Fig. 4. One could find that the degree distributions are pretty well matched with the double power-law function, in both the small and large degree regimes. The values of a, b, r_1, r_2, as well as the critical degree k_c, are presented in the Figure.

The double power-law degree distribution implies that, when $k > k_c$, at the second power-law regime, the large degree decrease very rapidly with little fluctuations, just like the cutoff. Thus, we may doubt whether the second power-law is real, or nothing but a finite size effect.

We tried on more examples, since the immature of the Chinese fund market, the funds with a relatively long trading period, just like the previous 88 funds, are not too many, and we only find 20 more new ones here. We show the degree distributions of the Chinese fund network, with 108 funds, in Fig. 5, compared with those in Fig. 4, one could find, the double power-law degree distributions in Fig. 5 might be more convincing, we therefore may judge that, the second power-law is not caused by the finite size effect, but a reality.

The results also suggest that, most individual funds have a small number of links with the other ones, while a few funds, namely the hubs, have a large number of links. In other words, there might exist some leading fund management companies, whose investment portfolios prevail, and whose actions may have strong influence on the fund network, which may affect the fluctuation of other funds prices. And this observation is consistent with that of our previous work.

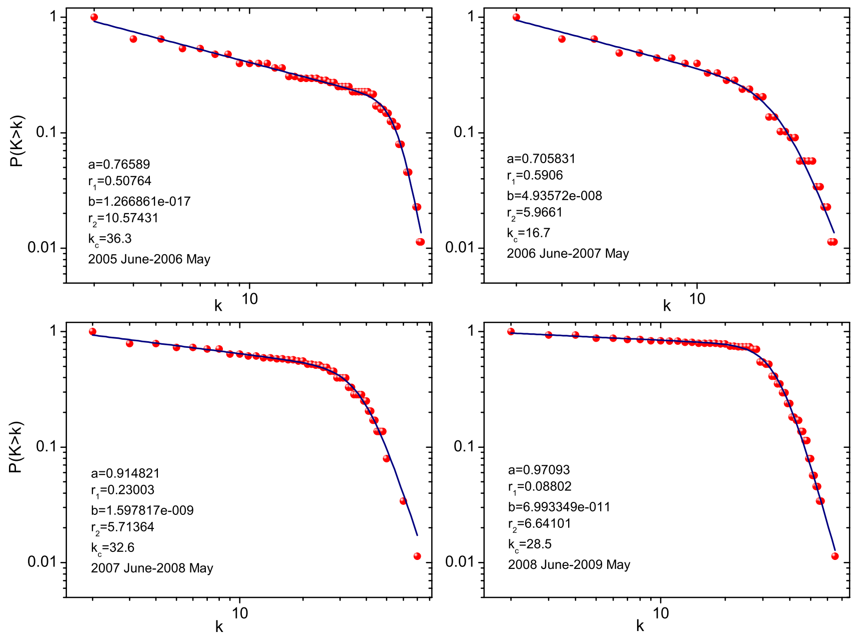

Fig. 4. The double power-law degree distributions of the Chinese fund network, with 88 funds, in different time periods, formed by a winner-take-all connection criterion, with threshold $\lambda = 0.75$.

3.3 Basic parameters of the network

Basic parameters which reflect the topological structure of the fund network are calculated in different time periods, with the threshold λ=0.7, 0.75 and 0.8, respectively.

From Fig. 6, one can see that, as λ decreases, the network becomes more compactly connected. The total number of connections L, the average degree $\langle K \rangle$ and the average clustering coefficient $\langle C \rangle$ increase, while the average shortest path length $\langle d \rangle$ and the diameter decrease accordingly. Also, the same trend is observed with the passing of time.

Moreover, no matter which threshold is chosen, the large average clustering coefficient and the small average shortest-path length have all been observed. These features may imply the compact topology of the Chinese fund network.

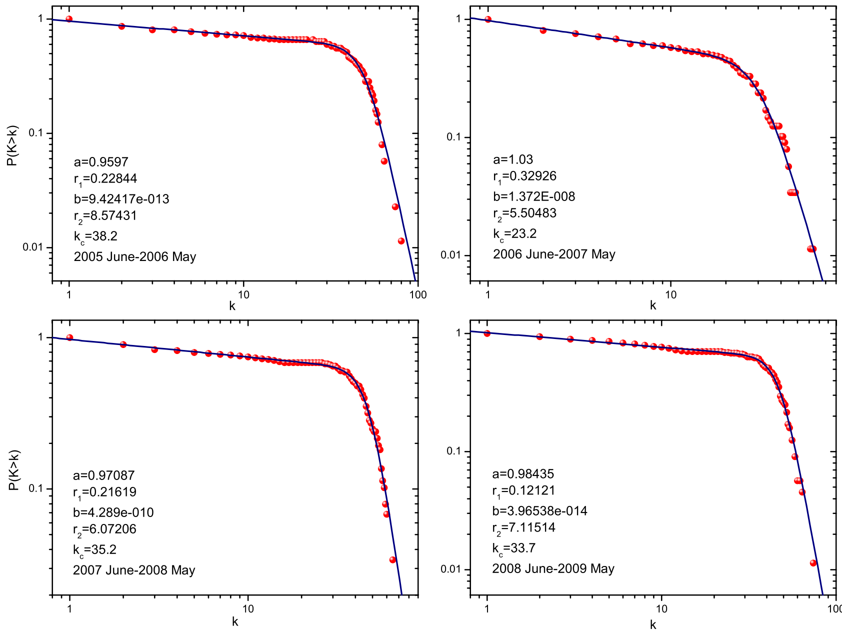

Fig. 5. The double power-law degree distributions of the Chinese fund network, with 108 funds, the previous 88 funds and 20 more new ones, the threshold λ also equals 0.75, to test whether the second power-law is real.

4 Conclusion

The self-similarity characteristics of the return series of the Chinese fund market have been investigated, in connection with the fractal structure and long-range correlation. It has been shown that these two properties are compatible with each other, and also commonly shared in the CFM.

To interpret the factors that influence the Hurst exponents, we defined the deviation D, which reflects the fluctuations of the price. A power-law relationship between the deviation D and the Hurst exponent H is observed. Therefore, if the Hurst exponent in a certain period is obtained, the deviation D can be calculated correspondingly, which may be applied to predict the risk.

The cross-correlations of the CFM have been studied from the complex network perspective with funds as nodes. A winner-take-all approach has been used to determine if two nodes are connected. The double power-law degree distributions have been found for different time periods. This law suggests that, at the second regimes of power-law, when $k > k_c$, the large degree decrease very rapidly, which means a relatively small number of funds may exert

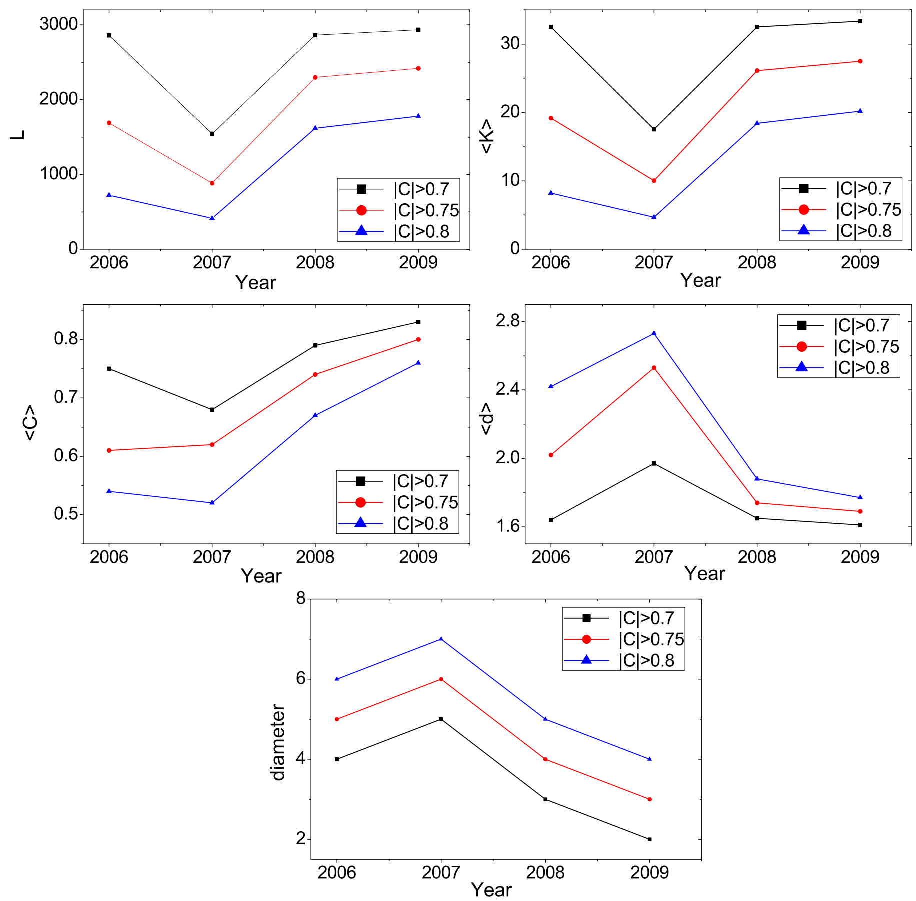

Fig. 6. The basic network parameters of the Chinese fund network from 2006 to 2009, including the total number of connections L, the average degree $\langle K \rangle$, the average clustering coefficient $\langle C \rangle$, the average shortest-path length $\langle d \rangle$ and the diameter, with the threshold $|C| > 0.7$, $|C| > 0.75$ and $|C| > 0.8$.

much influence on the fluctuations of the other funds' prices. Basic quantities which reflect the topological structure are also calculated. Our future work might pay attention to the relationship between the fund performance and its topological quantities.

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