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by

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Abstract

We study the quantum separability problem by using general symmetric informationally complete measurements and present separability criteria for both d-dimensional bipartite and multipartite systems. The criteria are effective in detecting several well known classes of quantum states. For isotropic states, it becomes both necessary and sufficient. Furthermore, our criteria can be experimentally implemented and require less local measurements than the criterion based on mutually unbiased measurements.

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1 Introduction

One of the most fundamental and intriguing tasks in quantum information theory and quantum information processing is the detection of entanglement. It is widely known that entangled states are useful resources in many quantum cryptography protocols, and can be used to enhance efficiency of quantum computing (see reviews [1, 2] and the references therein). There have been numerous criteria to distinguish quantum entangled states from the separable ones, such as positive partial transposition criterion [3, 4, 5], realignment criterion [6, 7, 8, 9, 10], covariance matrix criterion [11], and correlation matrix criterion [12, 13]. In Ref. [14], Li *et al.* proposed a generalized form of the correlation matrix criterion which is more effective than the previous criteria.

While mathematical methods presented above have been extensively studied, experimental implementation of entanglement detection for unknown quantum states has fewer results [15, 16, 17, 18]. In Ref. [19], the authors linked the separability problem with the concept of mutually unbiased bases (MUBs) [20]. They presented separability criteria for two-qudit, multipartite and continuous-variable quantum systems. These separability criteria are shown to be very powerful, and can be implemented experimentally. After that, Chen *et al.* [21] generalized such idea and provided a separability criterion for two-qudit states by using mutually unbiased measurements (MUMs) [22]. It is shown that the criterion based on MUMs is more effective than the criterion based on MUBs, and for isotropic states this criterion becomes both necessary and sufficient. Very recently, Liu *et al.* [23] derived separability criteria for arbitrary high-dimensional bipartite and multipartite systems using sets of MUMs.

Besides mutually unbiased bases, another related topic in quantum information theory is the symmetric informationally complete positive operator-valued measures (SIC-POVMs). MUBs and SIC-POVMs have many interesting and useful connections, from both operational link [24] and applications in quantum information theory such as quantum state tomography [20, 25, 26, 27] and uncertainty relations [28]. In [29], the author introduced the concept of general symmetric informationally complete (SIC) measurements in which the elements need not be of rank one, and showed that such general SIC measurements exist in all finite dimensions. Recently, Gour and Kalev [30] constructed the set of all general SIC measurements from the generalized Gell-Mann matrices. Naturally, one may expect that these general symmetric informationally complete measurements can be also used to detect entanglement experimentally.

In this paper we present separability criteria for both d-dimensional bipartite and multipartite systems by using the general symmetric informationally complete measurements. The criteria are shown to be powerful in detecting some well known classes of quantum states. Moreover, this criteria require less local measurements than the criterion based on mutually unbiased measurements. The paper is organized as follows. In Section 2, we recall some basic notions of SIC-POVMs and the general symmetric informationally complete measurements. In Section 3, we provide separability criteria based on the general symmetric informationally complete measurements, and illustrate the power of entanglement detection via some examples. We also compare the criterion for bipartite systems to the one based on mutually unbiased measurements. We conclude the paper in Section 4.

2 SIC-POVMs and General SIC Measurements

Let us first review some basic definitions of SIC-POVMs and general symmetric informationally complete measurements. A POVM with d^2 rank one operators acting on $\mathbb{C}^d$ is symmetric informationally complete, if every operator is of the form

$$P_j = \frac{1}{d}|\phi_j\rangle\langle\phi_j|, \quad j = 1, 2, \ldots, d^2,$$

the vectors $|\phi_j\rangle$ satisfying

$$| \langle\phi_j|\phi_k\rangle |^2 = \frac{1}{d+1}, \quad j \neq k.$$

The existence of SIC-POVMs in arbitrary dimension d is an open problem. Only in a number of low-dimensional cases, the existence of SIC-POVMs has been proved analytically, and numerically for all dimensions up to 67 (see [31] and the references therein).

Recently, the concept and different constructions of general SIC measurements were introduced in Ref. [29, 30]. A set of d^2 positive-semidefinite operators $\{P_\alpha\}_{\alpha=1}^{d^2}$ on $\mathbb{C}^d$ is said to be a general SIC measurements, if

(1) $\sum_{\alpha=1}^{d^2} P_\alpha = I$,

(2) $\mathrm{Tr}(P_\alpha^2) = a,\ \mathrm{Tr}(P_\alpha P_\beta) = \frac{1-da}{d(d^2-1)},\ \forall\alpha, \beta \in \{1, 2, \ldots, d^2\},\ \alpha \neq \beta$,

where I is the identity operator, the parameter a satisfies $\frac{1}{d^3} < a \leq \frac{1}{d^2}$, $a = 1/d^2$ if and only if all P_α are rank one, which gives rise to a SIC-POVM. It can be shown that $\mathrm{Tr}(P_\alpha) = \frac{1}{d}$ for all α [30].

Like the mutually unbiased measurements, these general symmetric informationally complete measurements can be also explicitly constructed for arbitrary dimensional spaces [30]. Let $\{F_\alpha\}_{\alpha=1}^{d^2-1}$ be a set of d^2-1 Hermitian, traceless operators acting on $\mathbb{C}^d$, satisfying $\mathrm{Tr}(F_\alpha F_\beta) = \delta_{\alpha,\beta}$. Define $F = \sum_{\alpha=1}^{d^2-1} F_\alpha$, then the d^2 operators

$$\begin{aligned} P_\alpha &= \frac{1}{d^2} I + t[F - d(d+1)F_\alpha], \quad \alpha = 1, 2, \ldots, d^2 - 1, \\ P_{d^2} &= \frac{1}{d^2} I + t(d+1)F, \end{aligned} \tag{1}$$

form a general SIC measurements. Here t should be chosen such that $P_\alpha \geq 0$, and the parameter a is given by

$$a = \frac{1}{d^3} + t^2(d-1)(d+1)^3. \tag{2}$$

These general symmetric informationally complete measurements have many useful applications in quantum information theory. In Ref. [32], based on the calculation of the so-called index of coincidence, the author derived a number of uncertainty relation inequalities via general SIC measurements. In the following, we study entanglement detection using general symmetric informationally complete measurements.

3 General SIC-POVM Based Separability Criterion

The entanglement detection via SIC-POVMs has been briefly discussed in Ref. [28]. But the method is subject to the existence of SIC-POVMs, which is an open question. Fortunately, unlike the SIC-POVMs, general symmetric informationally complete measurements do exist for arbitrary dimension d, and the separability criterion for two-qudit states can be explicitly presented.

Theorem 1. Let ρ be a density matrix in $\mathbb{C}^d \bigotimes \mathbb{C}^d$. Let $\{P_j\}_{j=1}^{d^2}$ and $\{Q_j\}_{j=1}^{d^2}$ be any two sets of general symmetric informationally complete measurements on $\mathbb{C}^d$ with the same parameter a. Define $J_a(\rho) = \sum_{j=1}^{d^2} \mathrm{Tr}(P_j \bigotimes Q_j \rho)$. If ρ is separable, then $J_a(\rho) \leq \frac{ad^2+1}{d(d+1)}$.

Proof. It is obvious that $J_a(\rho)$ is a linear function of ρ. Hence we need only to consider pure separable states, $\rho = |\phi\rangle\langle\phi| \otimes |\psi\rangle\langle\psi|$. We have

$$\begin{aligned} J_a(\rho) &= \sum_{j=1}^{d^2} \mathrm{Tr}(P_j \bigotimes Q_j \rho) \\ &= \sum_{j=1}^{d^2} \mathrm{Tr}(P_j |\phi\rangle\langle\phi|)\mathrm{Tr}(Q_j|\psi\rangle\langle\psi|) \\ &\le \frac{1}{2}\sum_{j=1}^{d^2}\{[\mathrm{Tr}(P_j|\phi\rangle\langle\phi|)]^2 + [\mathrm{Tr}(Q_j|\psi\rangle\langle\psi|)]^2\}. \end{aligned}$$

Note that

$$\sum_{j=1}^{d^2}[\mathrm{Tr}(P_j\rho)]^2 = \frac{(ad^3-1)\mathrm{Tr}(\rho^2) + d(1-ad)}{d(d^2-1)}$$

for any density matrix ρ in $\mathbb{C}^d$ [32], and $\mathrm{Tr}(\rho^2) = 1$ as ρ is a pure state. Thus we have $J_a(\rho) \le \frac{ad^2+1}{d(d+1)}$. $\square$

Let $\{P_j\}_{j=1}^{d^2}$ be a set of general symmetric informationally complete measurements on $\mathbb{C}^d$ with the parameter a. Let $\overline{P_j}$ denote the conjugation of P_j. Then $\{\overline{P_j}\}_{j=1}^{d^2}$ is another set of general SIC-POVM with the same parameter a. To show effectiveness of our criterion, let us consider some examples in the following.

Example 1. We first consider the maximally entangled pure state $|\Phi^+\rangle = \frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}|ii\rangle$. We have

$$\begin{aligned} J_a(|\Phi^+\rangle) &= \sum_{j=1}^{d^2} \mathrm{Tr}(P_j \bigotimes \overline{P_j}|\Phi^+\rangle\langle\Phi^+|) \\ &= da > \frac{ad^2+1}{d(d+1)}, \end{aligned}$$

since $a > \frac{1}{d^3}$ from (2). Thus the criterion can detect all the maximally entangled pure states.

In Ref. [28], the maximally entangled pure states can be also detected by the criterion based on the SIC-POVMs, but the criterion depends on the existence of SIC-POVMs. Here we can detect all the maximally entangled pure states for arbitrary dimensions.

Example 2. Let us consider the isotropic states, which are locally unitarily equivalent to a maximally entangled state mixed with white noise:

$$\rho_{iso} = \alpha|\Phi^+\rangle\langle\Phi^+| + \frac{1-\alpha}{d^2}I,$$

where $0 \leq \alpha \leq 1$. One can easily get that

$$\begin{aligned} J_a(\rho_{iso}) &= \sum_{j=1}^{d^2} \mathrm{Tr}(P_j \bigotimes \overline{P_j} \rho_{iso}) \\ &= da\alpha + \frac{1-\alpha}{d^2}. \end{aligned}$$

If $\alpha > \frac{1}{d+1}$, then $J_a(\rho_{iso}) > \frac{ad^2+1}{d(d+1)}$ and ρ_{iso} must be entangled by our theorem. Thus the criterion can detect all the entanglement of the isotropic states, since it has been proven that ρ_{iso} is entangled for $\alpha > \frac{1}{d+1}$, and separable for $\alpha \leq \frac{1}{d+1}$ [33]. That is to say that our criterion is both necessary and sufficient for the separability of isotropic states, similar to the criterion based on mutually unbiased measurements [21].

Example 3. We consider now the Bell-diagonal states,

$$\rho_{Bell} = \sum_{s,t=0}^{d-1} p_{s,t} |\Phi^+_{s,t}\rangle\langle\Phi^+_{s,t}|,$$

where $p_{s,t} \geq 0$, $\sum_{s,t=0}^{d-1} p_{s,t} = 1$, $|\Phi^+_{s,t}\rangle = (U_{s,t} \bigotimes I)|\Phi^+\rangle$, and $U_{s,t} = \sum_{j=0}^{d-1} \zeta_d^{sj} |j\rangle\langle j \oplus t|$, $s,t = 0,1,\cdots,d-1$, are Weyl operators, $\zeta_d = e^{\frac{2\pi\sqrt{-1}}{d}}$ and $j \oplus t$ denotes $(j+t)$ mod d. Denoting $A \geq B$ if $A - B$ is positive for operators A and B, we have

$$\rho_{Bell} \geq c|\Phi_c\rangle\langle\Phi_c|,$$

where $c = \max\{p_{s,t} : s,t = 0,1,\cdots,d-1\}$, $\frac{1}{d^2} \leq c \leq 1$, and $|\Phi_c\rangle$ is the corresponding maximally entangled pure state. Thus we obtain

$$\begin{aligned} J_a(\rho_{Bell}) &= \sum_{j=1}^{d^2} \mathrm{Tr}(P_j \bigotimes \overline{P_j} \rho_{Bell}) \\ &\geq \sum_{j=1}^{d^2} \mathrm{Tr}[(P_j \bigotimes \overline{P_j}(c|\Phi_c\rangle\langle\Phi_c|)] \\ &= cda. \end{aligned}$$

If $c > (1+\frac{1}{ad^2})/(d+1)$, then $J_a(\rho_{Bell}) > \frac{ad^2+1}{d(d+1)}$ and ρ_{Bell} must be entangled by the theorem. It can be easily seen that the criterion detects more entanglement as a increases.

When $a = \frac{1}{d^2}$, i.e. the general SIC measurements is given by the rank one SIC-POVM, we get $c > \frac{2}{d+1}$. This condition is the same as the one obtained by the mutually unbiased measurements, in which the parameter $\kappa = 1$ [21], i.e. the mutually unbiased measurements is given by the mutually unbiased bases. But for general cases, we do not know which criterion is more effective in detecting the Bell-diagonal states, since we do not know the relation between the parameter a and κ for fixed d.

Example 4. In [34], the authors discussed the entanglement of a bipartite state in $\mathbb{C}^d \bigotimes \mathbb{C}^d$:

$$\rho = a_1|\Phi^+\rangle\langle\Phi^+| + \sum_{k=1,i=2}^{d} \frac{a_i}{d}|k\rangle\langle k| \otimes |k+i-1\rangle\langle k+i-1|,$$

where $a_i > 0, i = 1, 2, \ldots, d, \sum_{i=1}^{d} a_i = 1$. If $a_i \geq a_1 (i \neq 1)$, then the state is separable [34]. We now consider a special case where $a_i = a_2$, for $i \neq 1$. It is obvious that ρ must be entangled, when $a_1 > \frac{1}{d}$. We employ two sets of general SIC measurements $\{P_j\}_{j=1}^{d^2}$ and $\{\overline{P_j}\}_{j=1}^{d^2}$ as above, and denote the diagonal elements of P_j as $\{P_j^{(1)}, P_j^{(2)}, \ldots, P_j^{(d)}\}$. Note that $\sum_{k=1}^{d}[P_j^{(k)}]^2 \leq [\mathrm{Tr}(P_j)]^2$, since $P_j^{(k)}$'s are non-negative numbers. Then we have

$$\begin{aligned}
J_a(\rho) &= \sum_{j=1}^{d^2} \mathrm{Tr}(P_j \bigotimes \overline{P_j}\rho) \\
&= a_1 da + \frac{a_2}{d} \sum_{j=1}^{d^2} \sum_{k=1,i=2}^{d} P_j^{(k)} P_j^{(k+i-1)} \\
&= a_1 da + \frac{a_2}{d} \sum_{j=1}^{d^2} \sum_{k=1}^{d} P_j^{(k)} [\mathrm{Tr}(P_j) - P_j^{(k)}] \\
&= a_1 da + \frac{a_2}{d} \sum_{j=1}^{d^2} \{[\mathrm{Tr}(P_j)]^2 - \sum_{k=1}^{d} [P_j^{(k)}]^2\} \\
&\geq a_1 da.
\end{aligned}$$

Similar to the discussion of Bell-diagonal states, we can conclude that the state $\rho = a_1|\Phi^+\rangle\langle\Phi^+| + \frac{a_2}{d}\sum_{k=1,i=2}^{d} |k\rangle\langle k| \otimes |k+i-1\rangle\langle k+i-1|$ is entangled when $a_1 > (1 + \frac{1}{ad^2})/(d+1)$. Note that when d is large enough, the separability threshold $(1 + \frac{1}{ad^2})/(d+1)$ derived from our criterion approaches to $\frac{1}{d}$, independent of the exact value of a.

The criterion for two-qudit states can also be extended to d-dimensional multipartite states. We have the following theorem.

Theorem 2. Let ρ be a density matrix in $(\mathbb{C}^d)^{\otimes N}$. Let $\{P_j^{(i)}\}_{j=1}^{d^2}$, $i = 1, 2, \ldots, N$, be N sets of general symmetric informationally complete measurements on $\mathbb{C}^d$ with the parameters a_i, respectively. If ρ is fully separable, then

$$J(\rho) \leq \frac{1}{N}\sum_{i=1}^{N}\frac{a_i d^2+1}{d(d+1)},$$

where $J(\rho)=\sum_{j=1}^{d^2}\mathrm{Tr}(\bigotimes_{i=1}^{N}P_j^{(i)}\rho)$.

Proof. Similar to $J_a(\rho)$ defined in Theorem 1, $J(\rho)$ is also a linear function of ρ. Therefore we only need to consider fully separable pure states, $\rho=\bigotimes_{i=1}^{N}|\phi_i\rangle\langle\phi_i|$. We have

$$J(\rho) = \sum_{j=1}^{d^2}\prod_{i=1}^{N}\mathrm{Tr}(P_j^{(i)}|\phi_i\rangle\langle\phi_i|).$$

Note that $0\leq \mathrm{Tr}(P_j^{(i)}|\phi_i\rangle\langle\phi_i|)\leq\frac{1}{d}$. Using the inequality for N non-negative real numbers [23]: $x_1x_2\cdots x_N\leq(\frac{x_1^2+x_2^2+\cdots+x_N^2}{N})^{\frac{N}{2}}$, we obtain

$$\begin{aligned}
J(\rho) &\leq \sum_{j=1}^{d^2}\{\frac{1}{N}\sum_{i=1}^{N}[\mathrm{Tr}(P_j^{(i)}|\phi_i\rangle\langle\phi_i|)]^2\}^{\frac{N}{2}}\\
&\leq \sum_{j=1}^{d^2}\frac{1}{N}\sum_{i=1}^{N}[\mathrm{Tr}(P_j^{(i)}|\phi_i\rangle\langle\phi_i|)]^2\\
&= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{d^2}[\mathrm{Tr}(P_j^{(i)}|\phi_i\rangle\langle\phi_i|)]^2\\
&= \frac{1}{N}\sum_{i=1}^{N}\frac{a_i d^2+1}{d(d+1)}.
\end{aligned}$$

Therefore, $J(\rho)\leq\frac{1}{N}\sum_{i=1}^{N}\frac{a_i d^2+1}{d(d+1)}$ holds for all d-dimensional fully separable states ρ. □

Obviously, Theorem 1 is a special case of Theorem 2 for $N=2$ and $a_1=a_2$.

Let us make a comparison between the criterion for bipartite quantum states presented in this work and the one based on mutually unbiased measurements in Ref. [21]. Let $\{P_j\}_{j=1}^{d^2}$ be a set of general SIC measurements on $\mathbb{C}^d$ with the parameter a. By expanding a two-qudit state ρ in terms of the operator basis adopted in $\{P_j\}_{j=1}^{d^2}$, we get

$$\begin{aligned}
J_a(\rho) &= \sum_{j=1}^{d^2}\mathrm{Tr}(P_j\bigotimes P_j\rho)\\
&= \frac{1}{d^2}+\frac{2(ad^2-1/d)}{d^2-1}\mathrm{Tr}(T),
\end{aligned}$$

where T is the correlation matrix of ρ. If ρ is separable, then we obtain $\mathrm{Tr}(T) \leq \frac{d-1}{2d}$, which is the same as the inequality deduced from the criterion based on the mutually unbiased measurements in Ref. [21]. It is also a special case of the inequality satisfied by separable states [14]. However, the separability criterion based on general SIC-Measurement and mutually unbiased measurement can be experimentally implemented. One does not need to do tomography of an unknown state first. Moreover, the criterion for bipartite quantum states based on general SIC measurements is superior to the one based on mutually unbiased measurements, since the former only needs d^2 joint local measurements, while the later needs $d(d+1)$ joint local measurements, which greatly reduces the experimental implementation complexity.

4 Conclusion and Discussions

We have studied the separability problem via general symmetric informationally complete measurements. More experimentally feasible quantum separability criteria for two-qudit states and for d-dimensional multipartite states have been presented. The criterion for bipartite quantum states has been shown to be powerful in detecting the quantum entanglement for maximally entangled pure states, the isotropic states and the Bell-diagonal states. Especially, for isotropic states, this criterion is both necessary and sufficient. Comparing with the criterion based on the mutually unbiased measurements, our criterion based on the general symmetric informationally complete measurements requires much less measurements. It would be also worthwhile to generalize the present results to arbitrary high-dimensional multipartite systems.

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