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Multiqubit Quantum Teleportation Based on Generalized Cluster-like States

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by

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Multiqubit Quantum Teleportation

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Abstract

We provide a class of six-qubit states for three-qubit perfect teleportation. These states include the six-qubit cluster states as a special class. We generalize this class of six-qubit states to $2n$-qubit pure states for n-qubit teleportation, $n \geq 1$. These states can be also used for $2n$ bit classical information transmission in dense coding.

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I. INTRODUCTION

Quantum teleportation, employing classical communication and shared resource of entanglement, allows to transmit an unknown quantum state from a sender to a receiver that are spatially separated. Let $|\phi\rangle$ be an arbitrary unknown pure state that is to be sent from Alice to Bob, and $|\psi\rangle$ the entangled state shared by Alice and Bob. To carry out teleportation Alice needs to perform projective measurements on her two particles: one in the state $|\phi\rangle$ and one part of the entangled state $|\psi\rangle$. Learning the measurement results from Alice via the classical communication channel, Bob applies a corresponding unitary transformation on the other part of the entangled state $|\psi\rangle$, so as to transform the state of this part to the unknown state $|\phi\rangle$. In this scenario, Bennett *et. al.* [1] first demonstrates the teleportation of an arbitrary qubit state in terms of an entangled Einstein-Podolsky-Rosen pair. Then the three-qubit GHZ state and a class of W states are revealed to be the ideal resource for faithful teleportation of one-qubit state [2, 3]. For two-qubit teleportation, the tensor product of two Bell states [4], the genuine four-qubit entangled state [5] and one five-qubit state in Ref. [6] are showed to have the ability for faithful teleportation. Ref. [7] has also proposed a scheme for two-qubit teleportation. For three-qubit teleportation, Refs. [8, 9] have investigated the teleportation by one genuine entangled six-qubit state. Generally, Ref. [10] provides the necessary and sufficient condition that the genuine $2n$-qubit entanglement channels must satisfy to teleport an arbitrary n-qubit state, and Ref. [11] analyzes the criterion of multiqubit states for n-qubit teleportation. Then one kind of entangled $2n$-qubit states has been presented for n-qubit teleportation [12]. Besides potential applications in quantum communication, quantum error correction, and one way quantum computation, the cluster states are also of importance in quantum teleportation and dense coding [13].

In this paper, we propose a class of $2n$-qubit states for n-qubit ($n \geq 1$) perfect teleportation, which are also the ideal resource for transmission of $2n$ bit classical information in dense coding. This class of states are not equivalent to the tensor product of Bell states. For three-qubit and two-qubit cases, this class of states includes the cluster states as a special case.

II. THREE-QUBIT TELEPORTATION

We first consider the quantum teleportation of an arbitrary unknown three-qubit state $|\phi\rangle_{A'}^{(3)}$. Suppose Alice and Bob share a priori three pairs of qubits in the state

$$|\xi\rangle_{AB}^{(3,3)} = \tfrac{1}{2\sqrt{2}} \textstyle\sum_{K=0}^{7} |\overrightarrow{K}\rangle_A^{(3)} \otimes |\overrightarrow{K}'\rangle_B^{(3)}, \tag{1}$$

where $|\overrightarrow{K}\rangle^{(3)}$'s constitute an orthonormal basis,

$$\begin{aligned}
|\overrightarrow{0}\rangle_A^{(3)} &= |00\rangle \otimes (\cos\theta_1|0\rangle + \sin\theta_1|1\rangle), \\
|\overrightarrow{1}\rangle_A^{(3)} &= |00\rangle \otimes (-\sin\theta_1|0\rangle + \cos\theta_1|1\rangle), \\
|\overrightarrow{2}\rangle_A^{(3)} &= |01\rangle \otimes (\cos\theta_2|0\rangle + \sin\theta_2|1\rangle), \\
|\overrightarrow{3}\rangle_A^{(3)} &= |01\rangle \otimes (\sin\theta_2|0\rangle - \cos\theta_2|1\rangle), \\
|\overrightarrow{4}\rangle_A^{(3)} &= |10\rangle \otimes (\cos\theta_3|0\rangle + \sin\theta_3|1\rangle), \\
|\overrightarrow{5}\rangle_A^{(3)} &= |10\rangle \otimes (-\sin\theta_3|0\rangle + \cos\theta_3|1\rangle), \\
|\overrightarrow{6}\rangle_A^{(3)} &= |11\rangle \otimes (\cos\theta_4|0\rangle + \sin\theta_4|1\rangle), \\
|\overrightarrow{7}\rangle_A^{(3)} &= |11\rangle \otimes (-\sin\theta_4|0\rangle + \cos\theta_4|1\rangle),
\end{aligned}$$

and $|\overrightarrow{K}'\rangle^{(3)}$'s constitute another orthonormal basis,

$$\begin{aligned}
|\overrightarrow{0}'\rangle_B^{(3)} &= |00\rangle \otimes (\cos\theta_1'|0\rangle + \sin\theta_1'|1\rangle), \\
|\overrightarrow{1}'\rangle_B^{(3)} &= |00\rangle \otimes (-\sin\theta_1'|0\rangle + \cos\theta_1'|1\rangle), \\
|\overrightarrow{2}'\rangle_B^{(3)} &= |01\rangle \otimes (\cos\theta_2'|0\rangle + \sin\theta_2'|1\rangle), \\
|\overrightarrow{3}'\rangle_B^{(3)} &= |01\rangle \otimes (-\sin\theta_2'|0\rangle + \cos\theta_2'|1\rangle), \\
|\overrightarrow{4}'\rangle_B^{(3)} &= |10\rangle \otimes (\cos\theta_3'|0\rangle + \sin\theta_3'|1\rangle), \\
|\overrightarrow{5}'\rangle_B^{(3)} &= |10\rangle \otimes (-\sin\theta_3'|0\rangle + \cos\theta_3'|1\rangle), \\
|\overrightarrow{6}'\rangle_B^{(3)} &= |11\rangle \otimes (\cos\theta_4'|0\rangle + \sin\theta_4'|1\rangle), \\
|\overrightarrow{7}'\rangle_B^{(3)} &= |11\rangle \otimes (-\sin\theta_4'|0\rangle + \cos\theta_4'|1\rangle),
\end{aligned}$$

with $0 \leq \theta_1, \theta_2, \theta_3, \theta_1', \theta_2', \theta_3' \leq \frac{\pi}{2}$. Here these orthonormal basis $\{|\overrightarrow{K}\rangle^{(3)}\}_{K=0}^{7}$ and $\{|\overrightarrow{K}'\rangle^{(3)}\}_{K=0}^{7}$ can be viewed factually as the generalizations of the computational basis, and they will be shown to be able to give more resource for quantum teleportation. If we choose the basis for A and B as the three-qubit computational basis, then the resource for quantum teleportation in form of Eq. (1) is naturally the tensor product of Bell states.

The arbitrary unknown three-qubit state $|\phi\rangle_{A'}^{(3)}$ can be expressed as

$$|\phi\rangle_{A'}^{(3)} = \sum_{K=0}^{7} a_K |\overrightarrow{K}'\rangle_{A'}^{(3)}$$

with $\sum_{K=0}^{7} |a_K|^2 = 1$. Noting that $|\xi\rangle_{AB}^{(3,3)}$ is a maximally entangled state, we may construct the following basis of 64 orthonormal states:

$$|\Pi_{ijk}\rangle_{A'A}^{(3,3)} = (\sigma^{(i)} \otimes \sigma^{(j)} \otimes \sigma^{(k)})_{A'} |\Pi_{000}\rangle_{A'A}^{(3,3)}, \tag{2}$$

where $|\Pi_{000}\rangle_{A'A}^{(3,3)} = \frac{1}{2\sqrt{2}} \sum_{K=0}^{7} |K'\rangle_{A'}^{(3)} \otimes |K\rangle_A^{(3)}$, $\sigma^{(0)}$ is the 2×2 identity matrix, $\sigma^{(1)}$, $\sigma^{(2)}$ and $\sigma^{(3)}$ are three Pauli operators. If Alice performs a complete projective measurement jointly on $A'A$ in the above basis in Eq. (2) with the measurement outcome ijk, then Bob's sequences of qubits will be in the state $\sigma^{(i)} \otimes \sigma^{(j)} \otimes \sigma^{(k)} |\phi\rangle_B^{(3)}$. Bob will always succeed in recovering an exact replica of the original unknown state upon receiving 8 bits of classical information about measurement results from Alice. Namely

$$|\phi\rangle_{A'}^{(3)} \otimes |\xi\rangle_{AB}^{(3,3)} = \frac{1}{8} \sum_{ijk} |\Pi_{ijk}\rangle_{A'A}^{(3,3)} (\sigma^{(i)} \otimes \sigma^{(j)} \otimes \sigma^{(k)}) |\phi\rangle_B^{(3)}, \tag{3}$$

$${}_{A'A}^{(3,3)}\langle \Pi_{ijk}| (|\phi\rangle_{A'}^{(3)} \otimes |\xi\rangle_{AB}^{(3,3)}) = \frac{1}{8} (\sigma^{(i)} \otimes \sigma^{(j)} \otimes \sigma^{(k)}) |\phi\rangle_B^{(3)}. \tag{4}$$

These equations follow from the result given below, which also guarantees the success of the protocol.

$$ {}^{(3,3)}_{A'A}\langle\Pi_{000}|\xi\rangle^{(3,3)}_{AB} = \frac{1}{8}\sum_{J,K=0}^{7}({}^{(3)}_{A'}\langle K'| \otimes^{(3)}_{A} \langle K|)(|J\rangle^{(3)}_{A} \otimes |J'\rangle^{(3)}_{B}) = \frac{1}{8}\sum_{K=0}^{7}|K'\rangle^{(3)}_{B} \times^{(3)}_{A'} \langle K'|. $$

It can be verified that the reduced matrix $\rho_{A_3B_3} = tr_{A_1A_2B_1B_2}(|\xi\rangle^{(3,3)}_{AB}\langle\xi|)$ is not a pure state, as it is not rank one typically. This can be seen from its nonsingular submatrix spanned by $\{|00\rangle\langle 00|,\ |00\rangle\langle 11|\ |11\rangle\langle 00|,\ |11\rangle\langle 11|\}$: $[\cos^2(\theta_1-\theta_1')+\cos^2(\theta_2+\theta_2')+\cos^2(\theta_3-\theta_3')+\cos^2(\theta_4-\theta_4')](|00\rangle\langle 00|+|11\rangle\langle 11|)+[\cos^2(\theta_1-\theta_1')-\cos^2(\theta_2+\theta_2')+\cos^2(\theta_3-\theta_3')+\cos^2(\theta_4-\theta_4')](|00\rangle\langle 11|+|11\rangle\langle 00|)$. Therefore, $|\xi\rangle^{(3)}_{AB}$ is not equivalent to the tensor product of Bell states, $\bigotimes_{i=1}^{3}\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{A_iB_i}$. Furthermore, $\rho_{A_3B_3}$ is not maximally mixed state generally, so it is not equivalent to the genuine entangled six-qubit state in Refs. [8, 9]. When $\theta_1=\theta_1'$, $\theta_2+\theta_2'=\frac{\pi}{2}$, $\theta_3=\theta_3'$, $\theta_4=0$, and $\theta_4'=\frac{\pi}{2}$, the entangled state $|\xi\rangle^{(3,3)}_{AB}$ becomes

$$
\begin{aligned}
&|\xi_0\rangle^{(3,3)}_{AB}\\
=\ &\frac{1}{2\sqrt{2}}(|110111\rangle-|111110\rangle-|101101\rangle+|100100\rangle\\
&+|000000\rangle+|001001\rangle+|010011\rangle+|011010\rangle)_{A_1A_2A_3B_1B_2B_3}\\
=\ &\frac{1}{2\sqrt{2}}((|00\rangle+|11\rangle)|0\rangle(|000\rangle+|111\rangle)+(|00\rangle-|11\rangle)|1\rangle(|100\rangle+|011\rangle))_{A_1B_1A_3B_3A_2B_2},
\end{aligned}
$$

which is exactly the six-qubit cluster state [13].

Additionally, $|\xi\rangle^{(3,3)}_{AB}$ in Eq. (1) is also able to transmit 64 bit classical information. A dense coding scheme using $|\xi\rangle^{(3,3)}_{AB}$ is the following. Because $|\xi\rangle^{(3,3)}_{AB}$ is maximally entangled between A and B, $\{(\sigma^{(i_1)}\otimes\sigma^{(i_2)}\otimes\sigma^{(i_3)})_A|\xi\rangle^{(3,3)}_{AB}\}^{3}_{i_1,i_2,i_3=0}$ are 64 maximally entangled states and constitute an orthonormal basis for the 64 dimensional Hilbert space. The classical information sender Alice encodes her messages by using operators $\sigma^{(i_1)}$, $\sigma^{(i_2)}$, $\sigma^{(i_3)}$, and sends her qubits to Bob. Bob then decodes the messages by performing a joint measurement on all six qubits in the basis in Eq. (2). Here Alice may encode her three qubits locally and independently. While Bob is compelled to read the messages from all the six-qubit together. This is different from a straightforward extension of the original dense coding scheme in terms of two Bell states, where Bob can measure his qubits individually.

III. MULTIQUBIT QUANTUM TELEPORTATION

Now we investigate the teleportation of an arbitrary n-qubit state $|\phi\rangle^{(n)}_{A'}$ by extending the above "generalized cluster-like" six-qubit states to 2n-qubit ones. A priori sequences of qubits shared by Alice and Bob are in the state

$$|\xi\rangle_{AB}^{(n,n)} = \frac{1}{\sqrt{2^n}} \sum_{K=0}^{2^n-1} |\overrightarrow{K}\rangle_A^{(n)} \otimes |\overrightarrow{K'}\rangle_B^{(n)}, \tag{5}$$

with $|\overrightarrow{K}\rangle^{(n)}$'s an orthonormal basis,

$$\begin{aligned}
|\overrightarrow{0}\rangle_A^{(n)} &= |\overrightarrow{0}\rangle_A^{(n-1)} \otimes (\cos\theta_1|0\rangle + \sin\theta_1|1\rangle), \\
|\overrightarrow{1}\rangle_A^{(n)} &= |\overrightarrow{0}\rangle_A^{(n-1)} \otimes (-\sin\theta_1|0\rangle + \cos\theta_1|1\rangle), \\
|\overrightarrow{2}\rangle_A^{(n)} &= |\overrightarrow{1}\rangle_A^{(n-1)} \otimes (\cos\theta_2|0\rangle + \sin\theta_2|1\rangle), \\
|\overrightarrow{3}\rangle_A^{(n)} &= |\overrightarrow{1}\rangle_A^{(n-1)} \otimes (\sin\theta_2|0\rangle - \cos\theta_2|1\rangle), \\
|\overrightarrow{4}\rangle_A^{(n)} &= |\overrightarrow{2}\rangle_A^{(n-1)} \otimes (\cos\theta_3|0\rangle + \sin\theta_3|1\rangle), \\
|\overrightarrow{5}\rangle_A^{(n)} &= |\overrightarrow{2}\rangle_A^{(n-1)} \otimes (-\sin\theta_3|0\rangle + \cos\theta_3|1\rangle), \\
&\vdots \\
|\overrightarrow{2^n-2}\rangle_A^{(n)} &= |\overrightarrow{2^{n-1}-1}\rangle_A^{(n-1)} \otimes (\cos\theta_{2^{n-1}}|0\rangle + \sin\theta_{2^{n-1}}|1\rangle), \\
|\overrightarrow{2^n-1}\rangle_A^{(n)} &= |\overrightarrow{2^{n-1}-1}\rangle_A^{(n-1)} \otimes (-\sin\theta_{2^{n-1}}|0\rangle + \cos\theta_{2^{n-1}}|1\rangle),
\end{aligned}$$

and $|\overrightarrow{K'}\rangle^{(n)}$'s another orthonormal basis

$$\begin{aligned}
|\overrightarrow{0'}\rangle_B^{(n)} &= |\overrightarrow{0'}\rangle_B^{(n-1)} \otimes (\cos\theta'_1|0\rangle + \sin\theta'_1|1\rangle), \\
|\overrightarrow{1'}\rangle_B^{(n)} &= |\overrightarrow{0'}\rangle_B^{(n-1)} \otimes (-\sin\theta'_1|0\rangle + \cos\theta'_1|1\rangle), \\
|\overrightarrow{2'}\rangle_B^{(n)} &= |\overrightarrow{1'}\rangle_B^{(n-1)} \otimes (\cos\theta'_2|0\rangle + \sin\theta'_2|1\rangle), \\
|\overrightarrow{3'}\rangle_B^{(n)} &= |\overrightarrow{1'}\rangle_B^{(n-1)} \otimes (-\sin\theta'_2|0\rangle + \cos\theta'_2|1\rangle), \\
|\overrightarrow{4'}\rangle_B^{(n)} &= |\overrightarrow{2'}\rangle_B^{(n-1)} \otimes (\cos\theta'_3|0\rangle + \sin\theta'_3|1\rangle), \\
|\overrightarrow{5'}\rangle_B^{(n)} &= |\overrightarrow{2'}\rangle_B^{(n-1)} \otimes (-\sin\theta'_3|0\rangle + \cos\theta'_3|1\rangle), \\
&\vdots \\
|\overrightarrow{(2^n-2)'}\rangle_B^{(n)} &= |\overrightarrow{(2^{n-1}-1)'}\rangle_B^{(n-1)} \otimes (\cos\theta'_{2^{n-1}}|0\rangle + \sin\theta'_{2^{n-1}}|1\rangle), \\
|\overrightarrow{(2^n-1)'}\rangle_B^{(n)} &= |\overrightarrow{(2^{n-1}-1)'}\rangle_B^{(n-1)} \otimes (-\sin\theta'_{2^{n-1}}|0\rangle + \cos\theta'_{2^{n-1}}|1\rangle),
\end{aligned}$$

for $0 \leq \theta_1, \theta'_1, \cdots, \theta_{2^{n-1}}, \theta'_{2^{n-1}} \leq \frac{\pi}{2}$. $\{|\overrightarrow{K}\rangle_A^{(n-1)}\}_{K=0}^{2^{n-1}-1}$ and $\{|\overrightarrow{K'}\rangle_B^{(n-1)}\}_{K=0}^{2^{n-1}-1}$ are arbitrary orthonormal basis of the $n-1$ qubit systems. Hence we may express the state to be teleported $|\phi\rangle_A^{(n)}$ in the basis $\{|\overrightarrow{K'}\rangle_B^{(n)}\}_{K=0}^{2^n-1}$,

$$|\phi\rangle_A^{(n)} = \sum_{K=0}^{2^n-1} a_K |\overrightarrow{K'}\rangle_{A'}^{(n)}$$

with $\sum_{K=0}^{2^n-1}|a_K|^2 = 1$. By virtue of the fact that $|\xi\rangle_{AB}^{(n,n)}$ is a maximally entangled state between A and B, we may construct the following basis of 2^{2n} orthonormal states:

$$|\Pi_{i_1 i_2\cdots i_n}\rangle_{A'A}^{(n,n)} = (\sigma^{(i_1)} \otimes \sigma^{(i_2)} \otimes \cdots \otimes \sigma^{(i_n)})_{A'}|\Pi_{00\cdots 0}\rangle_{A'A}^{(n,n)}, \tag{6}$$

where

$$|\Pi_{00\cdots 0}\rangle_{A'A}^{(n,n)} = \frac{1}{\sqrt{2^n}}\sum_{K=0}^{2^n-1}|\overrightarrow{K'}\rangle_{A'}^{(n)} \otimes |\overrightarrow{K}\rangle_{A}^{(n)},$$

and $i_1, \cdots, i_n \in \{0, 1, 2, 3\}$.

Now Alice performs a complete projective measurement jointly on $A'A$ in the above basis in Eq. (6). If the measurement outcome is $i_1 i_2 \cdots i_n$, then Bob's sequences of qubits will be in the state $\sigma^{(i_1)} \otimes \sigma^{(i_2)} \otimes \cdots \otimes \sigma^{(i_n)}|\phi\rangle_B^{(n)}$. Bob can always recover an exact replica of the original unknown state, since

$$|\phi\rangle_{A'}^{(n)} \otimes |\xi\rangle_{AB}^{(n,n)} = \frac{1}{2^n}\sum_{i_1 i_2\cdots i_n}|\Pi_{i_1 i_2\cdots i_n}\rangle_{A'A}^{(n)}(\sigma^{(i_1)} \otimes \sigma^{(i_2)} \otimes \cdots \otimes \sigma^{(i_n)})|\phi\rangle_B^{(n)}, \tag{7}$$

$${}_{A'A}^{(n,n)}\langle\Pi_{00\cdots 0}|(|\phi\rangle_{A'}^{(n)} \otimes |\xi\rangle_{AB}^{(n,n)}) = \frac{1}{2^n}(\sigma^{(i_1)} \otimes \sigma^{(i_2)} \otimes \cdots \otimes \sigma^{(i_n)})|\phi\rangle_B^{(n)}. \tag{8}$$

These equations follow from the result given below, which also guarantees the success of the protocol.

$${}_{A'A}^{(n,n)}\langle\Pi_{00\cdots 0}|\xi\rangle_{AB}^{(n,n)} = \frac{1}{2^n}\sum_{J,K=0}^{2^n-1}({}_{A'}^{(n)}\langle K'| \otimes_A^{(n)} \langle K|)(|J\rangle_A^{(n)} \otimes |J'\rangle_B^{(n)}) = \frac{1}{2^n}\sum_{K=0}^{2^n-1}|K'\rangle_B^{(n)} \times_{A'}^{(n)} \langle K'|.$$

Therefore, $2n$-qubit state $|\xi\rangle_{AB}^{(n,n)}$ is an ideal resource for n-qubit teleportation.

As we know the tensor product of Bell state $|\eta\rangle = \bigotimes_{i=1}^n |\psi^+\rangle_{A_iB_i}$ with $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, is the ideal resource for the n qubit teleportation. Here one can verify that the reduced matrix $\rho_{A_nB_n} = tr_{A_1A_2\cdots A_{n-1}B_1B_2\cdots B_{n-1}}(|\xi\rangle_{A_1A_2\cdots A_nB_1B_2\cdots B_n}\langle\xi|)$ is not a pure state, therefore, $|\xi\rangle_{AB}^{(n,n)}$ is not equivalent to the tensor product of Bell states. This also shows $|\xi\rangle_{AB}^{(n,n)}$ is not equivalent to the entangled states used for n-qubit teleportation in Ref. [12]. Furthermore, when $n = 2$, one can verify that four-qubit cluster state is a special case of $|\xi\rangle_{AB}^{(n,n)}$.

The state $|\xi\rangle_{AB}^{(n,n)}$ in Eq. (5) can be again used for dense coding and is able to transmit $2n$ bit classical information. Since $|\xi\rangle_{AB}^{(n,n)}$ is maximally entangled between A and B, $\{(\sigma^{(i_1)} \otimes \sigma^{(i_2)} \otimes \cdots \otimes \sigma^{(i_n)})_A|\xi\rangle_{AB}^{(n,n)}\}_{i_1,i_2,\cdots,i_n=0}^3$ are 2^{2n} maximally entangled states and constitute an orthonormal basis for the 2^{2n} dimensional Hilbert space. Alice can encode the message by using the operators $\sigma^{(i_1)}$, $\sigma^{(i_2)}$, $\cdots$, and $\sigma^{(i_n)}$. Upon receiving the qubits from Alice, Bob can decode the message by performing a joint measurement in the basis in Eq. (6).

IV. CONCLUSIONS

We have presented a class of $2n$-qubit states which may serve as ideal resources for perfect teleportation of n-qubit states ($n \geq 1$). This class of states is also the ideal resource for transmission of $2n$ bit classical information in dense coding. This kind of states are not equivalent to the tensor product of Bell states. For three-qubit and two-qubit cases, it is easy to see that this class of states includes the cluster states as a special case. Here we just study the teleportation and dense coding in terms of these kinds of entangled states. It would be also interesting to consider one-way quantum computing according to these "generalized cluster-like" states.

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