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embedded networks

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ABSTRACT

This work is a modeling of evolutionary networks embedded in one or two dimensional configuration space. The evolution is based on two attachments depending on degree and spatial distance. The probability for a new node n to connect with a previous node i at distance r_{ni} follows $a \frac{k_i}{\sum_j k_j} + (1 - a) \frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}^{-\alpha}}$, where k_i is the degree of node i , α and a are tunable parameters. In spatial driven model ($a = 0$), the spatial distance distribution follows the power-law feature. The mean topological distance l and the clustering coefficient C exhibit phase transitions at same critical values of α which change with the dimensionality d of the embedding space. When $a \neq 0$, the degree distribution follows the “shifted power law” (SPL) which interpolates between exponential and scale-free distributions depending on the value of a .

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1. Introduction

In the recent development of network sciences [1–9], spatial constraint networks have become an object of extensive investigation [9–23]. These are the networks embedded in configuration space and influenced by spatial constraints. Recent findings have revealed that the spatial distance distribution follows power law or exponential distribution [6,7,9,17,18]. These distributions are quite natural since, for instance, people tend to have their friends and relatives in their neighborhood, transportation networks often favor shorter distance trips, and many communication networks are mainly dominated by short radio ranges [20]. To model these systems, scientists have proposed spatially constrained networks embedded in one- or two-dimensional space [10–13,15,19,21,22]. According to the generation rules, these networks can be categorized into three classes: scale-free (SF) networks with disadvantaged long-range links, SF networks embedded in lattices and space-filling networks [18].

The first class is an extension of the conventional SF models by adding competition between degree and spatial distance preferences of linking. In one of the extended models, the network grows with addition of nodes randomly positioned in space. The nodes are connected to each other with the probability $\Pi_i \sim k_i r^\alpha$, where r is the spatial distance between the new node and the node i with degree k_i . The distance distribution $p(r)$ is given by $p(r) \sim r^{-(\alpha-d+1)}$ as expected, where d is the dimension of the space. On the other hand, the degree distribution is a power law for $\alpha > -1$ and a stretched exponential law for $\alpha < -1$ [15,19]. Another extension uses the connection probability $\Pi_i \sim k_i^\beta r^\alpha$ and generates a power law degree distribution on a line in the $\alpha - \beta$ plane and in the zone limited by $\beta > 1$ and $\alpha < -0.5$ [11]. The second class is

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an extension of the SF networks by embedding them in regular one- or two-dimensional spatial lattices in which the links are added according to the probability $p(r) \sim r^{-\delta}$. The structure of the network is affected by the parameter δ [13]. The third class uses the method of space-filling packing in which a region is iteratively partitioned into subregions by adding new nodes which are connected to the closest neighbors [10,12,21,22].

In this work we are interested in the first class networks. We introduce a model with a kind of competition between the degree and the spatial distance preferences. While the degree preferential attachment produces connections free from spatial constraints, the spatial distance preference favors closer connections. This competition between short-range and long-range connections is modulated by a parameter a .

2. The model

To construct the networks, the nodes are embedded on a one-dimensional ($d = 1$) ring of radius $R = 1/\pi$ or on a two-dimensional ($d = 2$) sphere of radius $R = 1/\pi$. The spatial distance r between a pair of nodes is defined as the shortest distance between them.

The model is constructed in the following way:

- (1) *Initial condition*: We start with an initial state ($t = m_0$) of $m_0 + 1$ all-to-all connected nodes on the ring or the sphere.
- (2) *Growth*: At every time step, a new node is added, which is randomly placed on the ring or the sphere.
- (3) *Addition of edges*: The new node n connects with $m(m \leq m_0 + 1)$ previous nodes, which are selected with the probability π_i

$$\pi_i = a \frac{k_i}{\sum_j k_j} + (1 - a) \frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}^{-\alpha}} \quad (1)$$

where k_i is the degree of node i , r_{ni} is the Euclidean distance between a new node n and a previous node i , $0 \leq a \leq 1$ and $0 \leq \alpha$. The growing process repeats step (2) and (3) until the network reaches the desired size. Accordingly, at each step, the number of nodes increases by one, while the number of edges increases by m ($m = m_0 = 2$ in what follows if not mentioned). Hence at time t , the network contains $t + 1$ nodes and $m(t + 1)$ edges.

This model has two limit cases: when $a = 1$, the network recovers the SF network model, while the case of $a = 0$ and $\alpha = 0$ corresponds to the random growing process.

The numerical results described in this paper are the average of 20 simulations for different realization of networks under the same parameters with the network size of 10 000 nodes. We have also tried 50 000 nodes, but the result is almost the same.

3. Spatial driven model

In this section, we focus on the behavior of pure spatial-driven model with $a = 0$ and $\alpha \neq 0$ in Eq. (1). The connection probability is

$$\pi_i = \frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}^{-\alpha}}. \quad (2)$$

3.1. Degree distribution

The nodes are labeled by their birth times, $s = 0, 1, 2 \dots t$. $p(k, s, t)$ is the probability that the node s has degree k at time t . The master equation of $p(k, s, t)$ is given by

$$p(k, s, t + 1) = \frac{m}{t + 1} p(k - 1, s, t) + \left(1 - \frac{m}{t + 1}\right) p(k, s, t). \quad (3)$$

The initial conditions are $p(k, s = 0, 1 \dots m_0, t = m_0) = \delta_{k, m_0}$ and $p(k, t, t) = \delta_{k, m}$. $p(k, s, t + 1)$ contains two parts. The first one comes from the nodes having degree $k - 1$ at time t and selected to connect with the new node at time $t + 1$. The second one comes from the nodes having degree k at time t and not selected at time $t + 1$.

The degree distribution of the entire network can be written as

$$p(k, t) = \frac{1}{t + 1} \sum_{s=0}^t p(k, s, t). \quad (4)$$

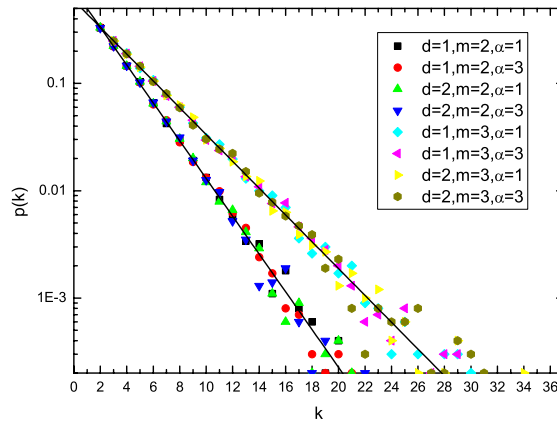


Fig. 1. The symbols represent the degree distribution $p(k)$ of a network of size $N = 10\,000$ grown according to our model for different $m, \alpha, d = 1$ and $d = 2$. The solid lines are analytic results given by Eq. (8).

Combining Eqs. (3) and (4), we get the following equation for the degree distribution

$$(t + 2)p(k, t + 1) - (t + 1)p(k, t) = mp(k - 1, t) - mp(k, t) + \delta_{k,m} \tag{5}$$

let $t \rightarrow \infty$, $p(k, t)$ will approach a stationary distribution $p(k)$ [12]. Eq. (5) becomes

$$(m + 1)p(k) - mp(k - 1) = \delta_{k,m} \tag{6}$$

which means

$$p(k) = \begin{cases} \frac{m}{m+1} p(k-1) & \text{if } k > m \\ \frac{1}{m+1} & \text{if } k = m. \end{cases} \tag{7}$$

The final degree distribution turns out to be

$$p(k) = \frac{1}{m+1} \left(\frac{m}{m+1} \right)^{k-m}, \quad (k \geq m) \tag{8}$$

which decays exponentially with k ($p(k) = 0$ for $k < m$), in agreement with the results of Ref. [12,23]. Thus the spatial driven network is an exponential network like most small-world networks [12,22,23].

Fig. 1 shows the results of numerical simulation compared to the analytical results of Eq. (8) with a good agreement for different α and m . The degree distribution is only affected by the number of new edges m added at every time step.

3.2. Spatial distribution of link

When networks are embedded in space, the spatial distance r between the nodes is well-defined. The network evolution follows the purely spatial motivation as in Eq. (2). When t is large enough, the nodes are homogeneously located on the one-dimension ring or two-dimension surface. The number of previous nodes at distance r from the new node is 2 when $d = 1$ and is proportional to $2\pi \sin \frac{r}{R}$ when $d = 2$. We define $\pi_t(r)$ as the probability of an added link between two specific nodes with distance r at time t , and $\Delta N(r, t)$ as the number of new links of length r that the network has at time t . $\Delta N(r, t)$ is given by the number of “neighbors” at distance r multiplied by the probability of link addition and by the number of new links, i.e., $\Delta N(r, t) = 2m\pi_t(r)$ for $d = 1$ and $\Delta N(r, t) \sim 2\pi m\pi_t(r) \sin \frac{r}{R}$ for $d = 2$.

In one-dimension, the number of links of length r at time t is

$$N(r, t) = \sum_{s=0}^t \Delta N(r, s) = 2m(\pi_0(r) + \pi_1(r) + \dots + \pi_t(r)). \tag{9}$$

From Eq. (2), we get

$$\pi_s(r) \sim r^{-\alpha} \tag{10}$$

and

$$N(r, t) \sim r^{-\alpha}. \tag{11}$$

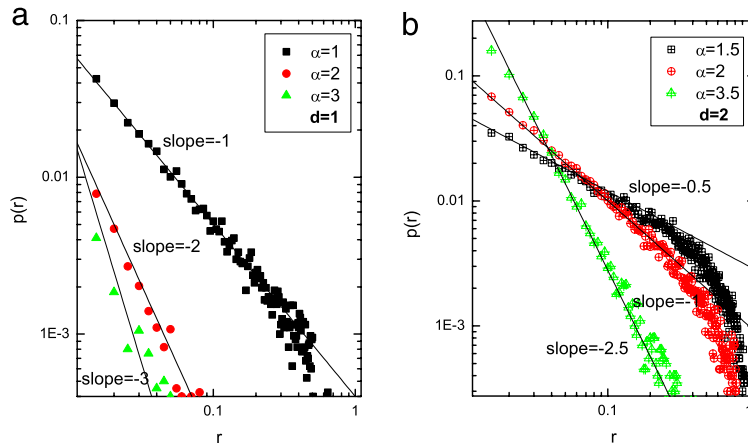


Fig. 2. (a) Probability $p(r)$ that a node has a connection at distance r for $\alpha = 1, 2, 3$ and $d = 1$. (b) Probability $p(r)$ that a node has a connection at distance r for $\alpha = 1.5, 2, 3.5$ and $d = 2$. The straight lines are the results of analytic calculation with the slope $-(\alpha - d + 1)$.

The spatial distribution of links is given by

$$p(r) = \frac{N(r, t)}{m(t + 1)} \sim r^{-\alpha}. \tag{12}$$

For the case of two-dimension, we have

$$N(r, t) = \sum_{s=0}^t \Delta N(r, s) \sim 2\pi m(\pi_0(r) + \pi_1(r) + \dots + \pi_t(r)) \sin \frac{r}{R}. \tag{13}$$

According to the Taylor series, the spatial distribution is given by

$$p(r) = \frac{N(r, t)}{m(t + 1)} \sim r^{-\alpha+1}. \tag{14}$$

From the above results, we can derive $p(r) \sim r^{-(\alpha-d+1)}$. This result has also been found by Kosmidis and Manna using numerical simulation [13,15]. Fig. 2 shows a good agreement between the results of numerical simulation and analytical calculation from Eqs. (12) and (14) for different values of α .

3.3. Clustering coefficient

The clustering coefficient of a single node i in network is defined as $c_i = \frac{2e_i}{k_i(k_i-1)}$ [1], where e_i is the total number of edges between all the k_i neighbors. The clustering coefficient C of the whole network is the average of c_i over all nodes.

Fig. 3 shows the behavior of the clustering coefficient C as a function of $1/N$ for different values of α . The variation of C follows $C \sim (1/N)^\delta$ where δ depends on α as shown in Fig. 4 for $d = 1$ and $d = 2$. The error bars are determined from the fitting with $C \sim (1/N)^\delta$. There are three regimes separated by $\alpha = d/2$ and $\alpha = 3d$ with different C behaviors. In the first regime $0 < \alpha \leq d/2$, the data follow $C \sim 1/N$, similar to the Erdős–Rényi (ER) random graph [24]. In the second regime $d/2 < \alpha < 3d$, the exponent δ decreases from 1 to 0 (see Fig. 4). In the third regime $3d \leq \alpha$, C is independent from N ($\delta = 0$), similar to the *OHO* model presented by Ozik et al. [12,23]. These regimes can be expressed as follows:

$$C \sim \begin{cases} 1/N & 0 < \alpha \leq d/2 \\ (1/N)^\delta & d/2 < \alpha < 3d \\ constant & 3d \leq \alpha. \end{cases} \tag{15}$$

Now let us see how the clustering coefficient C depends on α when the network size is fixed ($N = 10\,000, N = 15\,000, N = 20\,000$). Fig. 5 shows that the critical point at $\alpha = d/2$ separates a phase of vanishing clustering ($0 < \alpha \leq d/2$) from a phase of increasing clustering ($d/2 < \alpha < 3d$), and that the point at $\alpha = 3d$ separates the phase of increasing clustering from a phase of constant clustering ($3d \leq \alpha$). The increasing clustering coefficient with increasing α is expected from the model because larger α favors smaller distance connection and higher clustering. This spatial effect on C is specifically notable in the second phase $d/2 < \alpha < 3d$.

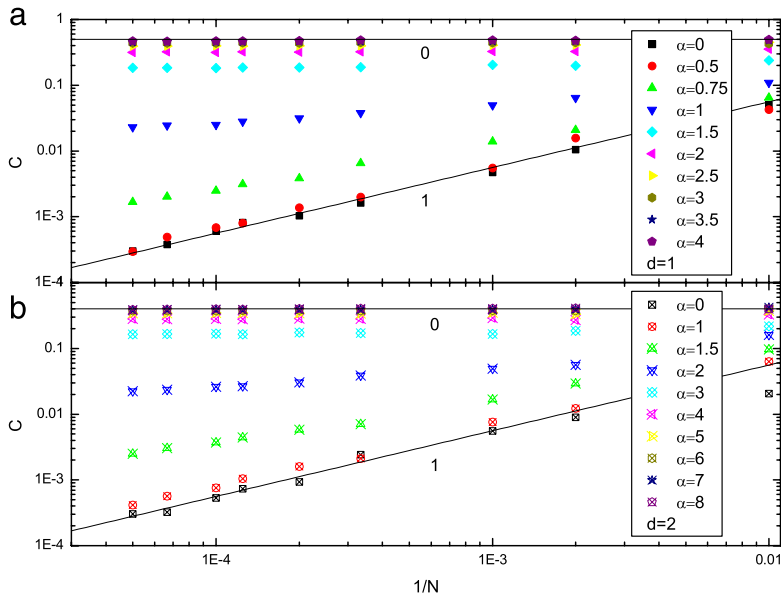


Fig. 3. (a) Clustering coefficient C as a function of $1/N$ for different α and $d = 1$. (b) Clustering coefficient C as a function of $1/N$ for different α and $d = 2$. Straight lines of slope 0 and 1 are best fits of the data.

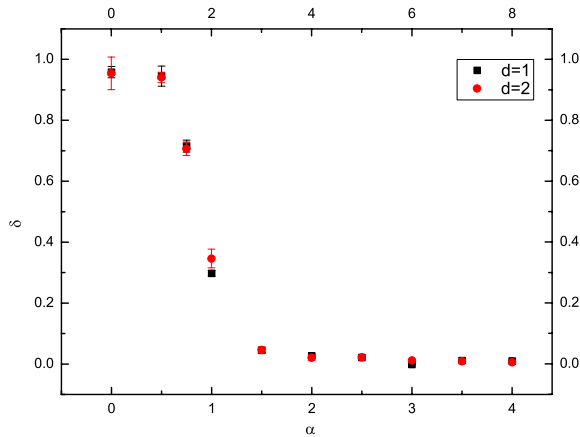


Fig. 4. Exponent δ as a function of α for $d = 1$ and $d = 2$. The left and the lower coordinates correspond to $d = 1$, the right and the upper coordinates correspond to $d = 2$. This figure shows that the α/d dependence of δ is free from dimension.

3.4. Topological distance

We can see in Fig. 6 that the mean topological distance l of the network follows the function $l = \gamma \log N$ for different α , which is a typical small world network behavior and different from what observed in Kosmidis's model embedded in regular lattices [13]. The α dependence of the slope γ is depicted in Fig. 7. The error bars are determined by the fitting with $l = \gamma \log N$. We are interested in the behavior at the regime transition points $\alpha = d/2$ and $\alpha = 3d$. In the first regime $0 < \alpha \leq d/2$ and the third one $3d \leq \alpha$, γ is independent from α . In the second regime $d/2 < \alpha < 3d$, γ increases from the first one to the third one. It is worth mentioning that, in the ER random model, $l = \log N / \log \langle k \rangle$ or $\gamma = 1 / \log \langle k \rangle$. In the present case, $\langle k \rangle = 2m = 4$, leading to $\gamma \approx 1.66$, close to the value of the first regime.

4. Network structure when $a \neq 0$

In many real space networks, the spatial distance between the nodes plays an important role in the formation of links, while the degree preferential attachment is also a natural property of linking. When $a \neq 0$ in our model, there is an interplay between the preference of larger degree and the preference of smaller distance in the evolution of the network. We note that in many models of network growth, the probability to get connected to a node at distance r is proportional to $r^{-\alpha}$.

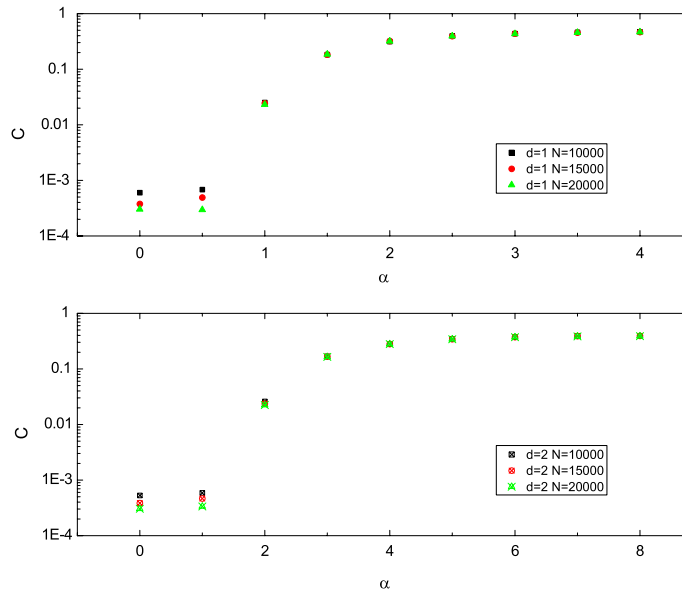


Fig. 5. Clustering coefficient C as a function of α with $N = 10\,000$, $15\,000$ and $20\,000$, for $d = 1$ and $d = 2$.

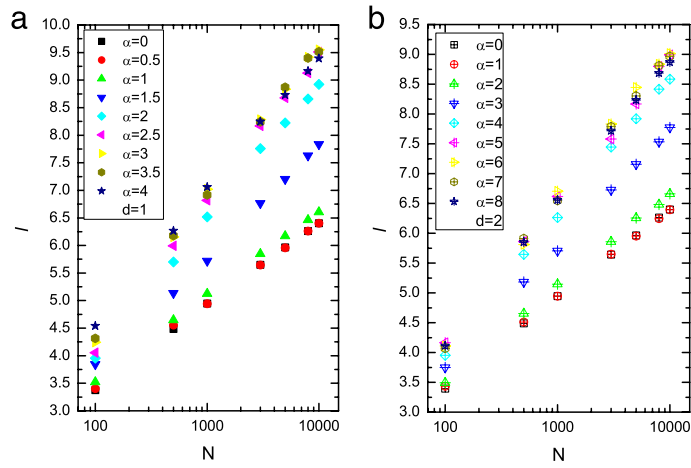


Fig. 6. (a) Topological distance l versus N for different α and $d = 1$. (b) Topological distance l versus N for different α and $d = 2$.

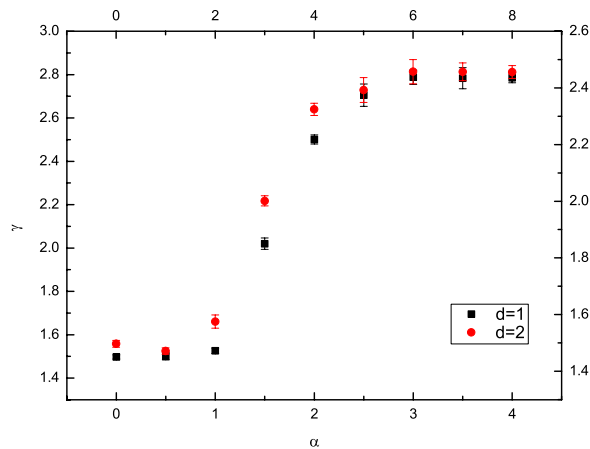


Fig. 7. α dependence of the slope γ . The left and the lower coordinates correspond to $d = 1$, the right and the upper coordinates correspond to $d = 2$.

From the simulation results, when α increases, the degree distribution of the network gradually changes from power-law to stretched exponential [15,19].

The degree increases each time when a new node n is added into the system and connects to a previous node i with probability π_i . Assume that k_i is a continuous variable, its increasing rate should be proportional to π_i and satisfy the following equation:

$$\frac{\partial k_i}{\partial t} = m\pi_i = ma \sum_j k_j + m(1-a) \frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}^{-\alpha}}. \tag{16}$$

The sum in the first term on the right-hand side goes over all nodes except the new one, giving $\sum_j k_j = 2mt$. Since the new node is randomly located, and the existing nodes are uniformly distributed, when t is large, the change of the degree of node i must be independent of where node i is on the circle or the sphere, so that the second term reads $\frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}^{-\alpha}} \approx \frac{1}{t}$ [25]. To prove

this relationship we use the mean-field approximation in which the spatial distance preference probability $\pi_i^s = \frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}^{-\alpha}}$ is represented by its average value. As we will show, the mean-field approximation turns out to be exact in the limit of large system size.

When t is large enough, the existing nodes are homogeneously located on the one-dimension ring or two-dimension surface. At time step t , we calculate π_i^s for T times, for each calculation the new node is placed randomly and labeled by $n_s = n_1, n_2, \dots, n_T$. The average value of π_i^s can be written as

$$\langle \pi_i^s \rangle = \frac{1}{T} \left(\frac{r_{n_1 i}^{-\alpha}}{\sum_{j=0}^{t-1} r_{n_1 j}^{-\alpha}} + \frac{r_{n_2 i}^{-\alpha}}{\sum_{j=0}^{t-1} r_{n_2 j}^{-\alpha}} + \dots + \frac{r_{n_T i}^{-\alpha}}{\sum_{j=0}^{t-1} r_{n_T j}^{-\alpha}} \right). \tag{17}$$

Due to the uniform distribution of the existing nodes, we have $\sum_{j=0}^{t-1} r_{n_1 j}^{-\alpha} = \sum_{j=0}^{t-1} r_{n_2 j}^{-\alpha} = \dots = \sum_{j=0}^{t-1} r_{n_T j}^{-\alpha}$. Hence

$$\langle \pi_i^s \rangle = \frac{1}{T} \frac{\sum_{l=1}^T r_{n_l i}^{-\alpha}}{\sum_{j=0}^{t-1} r_{n_s j}^{-\alpha}}. \tag{18}$$

In the denominator, $r_{n_s j}$ can be considered as a continuous variable between r_{min} and r_{max} , thus

$$\begin{aligned} \sum_{j=0}^{t-1} r_{n_s j}^{-\alpha} &= t \langle r_{n_s j}^{-\alpha} \rangle \\ &= t \int_{r_{min}}^{r_{max}} r_{n_s j}^{-\alpha} f(r_{n_s j}) dr_{n_s j} \end{aligned} \tag{19}$$

where $f(r_{n_s j})$ is the probability density of $r_{n_s j}$, satisfying $\int_{r_{min}}^{r_{max}} f(r_{n_s j}) dr_{n_s j} = 1$. For uniform distribution of nodes, $f(r_{n_s j})$ must be a constant, let it be c , and Eq. (19) can be written as

$$\sum_{j=0}^{t-1} r_{n_s j}^{-\alpha} = tc \int_{r_{min}}^{r_{max}} r_{n_s j}^{-\alpha} dr_{n_s j}. \tag{20}$$

This calculation also works for the sum over l from 1 to T , in the numerator. Since for large T , the new nodes are also uniformly distributed over $r_{n_l i}$, so the probability density $f(r_{n_l i})$ should be equal to c . Hence

$$\sum_{l=1}^T r_{n_l i}^{-\alpha} = Tc \int_{r_{min}}^{r_{max}} r_{n_l i}^{-\alpha} dr_{n_l i}. \tag{21}$$

Take Eqs. (20) and (21) to Eq. (18), Eq. (18) can be written as

$$\langle \pi_i^s \rangle = \frac{1}{T} \frac{Tc \int_{r_{min}}^{r_{max}} r_{n_l i}^{-\alpha} dr_{n_l i}}{tc \int_{r_{min}}^{r_{max}} r_{n_s j}^{-\alpha} dr_{n_s j}} = \frac{1}{t}. \tag{22}$$

From the above derivations and mean-field approximation, we can get

$$\pi_i^s \approx \langle \pi_i^s \rangle = \frac{1}{t}. \tag{23}$$

Substituting this result back into Eq. (16), the evolution of node i 's degree follows

$$\frac{\partial k_i}{\partial t} \approx a \frac{k_i}{2t} + (1-a) \frac{m}{t} = \frac{ak_i + 2(1-a)m}{2t}. \quad (24)$$

In the view of the initial condition $k_i(t_i) = m$ for the degree of a node added at time t_i , Eq. (24) has the solution

$$k_i = \frac{(2m - am) \left(\frac{t}{t_i}\right)^{\frac{a}{2}} - 2(1-a)m}{a} \quad (25)$$

from which the probability $P(k_i(t) < k)$ that a node has degree $k_i(t)$ smaller than k is given by

$$P(k_i(t) < k) = P\left(t_i > \frac{t}{\left(\frac{ak+2(1-a)m}{2m-am}\right)^{\frac{2}{a}}}\right). \quad (26)$$

Since the addition of nodes and links are carried out at equal time interval, the probability density at t_i is: $P_i(t_i) = 1/t_i$. Thus

$$P(k_i(t) < k) = 1 - \left(\frac{ak + 2(1-a)m}{2m - am}\right)^{-\frac{2}{a}}. \quad (27)$$

Then the probability density $p(k)$ reads

$$p(k) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{2}{2m - am} \left(\frac{a}{2m - am}\right)^{-\frac{2+a}{a}} \left[k + \frac{2(1-a)m}{a}\right]^{-\frac{2+a}{a}}. \quad (28)$$

This is the “shifted power law” (SPL) function. When a changes from 0 to 1, the degree distribution gradually changes from an exponential law to a power law. In Fig. 8, the symbols represent the degree distribution $p(k)$ of our model for different values of a and α ($d = 1$ and 2). The solid lines are given by Eq. (28) with $m = 2$, $a = 0.1, 0.5$ and 0.9 . The agreement between simulation results and analytical results means that the analysis from Eq. (16) to Eq. (28) is close to the numerical simulation with the model. SPL degree distribution is confirmed by empirical data as well [26,27].

Fig. 9 shows the behavior of the clustering coefficient C for different a and α . In the first regime $0 < \alpha \leq d/2$ (see Fig. 9a for $d = 1$ and Fig. 9d for $d = 2$), $\ln C$ increases with a independently from α . Fig. 9b and Fig. 9e correspond to the second regime $d/2 < \alpha < 3d$ in which $\ln C$ decreases linearly with a first and then increases slightly until $a = 1$. On the other hand, $\ln C$ increases with increasing α and its minimum seems to be dependent on α . In the third regime $3d \leq \alpha$ (see Fig. 9c for $d = 1$ and Fig. 9f for $d = 2$), $\ln C$ decreases with a independently from α .

5. Comparison with empirical data

Previous works have shown that the establishment of friendship and relationship is influenced by spatial constraints [7,18,28,29]. In Ref. [29], Goldenberg and Levy collected data on the location of the receivers of more than 4400 email messages and found that the spatial distribution of the communication was a power law. This motivates us to compare the topological structure of our model to the structure of the empirical data on email communication network [30], which consists of 1133 nodes and 5451 edges. In order to apply the two-dimensional model to the empirical data, we first set $N = 1133$ and $m = m_0 = 5$, then adjust $a = 0.02$ to match the degree distribution, finally set $\alpha = 5$ to match $\langle c \rangle$ and $\langle l \rangle$. The comparison of degree distribution is shown in Fig. 10. We employ the Kolmogorov–Smirnov test [31] in the software SPSS to compare the degree distribution of empirical data and that of our model. The result is $P - value = 0.826$ (when $P - value > 0.05$ means that the two comparative samples come from the same kind of distribution). Therefore we can conclude the email network [30] and our model follow the same kind of degree distribution. The topological parameters are listed in Table 1 with good agreement. The small value of a implies that the competition between degree and spatial distance preference for attachment may be present in the evolution of email systems and that the degree preference plays a much less important role than spatial distance. On the other hand, recent works [28,32] have shown that the degree dependence of clustering coefficient of several real networks follows a power law $C(k) \sim k^{-1}$. This tendency is confirmed by the email network data of Ref. [30] and our simulation (see Fig. 11).

6. Summary

In summary, we have studied an evolutionary network in the configuration space with a model in which the probability of attachment is controlled by two competing factors: degree preference and spatial distance preference. These two factors are modulated by two parameters a and α .

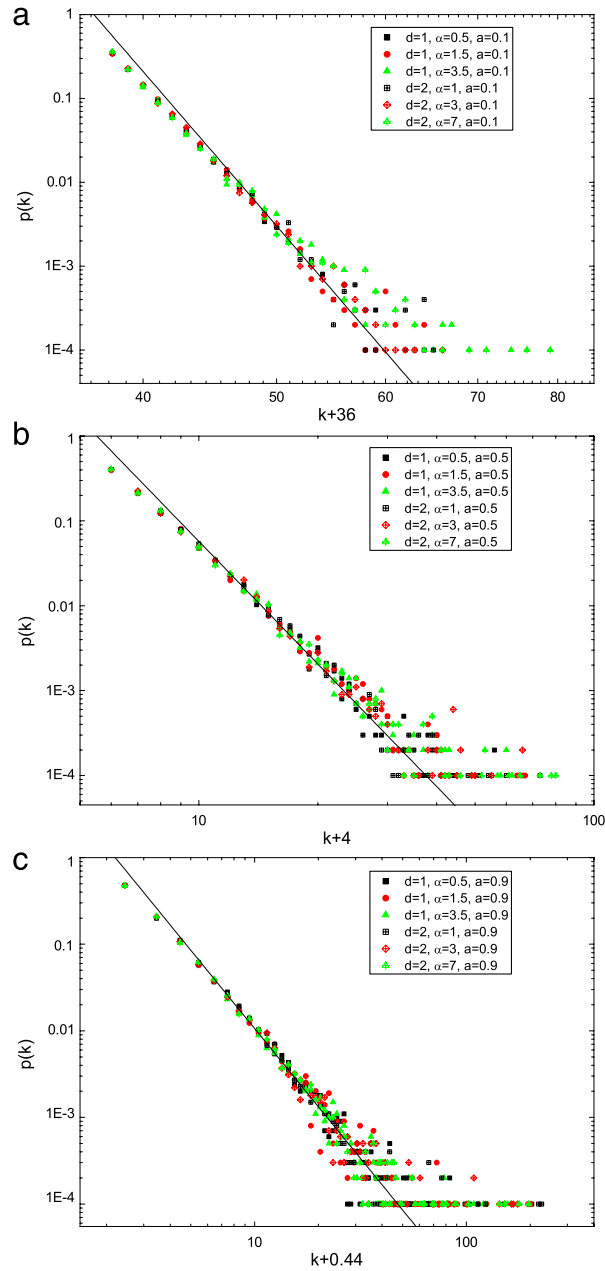


Fig. 8. (a) Degree distribution for $d = 1, a = 0.1, \alpha = 0.5, 1.5, 3.5$ and $d = 2, a = 0.1, \alpha = 1, 3, 7$. (b) Degree distribution for $d = 1, a = 0.5, \alpha = 0.5, 1.5, 3.5$ and $d = 2, a = 0.5, \alpha = 1, 3, 7$. (c) $d = 1, a = 0.9, \alpha = 0.5, 1.5, 3.5$ and $d = 2, a = 0.9, \alpha = 1, 3, 7$. The straight lines are analytic results given by Eq. (28) with the same a values as in the simulation, i.e. $a = 0.1, 0.5$ and 0.9 , respectively.

Table 1

Comparison of empirical data from email network and simulation results. N is the number of nodes, $\langle k \rangle$ is the average degree, $\langle c \rangle$ is the mean clustering coefficient, $\langle l \rangle$ is topological distance. The simulation results come from our model with parameters $\alpha = 5, a = 0.02, N = 1133, m = m_0 = 5$.

	N	$\langle k \rangle$	$\langle c \rangle$	$\langle l \rangle$
Email	1133	9.6222	0.2211	3.6060
Model	1133	10.0000	0.2214	3.5571

When $a = 0$ and $\alpha \neq 0$, the model reduces to the spatial driven model, exhibiting phase transitions at some critical values of α . For the regime $0 < \alpha \leq d/2$, where d is the dimension of the embedding space, the network has short topological

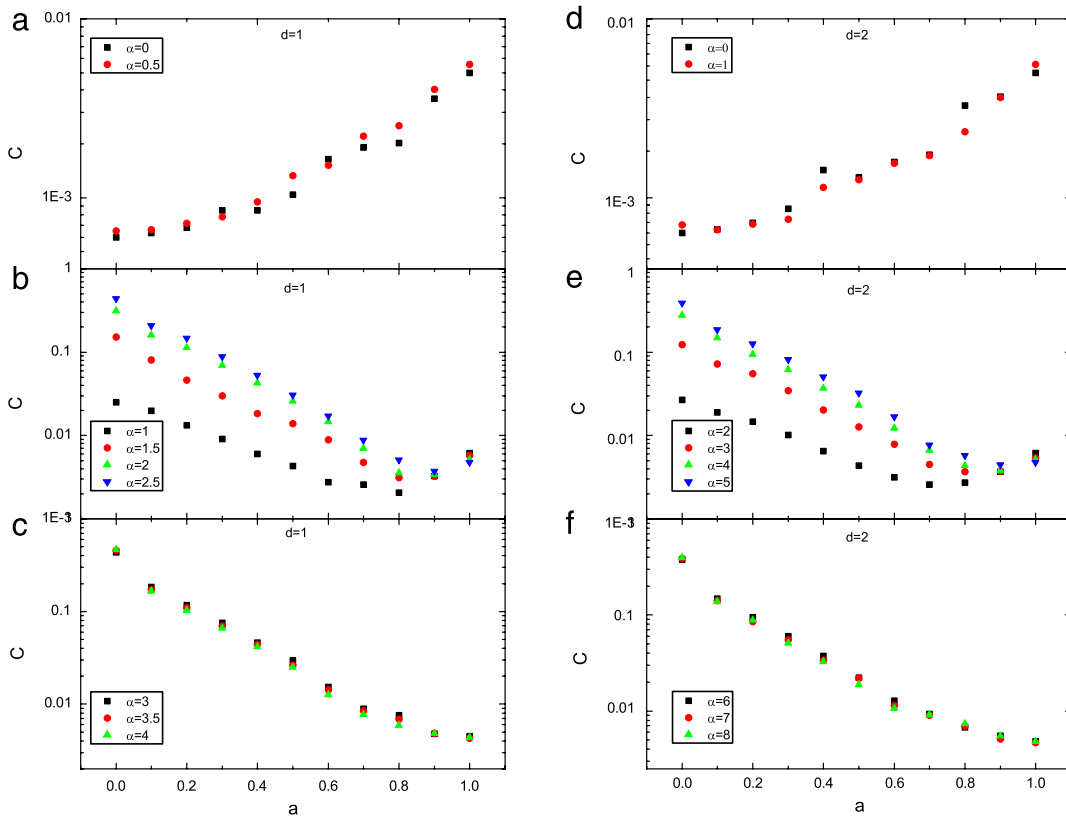


Fig. 9. (a) Clustering coefficient C as a function of a for $d = 1$ in the first regime for $\alpha = 0$ and 0.5 . (b) C as a function of a for $d = 1$ in the second regime for $\alpha = 1, 1.5, 2, 2.5$. (c) C as a function of a for $d = 1$ in the third regime for $\alpha = 3, 3.5, 4$. (d) C as a function of a for $d = 2$ in the first regime for $\alpha = 0, 1$. (e) C as a function of a for $d = 2$ in the second regime for $\alpha = 2, 3, 4, 5$. (f) C as a function of a for $d = 2$ in the third regime for $\alpha = 6, 7, 8$.

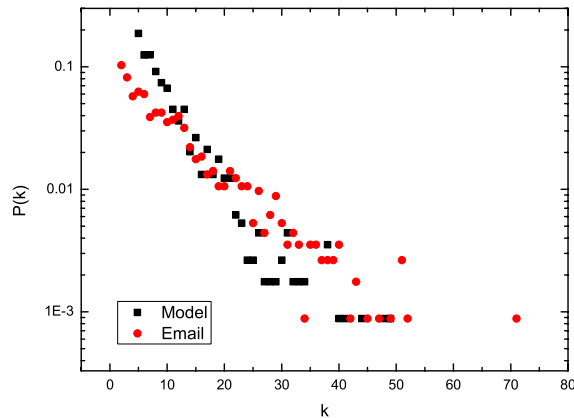


Fig. 10. Comparison of empirical data from email network and simulation results of the degree distribution. The simulation results come from our model with parameters $\alpha = 5, a = 0.02, N = 1133, m = m_0 = 5$.

distance and vanishing clustering as in the *ER* random model. For $d/2 < \alpha < 3d$, the network has increasing clustering coefficient with increasing α . The topological distance is short as well. For $3d \leq \alpha$, the network has a constant clustering, similar to the *OHO* model. In all these regimes, the spatial distribution follows the power law $p(r) \sim r^{-(\alpha-d+1)}$, the degree distribution follows exponential law.

When $a \neq 0$, there will be more long-range links caused by degree preferential attachment. The degree distribution follows shifted power law. When a changes from 0 to 1, the network property changes from the property of spatial driven network to the property of scale-free network.

The qualitatively consistent with empirical results reveals that the model has captured some basic mechanisms for the evolution of social communication networks. We hope that it will be helpful for further study and understanding of real

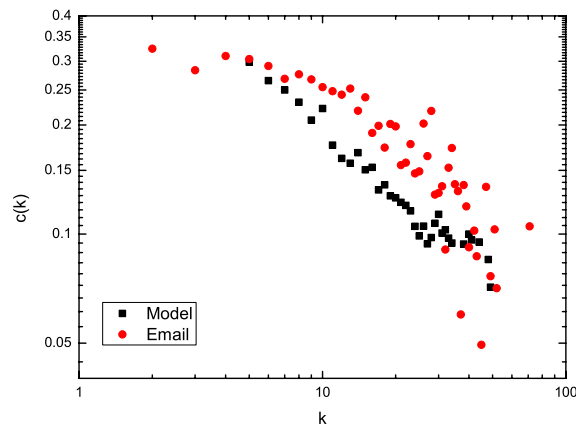


Fig. 11. Comparison of empirical data and simulation result of the k dependence of clustering coefficient. The simulation results come from our model with parameters $\alpha = 5$, $a = 0.02$, $N = 1133$, $m = m_0 = 5$.

networks whose evolution is influenced by the interplay of different even competing dynamics. The tunable parameter a can also become an object of investigation and of optimization when true physical processes such as epidemic diffusion, informational diffusion, opinion spreading, culture propagation, transport of matter and so forth, are considered in a spatial network.

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