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by

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(Dated:)

Super quantum discord is a kind of quantum correlations defined in terms of weak measurements for obtaining the maximal classical correlations in a bipartite systems. We derive explicitly the analytical formula for general X-type two-qubit states. The evolution of these states under local and global bit-flip and phase-flip channels are investigated. The relations among the super quantum discord, its evolution and the week measurement strength have been analyzed in detail.

I. INTRODUCTION

The quantum entanglement plays important roles in quantum information processing [1]. However, besides quantum entanglement there are other quantum correlations that is also useful for quantum information processing. It is found that many tasks can be carried out with quantum correlations other than entanglement [2–4]. In particular, the quantum discord [5–19]. plays an important role in some quantum information processing like assisted optimal state discrimination, in which only one side discord is required in the optimal process of assisted state discrimination, while another side discord and entanglement is not necessary [20].

The quantum discord is defined by the POVM quantum measurement. When we measure a quantum state in some orthogonal basis, the coherence of the state has been loosen. It is reasonable to replace the POVM measurement by weak measurement [21] in the definition of quantum discord, which gives rise to so called super quantum discord (SQD) [22]. The super quantum discord of Wenner states and Bell-diagonal states have been studied already. In this paper, we derive explicit formula for a seven-parameter family of states called X-state. We analyze the dynamics of two-qubit X-state under non-dissipative channels. In section II, we derive the analytical formulae of super quantum discord for X-state. In section III we compute the super quantum discord of some detailed examples, and analyze their dynamics under nondissipative channels.

II. SUPER QUANTUM DISCORD FOR TWO-QUBIT X-STATES

We consider the two-qubit X-state:

$$\rho_X = \begin{pmatrix} a_{11} & 0 & 0 & a_{14} \\ 0 & a_{22} & a_{23} & 0 \\ 0 & a_{23}^* & a_{33} & 0 \\ a_{14}^* & 0 & 0 & a_{44} \end{pmatrix}, \tag{1}$$

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where $\sum_{i=1}^{4} a_{ii}=1$, $|a_{23}^2|\leq a_{22}a_{33}$, $|a_{14}^2|\leq a_{11}a_{44}$. Density matrix.(1) can be written as [23]:

$$\rho_X=\frac{1}{4}\begin{pmatrix} 1+d_1 & 0 & 0 & c_1-c_2 \\ 0 & 1+d_2 & c_1+c_2 & 0 \\ 0 & c_1^*+c_2^* & 1+d_3 & 0 \\ c_1^*-c_2^* & 0 & 0 & 1+d_4 \end{pmatrix}, \tag{2}$$

where c_1 and c_2 are complex, d_1, d_2, d_3 and d_4 are real, $d_1=c_3+a_3+b_3$, $d_2=-c_3+a_3-b_3$, $d_3=-c_3-a_3+b_3$, $d_4=c_3-a_3-b_3$. These parameters are determined by the entries of the density matrix, $a_3=a_{11}-a_{44}+a_{22}-a_{33}$, $b_3=a_{11}-a_{44}-a_{22}+a_{33}$, $c_3=a_{11}+a_{44}-a_{22}-a_{33}$, $c_1=2(a_{23}+a_{14})$, $c_2=2(a_{23}-a_{14})$.

Let $\{\Pi_i^B\}$, $i=0,1$, be the projective measurement. The discord of a bipartite quantum state ρ_{AB} with the measurement $\{\Pi_i^B\}$ on the subsystem B is the dissimilarity between the mutual information $I(\rho_{AB})$ [24] and the classical correlation $J_B(\rho_{AB})$ [25]:

$$D(\rho_{AB})=\min_{\{\Pi_i^B\}}\sum_i p_i S(\rho_{A|i})+S(\rho_B)-S(\rho_{AB}), \tag{3}$$

where the minimization goes over all projective measurements $\{\Pi_i^B\}$, $S(\rho)=-tr(\rho\log_2\rho)$ is the von Neumann entropy of a quantum state ρ, ρ_B is the reduced density matrices of ρ_{AB} and

$$p_i=tr_{AB}[(I_A\otimes\Pi_i^B)\rho_{AB}(I_A\otimes\Pi_i^B)], \tag{4}$$

$$\rho_{A|i}=\frac{1}{p_i}tr_B[(I_A\otimes\Pi_i^B)\rho_{AB}(I_A\otimes\Pi_i^B)]. \tag{5}$$

The weak measurement operations are given by [26],

$$P(+x)=\sqrt{\frac{1-\tanh x}{2}}\Pi_0+\sqrt{\frac{1+\tanh x}{2}}\Pi_1, \tag{6}$$

$$P(-x)=\sqrt{\frac{1+\tanh x}{2}}\Pi_0+\sqrt{\frac{1-\tanh x}{2}}\Pi_1, \tag{7}$$

where Π_0 and Π_1 are two orthogonal projectors satisfying $\Pi_0+\Pi_1=I$, x is the strength parameter of measurement. The weak measurement operators satisfy: (i) $P^\dagger(+x)P(+x)+P^\dagger(-x)P(-x)=I$, (ii) $\lim_{x\to\infty}P(+x)=\Pi_0$ and $\lim_{x\to\infty}P(-x)=\Pi_1$.

The super quantum discord is defined by [22]:

$$D_w(\rho_{AB})=\min_{\{\Pi_i^B\}}S_w(A|\{P^B(x)\})+S(\rho_B)-S(\rho_{AB}),$$

where

$$S_w(A|\{P^B(x)\})=p(+x)S(\rho_{A|P^B(+x)})+p(-x)S(\rho_{A|P^B(-x)}),$$

$$p(\pm x)=tr_{AB}[(I_A\otimes P^B(\pm x))\rho_{AB}(I_A\otimes P^B(\pm x))],$$

$$\rho_{A|P^B(\pm x)}=\frac{tr_B[(I_A\otimes P^B(\pm x))\rho_{AB}(I_A\otimes P^B(\pm x))]}{tr_{AB}[(I_A\otimes P^B(\pm x))\rho_{AB}(I_A\otimes P^B(\pm x))]},$$

where $\{P^B(x)\}$ is the weak measurement operators on subsystem B.

The weak measurement operators on subsystem B can be expressed as,

$$I_A\otimes P^B(\pm x)=\sqrt{\frac{1\mp\tanh x}{2}}I\otimes V\Pi_0V^\dagger+\sqrt{\frac{1\pm\tanh x}{2}}I\otimes V^\dagger\Pi_1V^\dagger, \tag{8}$$

where $\Pi_k = |k\rangle\langle k|$, $k = 0, 1$, $|k\rangle$ is the computational base, and $V \in U(2)$. $V \in U(2)$ can be generally expressed as [27]:

$$V = tI + i\overrightarrow{y} \cdot \overrightarrow{\sigma}, \tag{9}$$

where $\overrightarrow{y} = (y_1, y_2, y_3)$ and $t, y_1, y_2, y_3 \in R^1$, $t^2 + y_1^2 + y_2^2 + y_3^2 = 1$.

To evaluate the super quantum discord of ρ_X, let us first express ρ_X in terms of the bases $I \otimes I, \sigma_i \otimes \sigma_j$, $i, j = 0, 1, 2$.

$$\rho_X = \frac{1}{4}(I + \sum_i \Re(c_i)\sigma_i \otimes \sigma_i) + \frac{1}{4}[(b_3 - a_3)I \otimes \sigma_3 + (\Im(c_1) + \Im(c_2)\sigma_1 \otimes \sigma_2)],$$

where $\Re(c_i), \Im(c_i)$ are the real and complex parts of c_i. By using the relations [27]

$$\begin{aligned}
V^\dagger\sigma_1 V &= (t^2 + y_1^2 - y_2^2 - y_3^2)\sigma_1 + 2(ty_3 + y_1y_2)\sigma_2 + 2(-ty_2 + y_1y_3)\sigma_3,\\
V^\dagger\sigma_2 V &= (t^2 + y_2^2 - y_1^2 - y_3^2)\sigma_2 + 2(ty_1 + y_2y_3)\sigma_1 + 2(-ty_3 + y_1y_2)\sigma_3,\\
V^\dagger\sigma_1 V &= (t^2 + y_3^2 - y_1^2 - y_2^2)\sigma_3 + 2(ty_2 + y_1y_3)\sigma_1 + 2(-ty_1 + y_2y_3)\sigma_3,
\end{aligned}$$

and $\Pi_0\sigma_3\Pi_0 = \Pi_0$, $\Pi_1\sigma_3\Pi_1 = -\Pi_1$, $\Pi_j\sigma_k\Pi_j = \Pi_0$ for $j = 0, 1$, $k = 1, 2$. Set $a_1 = z_1\Re(c_1) + z_2\Im(c_2)$, $a_2 = z_2\Re(c_2) - z_1\Im(c_1)$, with $z_1 = 2(-ty_2 + y_1y_3)$, $z_2 = 2(ty_1 + y_2y_3)$, $z_3 = t^2 + y_3^2 - y_1^2 - y_2^2$. After weak measurement, from Eq.(4) and Eq.(5) we have the ensemble $\{\rho_{A|P^B(\pm x)}, p(\pm x)\}$,

$$p(+x) = \frac{1}{2}(1 + b_3z_3), \qquad \rho_{A|P^B(+x)} = \frac{1}{2}[I - \frac{\tanh x(a_1\sigma_1 + a_2\sigma_2 + (a_3 + c_3z_3)\sigma_3)}{1 + b_3z_3}], \tag{10}$$

$$p(-x) = \frac{1}{2}(1 - b_3z_3), \qquad \rho_{A|P^B(-x)} = \frac{1}{2}[I + \frac{\tanh x(-a_1\sigma_1 - a_2\sigma_2 + (a_3 - c_3z_3)\sigma_3)}{1 - b_3z_3}]. \tag{11}$$

The eigenvalues of $\rho_{A|P^B(+x)}$ and $\rho_{A|P^B(-x)}$ are given by

$$\frac{1}{2}[1 \pm \frac{\tanh x}{1 + b_3z_3}\sqrt{(a_3 + c_3z_3)^2 + (a_2^2 + a_1^2)}], \tag{12}$$

$$\frac{1}{2}[1 \pm \frac{\tanh x}{1 - b_3z_3}\sqrt{(a_3 - c_3z_3)^2 + (a_2^2 - a_1^2)}]. \tag{13}$$

We compute now the minimum value of $S(\rho_{A|P^B(+x)})$ and the corresponding $p(+x)$. By using the method of Hessian matrix and the symmetries in Eqs.(9)-(13), we have that the minimum value lies at the $z_3 = 0$ or $z_3 = 1$. The extremum lies at the following points:

(z_3, z_2, z_1)	$p(+x)$	$\lambda_\pm$
(1,0,0)	$\frac{1+b_3}{2}$	$\frac{1}{2}[1 \pm \sqrt{\frac{(c_3+a_3)^2}{(1+b_3)^2}} \cdot \tanh x]$
(0,0,1)	$\frac{1}{2}$	$\frac{1}{2}[1 \pm \sqrt{a_3^2 + \lvert c_1\rvert^2} \cdot \tanh x]$
(0,1,$\frac{1}{2}$)	$\frac{1}{2}$	$\frac{1 \pm \sqrt{a_3^2 + \lvert c_2\rvert^2 - \lvert c_1 - c_2\rvert^2 - \lvert c_1 + c_2\rvert^2 + (\Re(c_1-c_2) + \Im(c_1+c_2))^2 + (\Re(c_1+c_2) + \Im(c_1-c_2))^2} \cdot \tanh x}{2}$
(0,-1,$-\frac{1}{2}$)	$\frac{1}{2}$	$\frac{1 \pm \sqrt{a_3^2 + \lvert c_2\rvert^2 - \lvert c_1 - c_2\rvert^2 - \lvert c_1 + c_2\rvert^2 + (\Re(c_1-c_2) - \Im(c_1+c_2))^2 + (\Re(c_1+c_2) - \Im(c_1-c_2))^2} \cdot \tanh x}{2}$

Similarly for the minimum value of $S(\rho_{A|P^B(-x)})$ and $p(-x)$, we have:

(z_3, z_2, z_1)	$p(-x)$	$\lambda'_{\pm}$
(1,0,0)	$\frac{1-b_3}{2}$	$\frac{1}{2}[1 \pm \sqrt{\frac{(c_3-a_3)^2}{(1-b_3)^2}} \cdot \tanh x]$
(0,0,1)	$\frac{1}{2}$	$\frac{1}{2}[1 \pm \sqrt{a_3^2 + \lvert c_1\rvert^2} \cdot \tanh x]$
(0,1,$\frac{1}{2}$)	$\frac{1}{2}$	$\frac{1\pm\sqrt{a_3^2+\lvert c_2\rvert^2-\lvert c_1-c_2\rvert^2-\lvert c_1+c_2\rvert^2+(\Re(c_1-c_2)+\Im(c_1+c_2))^2+(\Re(c_1+c_2)+\Im(c_1-c_2))^2}\cdot\tanh x}{2}$
(0,-1,$-\frac{1}{2}$)	$\frac{1}{2}$	$\frac{1\pm\sqrt{a_3^2+\lvert c_2\rvert^2-\lvert c_1-c_2\rvert^2-\lvert c_1+c_2\rvert^2+(\Re(c_1-c_2)-\Im(c_1+c_2))^2+(\Re(c_1+c_2)-\Im(c_1-c_2))^2}\cdot\tanh x}{2}$

From the above formulae, for a given state ρ_X, one can get the minimum values of $\lambda_\pm$ and $\lambda'_\pm$, which give rise to

$$S_w(\rho_{A|P^B(+x)}) = -\lambda_+ \log_2 \lambda_+ - \lambda_- \log_2 \lambda_-, \tag{14}$$

$$S_w(\rho_{A|P^B(-x)}) = -\lambda'_+ \log_2 \lambda'_+ - \lambda'_- \log_2 \lambda'_-, \tag{15}$$

and the super quantum discord

$$D_w(\rho_X) = p(+x)S_w(\rho_{A|P^B(+x)}) + p(-x)S_w(\rho_{A|P^B(-x)}) + S(\rho_X^B) - S(\rho_X). \tag{16}$$

III. DYNAMICS OF SUPER QUANTUM DISCORD UNDER NONDISSIPATIVE CHANNELS

First let us consider a special X-state, the Werner state [28],

$$\rho_W = \begin{pmatrix} \frac{1+z}{4} & 0 & 0 & \frac{z}{2} \\ 0 & \frac{1-z}{4} & 0 & 0 \\ 0 & 0 & \frac{1-z}{4} & 0 \\ \frac{z}{2} & 0 & 0 & \frac{1+z}{4} \end{pmatrix},$$

From Eqs.(10)-(16) we get $\lambda_{\pm i} = \lambda'_{\pm i} = (1 \pm z \tanh x)/2$, $\forall$ i. The eigenvalues of ρ_W^B are $\frac{1}{2}, \frac{1}{2}$, and the eigenvalues of ρ_W are $\frac{1+3z}{4}, \frac{1-z}{4}, \frac{1-z}{4}, \frac{1-z}{4}$. From Eq.(16) we have the super quantum discord of ρ_W,

$$D_w = -\frac{1-z\tanh x}{2}\log\frac{1-z\tanh x}{2} - \frac{1+z\tanh x}{2}\log\frac{1+z\tanh x}{2} + 1 + \frac{3(1-z)}{4}\log\frac{(1-z)}{4} + \frac{(1+3z)}{4}\log\frac{(1+3z)}{4},$$

which is in coincident with the result in [22]. Here as $\lambda_{\pm i} = \lambda'_{\pm i}$ for the Wener state, in fact, one gets the same results for any measurement basis.

For the Bell-diagonal states [29],

$$\rho = \begin{pmatrix} \frac{1+c_3}{4} & 0 & 0 & \frac{c_1-c_2}{4} \\ 0 & \frac{1-c_3}{4} & \frac{c_1+c_2}{4} & 0 \\ 0 & \frac{c_1+c_2}{4} & \frac{1-c_3}{4} & 0 \\ \frac{c_1-c_2}{4} & 0 & 0 & \frac{1+c_3}{4} \end{pmatrix},$$

we get $\lambda_{\pm 1} = \lambda'_{\pm 1} = \frac{1\pm c_1 \tanh x}{2}$, $\lambda_{\pm 2} = \lambda'_{\pm 2} = \frac{1\pm c_2 \tanh x}{2}$, $\lambda_{\pm 3} = \lambda'_{\pm 3} = \frac{1\pm c_3 \tanh x}{2}$, $\lambda_{\pm 4} = \lambda'_{\pm 4} = \frac{1\pm c_3 \tanh x}{2}$, the eigenvalues of ρ^B are $\frac{1}{2}, \frac{1}{2}$ and the eigenvalues of ρ are $\frac{1-c_1-c_2-c_3}{4}, \frac{1-c_1+c_2+c_3}{4}, \frac{1+c_1-c_2+c_3}{4}, \frac{1+c_1+c_2-c_3}{4}$. Let $c =$

$\max\{c_1, c_2, c_3\}$. From Eq.(16) we have the super quantum discord:

$$
\begin{aligned}
D_w = &-\frac{1-c\tanh x}{2}\log_2\frac{1-c\tanh x}{2} - \frac{1+c\tanh x}{2}\log_2\frac{1+c\tanh x}{2} + 1 \\
&+ \frac{1-c_1-c_2-c_3}{4}\log_2\frac{1-c_1-c_2-c_3}{4} + \frac{1-c_1+c_2+c_3}{4}\log_2\frac{1-c_1+c_2+c_3}{4} \\
&+ \frac{1+c_1-c_2+c_3}{4}\log_2\frac{1+c_1-c_2+c_3}{4} + \frac{1+c_1+c_2-c_3}{4}\log_2\frac{1+c_1+c_2-c_3}{4},
\end{aligned}
$$

which supported by result [30].

In the following we study the evolution of super quantum discord under bit flip noise, characterized by Kraus operators,

$$E_0 = \sqrt{p}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad E_1 = \sqrt{1-p}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{17}$$

We consider two channels: the "local bit-flip(Λ_{lbf})" and the "global bit-flip(Λ_{gbf})":

$$\Lambda_{lbf}(\rho_X) = (I\otimes E_0)\rho_X(I\otimes E_0)^\dagger + (I\otimes E_1)\rho_X(I\otimes E_1)^\dagger, \tag{18}$$

$$
\begin{aligned}
\Lambda_{gbf}(\rho_X) = &(E_0\otimes E_0)\rho_X(E_0\otimes E_0)^\dagger + (E_0\otimes E_1)\rho_X(E_0\otimes E_1)^\dagger \\
&+(E_1\otimes E_0)\rho_X(E_1\otimes E_0)^\dagger + (E_1\otimes E_1)\rho_X(E_1\otimes E_1)^\dagger.
\end{aligned} \tag{19}
$$

Under these channels, the netries of the density matrix have the following transformations:

channel	a_{11}	a_{14}
local bit-flip	$a_{22}+pa_{11}-pa_{22}$	$a_{23}+pa_{14}-pa_{23}$
global bit-flip	$(-1+p)^2a_{44}+p(a_{22}+a_{33}+pa_{11}-pa_{22}-pa_{33})$	$(-1+p)^2a^*_{14}+p(a_{23}+pa_{14}-pa_{23}+a^*_{23}-pa^*_{23})$
channel	a_{22}	a_{23}
local bit-flip	$a_{11}-pa_{11}+pa_{22}$	$a_{14}-pa_{14}+pa_{23}$
global bit-flip	$(-1+p)^2a_{33}+p(a_{11}+a_{44}-pa_{11}+pa_{22}-pa_{44})$	$p(a_{14}-pa_{14}+pa_{23})-(-1+p)pa^*_{14}+(-1+p)^2a^*_{23}$
channel	a_{33}	a_{44}
local bit-flip	$a_{44}+pa_{33}-pa_{44}$	$a_{33}-pa_{33}+pa_{44}$
global bit-flip	$(-1+p)^2a_{22}+p(a_{11}+a_{44}-pa_{11}+pa_{33}-pa_{44})$	$(-1+p)^2a_{11}+p(a_{22}+a_{33}-pa_{22}-pa_{33}+pa_{44})$
channel	a^*_{23}	a^*_{14}
local bit-flip	$a^*_{14}-pa^*_{14}+pa^*_{23}$	$a^*_{23}+pa^*_{14}-pa^*_{23}$
global bit-flip	$(-1+p)((-1+p)a_{23}-pa_{14})-(-1+p)pa^*_{14}+p^2a^*_{23}$	$p^2a^*_{14}+(-1+p)((-1+p)a_{14}-pa_{23}-pa^*_{23})$

As example, let us consider

$$\rho_X = \begin{pmatrix} 0.25 & 0 & 0 & 0.0625 \\ 0 & 0.25 & 0.125 & 0 \\ 0 & 0.125 & 0.25 & 0 \\ 0.0625 & 0 & 0 & 0.25 \end{pmatrix}. \tag{20}$$

Form Eqs.(10)-(13) and above charts, we get $p(+x) = p(-x) = 0.5$, $\lambda_+ = \lambda'_+ = \max\{0.5,\ 0.5+0.1875\cdot\tanh x,\ 0.5+0.0625\cdot\tanh x\}$, $\lambda_- = \lambda'_- = \min\{0.5,\ 0.5-0.1875\cdot\tanh x,\ 0.5-0.0625\cdot\tanh x\}$. Due to the symmetry of $\tanh x$, we take $x>0$. Namely, $p(+x) = p(-x) = 0.5$, $\lambda_+ = \lambda'_+ = 0.5+0.1875\cdot\tanh x$, $\lambda_- = \lambda'_- = 0.5-0.1875\cdot\tanh x$.

Fig. 1 shows that the super quantum discord attains the maximum value at $x=0$, where the weak measurement is most weak. When $x\to\infty$ the super quantum discord approaches the discord.

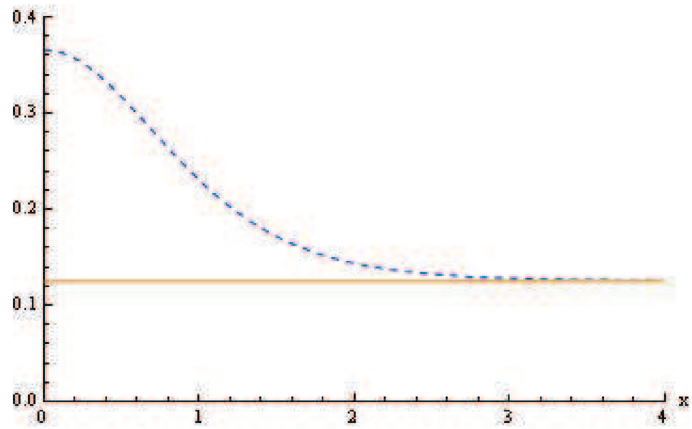

FIG. 1: Dashed line for super quantum discord as a function of x; solid line for discord.

Under the local bit-flip channel, we have $p(+x) = p(-x) = 0.5$, $\lambda_+ = \lambda'_+ = 0.5 + 0.1875 \cdot \tanh x$, $\lambda_- = \lambda'_- = 0.5 - 0.1875 \cdot \tanh x$, see Fig. 2.

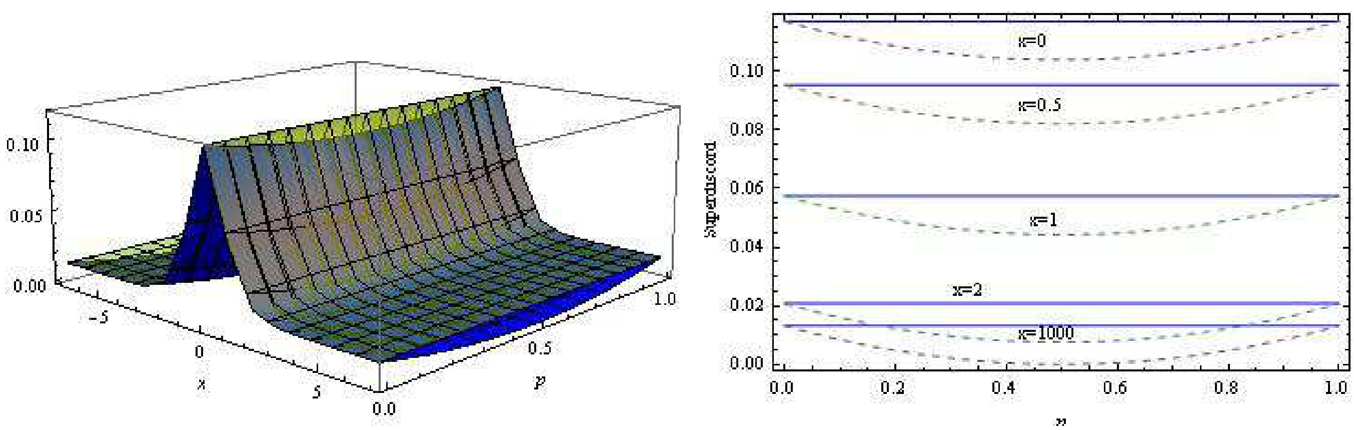

FIG. 2: Dotted line: super quantum discord under local bit-flip channel; solid line: super quantum discord of (20).

Under the global bit-flip channel, one has $p(+x) = p(-x) = 0.5$, $\lambda_+ = \lambda'_+ = 0.5 + 0.1875 \cdot \tanh x$, $\lambda_- = \lambda'_- = 0.5 - 0.1875 \cdot \tanh x$, see Fig. 3.

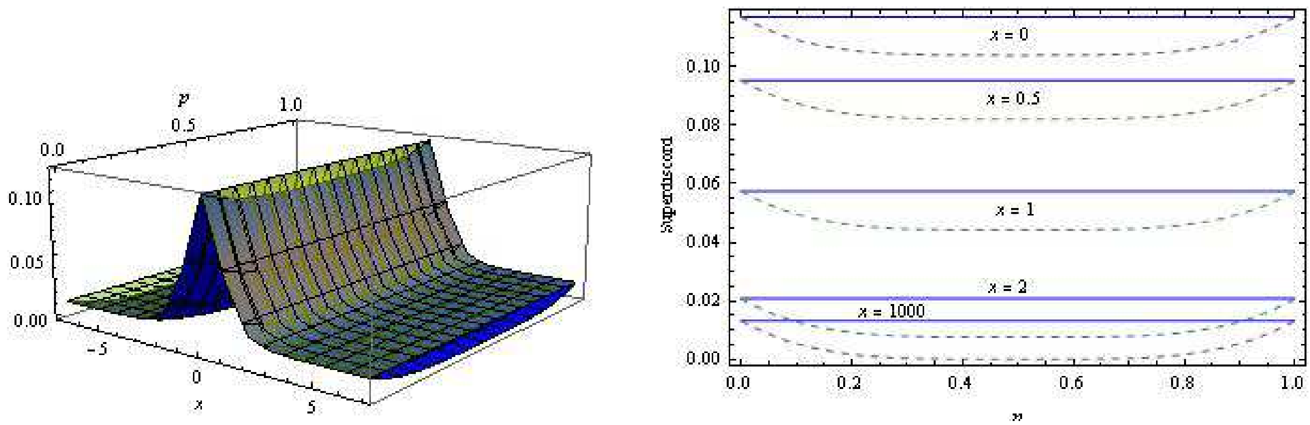

FIG. 3: Dotted line: super quantum discord under global bit-flip channel; solid line: super quantum discord of (20).

From Figs. 2 and 3 we can see at the $p = 0$ or $p = 1$, the super quantum discord is invariant under local or global bit-flip channels. In fact, p stands for the noise probability. The bit-flip channel flips the state of a qubit from $|0\rangle$ to $|1\rangle$ for $p = 0$. And for $p = 1$ the state keeps invariant. As the state (20) does not change flipping $|0\rangle$ to $|1\rangle$, the super quantum discord foes not change whenever $p = 0$ or $p = 1$. The super quantum discord attains the minimum at $p = 0.5$.

We see that the dynamics of super quantum discord for the X-state under bit-flip is symmetric. Let us now consider the local phase-flip channel (Λ_{lpf}) and the global phase-flip channel (Λ_{gpf}),

$$\Lambda_{lpf}(\rho_X) = (I \otimes E_0)\rho_X(I \otimes E_0)^\dagger + (I \otimes E_1)\rho_X(I \otimes E_1)^\dagger, \tag{21}$$

$$\begin{aligned}\Lambda_{gpf}(\rho_X) = {} & (E_0 \otimes E_0)\rho_X(E_0 \otimes E_0)^\dagger + (E_0 \otimes E_1)\rho_X(E_0 \otimes E_1)^\dagger \\ & +(E_1 \otimes E_0)\rho_X(E_1 \otimes E_0)^\dagger + (E_1 \otimes E_1)\rho_X(E_1 \otimes E_1)^\dagger,\end{aligned} \tag{22}$$

where

$$E_0 = \sqrt{p}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad E_1 = \sqrt{1-p}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{23}$$

Under these channels we have the following transformations of the entries of the density matrix:

channel	a_{11}	a_{14}	a_{22}	a_{23}	a_{33}	a_{44}	a^*_{23}	a^*_{14}
local phase-flip	a_{11}	$(-1+2p)a_{14}$	a_{22}	$(-1+2p)a_{23}$	a_{33}	a_{44}	$(-1+2p)a^*_{23}$	$(-1+2p)a^*_{14}$
global phase-flip	a_{11}	$(1-2p)^2a_{14}$	a_{22}	$(1-2p)^2a_{23}$	a_{33}	a_{44}	$(1-2p)^2a^*_{23}$	$(1-2p)^2a^*_{14}$

Take again the state (20) as an example. Under the local phase-flip channel, we get $p(+x) = p(-x) = 0.5$, and $\lambda_+ = \lambda'_+ = 0.5 + 0.1875(-1+2p)\cdot\tanh x$, $\lambda_- = \lambda'_- = 0.5 - 0.1875(-1+2p)\cdot\tanh x$ when $p > \frac{1}{2}$, $\lambda_+ = \lambda'_+ = 0.5 + 0.0625(-1+2p)\cdot\tanh x$, $\lambda_- = \lambda'_- = 0.5 - 0.0625(-1+2p)\cdot\tanh x$ when the $p < \frac{1}{2}$. From Fig. 4 we see that for $p < 0.5$, the super quantum discord is not sensitive to weak measurement strength. For $p > 0.5$, the super quantum discord is strongly related to the weak measurement operator. Here as the local phase-flip channel completely flip phase of the state for $p = 0$, while the state is not symmetric under such flip phase, the situation is different from the case in Fig. 2.

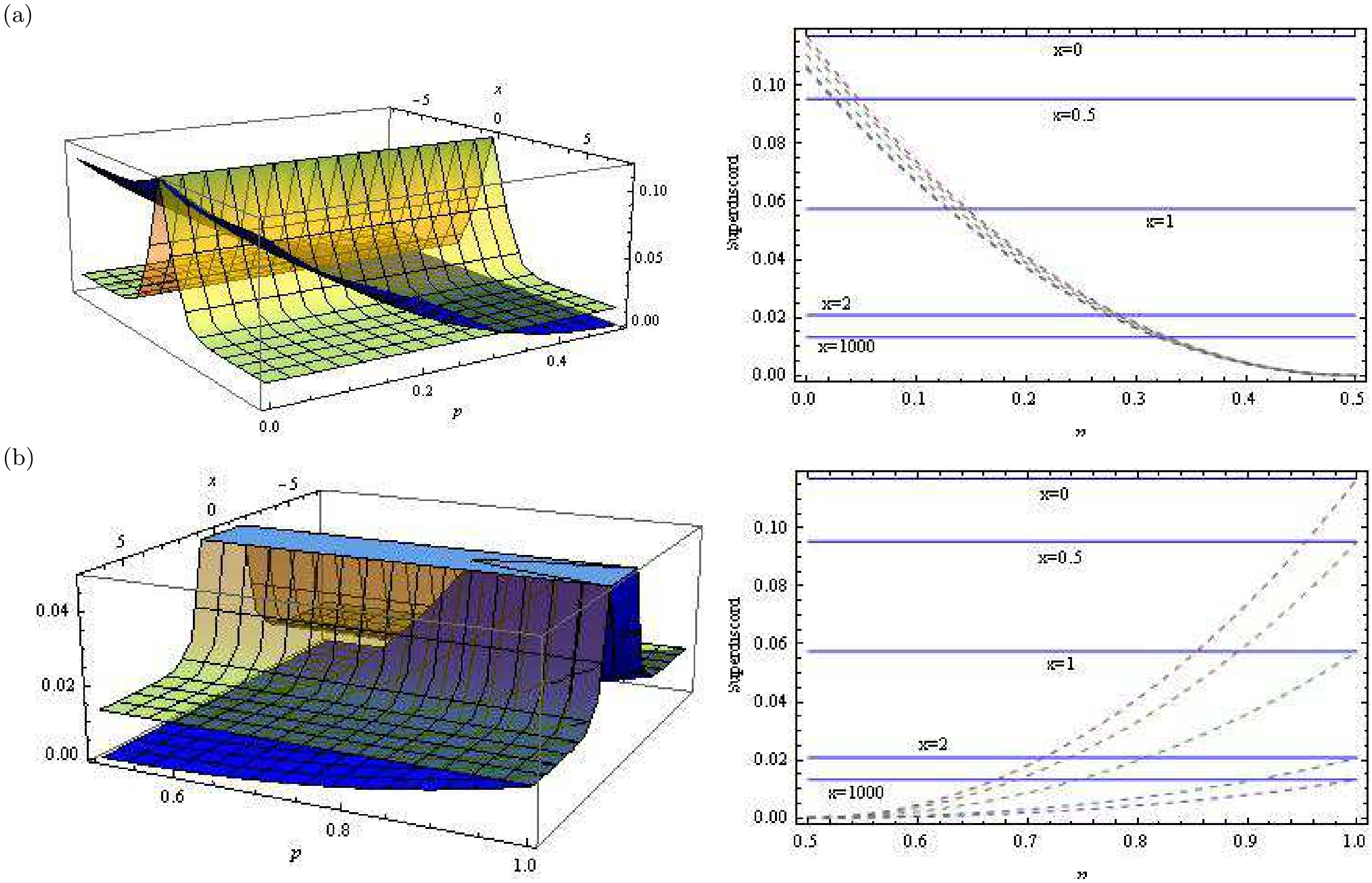

FIG. 4: a). $p < 0.5$: Dotted line for super quantum discord under local phase-flip channel, solid line for super quantum discord of (20); b). $p > 0.5$: Dotted line for super quantum discord under local phase-flip channel, solid for super quantum discord of (20).

Under global phase-flip channel, we get $p(+x) = p(-x) = 0.5$, $\lambda_+ = \lambda'_+ = 0.5 + 0.1875(1-2p)^2 \cdot \tanh x$, $\lambda_- = \lambda'_- = 0.5 - 0.1875(1-2p)^2 \cdot \tanh x$. From Fig. 5 one sees that at the points of $p = 0$ and $p = 1$, the dotted lines have same endpoints as the solid lines, since at $p = 0$ the global phase-flip channel becomes an identical one, and at $p = 1$ both parts of the state are flipped at the same time. It is interesting to note taht in the neighborhood of $p = 0.5$, the super quantum discord approaches to 0 for every weak measurement parameter x. In this neighborhood the global phase-flip channel annihilates the quantum correlations.

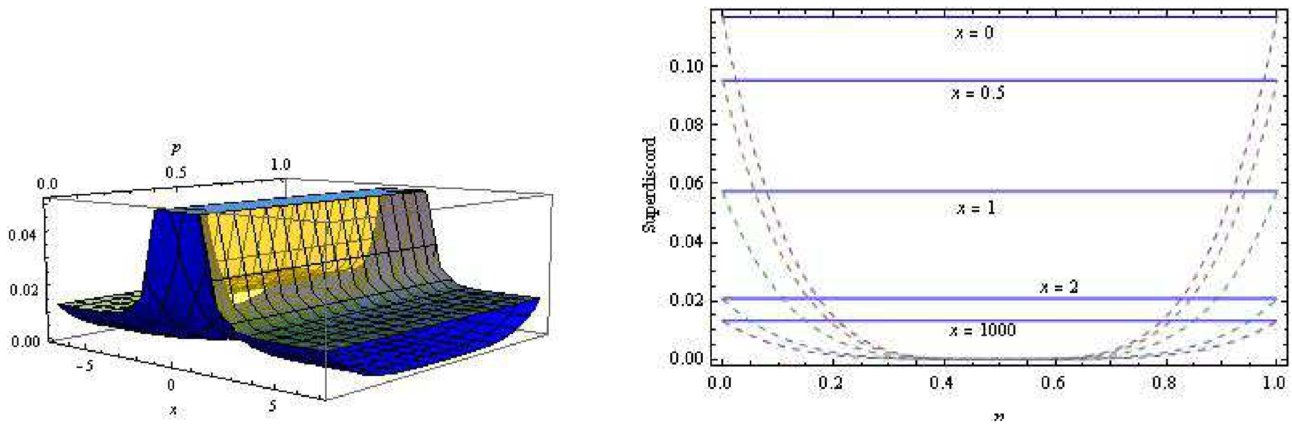

FIG. 5: Dotted line for super quantum discord under global phase-flip channel; solid line forsuper quantum discord of (20).

From Fig. 2 and 5 we can find that the noise operators on $S_w(\rho_{A|P^B(+x)})$ and $S_w(\rho_{A|P^B(-x)})$ play an important role in super quantum discord. The super quantum discord changes not much in Fig. 2, since under the local bit-flip channel and the global bit-flip channel, $S_w(\rho_{A|P^B(+x)})$ and $S_w(\rho_{A|P^B(-x)})$ are not changed. In Fig. 3, the variation of super quantum discord is nearly linear, because $\lambda_+ = \lambda'_+ = 0.5 + 0.1875(-1+2p)\cdot\tanh x$ and $\lambda_- = \lambda'_- = 0.5 - 0.1875(-1+2p)\cdot\tanh x$ are linear of p. Under global phase-flip, $\lambda_+ = \lambda'_+ = 0.5+0.1875(1-2p)^2\cdot\tanh x$ $\lambda_- = \lambda'_- = 0.5-0.1875(1-2p)^2\cdot\tanh x$ are parabolic function of p. Hence the total change shows a parabolic relation in Fig. 5.

IV. CONCLUSIONS AND DISCUSSIONS

We have studied the super quantum discord for X-type states. Explicit formulae of super quantum discord for X-type states have been derived. The evolution of these states under under local and global bit-flip and phase-flip channels has been investigated. The relations among the super quantum discord, the evolution of super quantum discord and the week measurement strength have been analyzed. However, for non X-type states, these problems still remain open.

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[1] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
[2] C.H. Bennett, D.P. DiVincenzo, C.A. Fuchs, T. Mor, E. Rains, P.W. Shor, J.A. Smolin, and W.K. Wootters, Phys. Rev. A **59**, 1070 (1999).
[3] M. Horodecki, P. Horodecki, R. Horodecki, J. Oppenheim, A. Sen, U. Sen, and B. Synak-Radke, Phys. Rev. A **71**, 062307 (2005).
[4] J. Niset and N.J. Cerf, Phys. Rev. A. **74**, 052103 (2006).
[5] H. Ollivier and W.H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
[6] B. Bylicka and D. Chruściński, Phys. Rev. A **81**, 062102 (2010).
[7] T. Werlang, S. Souza, F.F. Fanchini, and C.J. Villas Boas, Phys. Rev. A **80**, 024103 (2009).
[8] M.S. Sarandy, Phys. Rev. A **80**, 022108 (2009).
[9] A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acín, Phys. Rev. A **81**, 052318 (2010).

[10] F.F. Fanchini, T. Werlang, C.A. Brasil, L.G.E. Arruda, and A.O. Caldeira, Phys. Rev. A **81**, 052107 (2010).
[11] B. Dakíc, V. Vedral, and Č. Brukner, Phys. Rev. Lett. **105**, 190502 (2010).
[12] K. Modi, T. Paterek, W. Son, V. Vedral, and M. Williamson, Phys. Rev. Lett. **104**, 080501 (2010).
[13] N. Li and S. Luo, Phys. Rev. A **76**, 032327 (2007);S. Luo, *ibid* **77**, 022301 (2008).
[14] S. Luo, Phys. Rev. A **77**, 042303 (2008).
[15] B. Li, Z.X. Wang and S.M. Fei, Phys. Rev. A **83** (2011) 022321.
[16] M.D. Lang, and C.M. Caves, Phys. Rev. Lett. **105**, 150501 (2010).
[17] M. Ali, A.R.P. Rau, and G. Alber, Phys. Rev. A **81**, 042105 (2010); M. Ali, A.R.P. Rau, and G. Alber, *ibid* **82**, 069902 (2010).
[18] L. Mazzola, J. Piilo, and S. Maniscalco, Phys. Rev. Lett. **104**, 200401 (2010).
[19] J. Maziero, L. C. Céleri, R. M. Serra, and V. Vedral, Phys. Rev. A. **80**, 044102 (2009).
[20] L. Roa, J. C. Retamal, M. Alid-Vaccarezza, Phys. Rev. Lett. **107**, 080401 (2011). B. Li, S. M. Fei, Z. X. Wang and H. Fan, Phys. Rev. A **85**, 022328 (2012).
[21] Y, Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1998).
[22] U. Singh and A. K. Pati, arXiv: 1211. 0939.
[23] M. Shi, C. Sun, F. Jiang, X. Yan and J. Du, Phys. Rev. A. **85**, 064104 (2012).
[24] M. H. Partovi, Phys. Lett. A. **137**, 455 (1989).
[25] L. Henderson and V. Vedral, J. Phys. A. **34**, 6899 (2001).
[26] O. Oreshkov and T. A. Brun, Phys. Rev. Lett. **95**, 110409 (2005).
[27] S. Luo, Phys. A. **77**, 042303 (2008).
[28] R. F. Werner, Phys. Rev. A. **40**, 4277 (1989).
[29] C. H. Bennett, David P. DiVincenzo, John A. Smolin, and William K. Wootters,Phys. Rev. A. **54**, 3824 (1996).
[30] Y. K. Wang, T. Ma, H. Fan, S. M. Fei, and Z. X. Wang, arXiv: 1302. 4039.