Max-Planck-Institut für Mathematik in den Naturwissenschaften Leipzig

Stability analysis of a constant time-headway driving strategy with driver memory effects modeled with distributed delays

by

Rifat Sipahi, Fatihcan M. Atay, and Silviu-Iulian Niculescu

Preprint no.: 71 2015

Stability Analysis of a Constant Time-Headway Driving Strategy with Driver Memory Effects Modeled with Distributed Delays

Rifat Sipahi * **Fatihcan M. Atay** ** **Silviu-Iulian Niculescu** ***

* *Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA 02115 USA (e-mail: rifat@coe.neu.edu).*
** *Max Planck Institute for Mathematics in the Sciences, Inselstr. 22, Leipzig 04103, Germany (email:atay@member.ams.org).*
*** *Laboratoire de Signaux et Systèmes (L2S), Supélec, 3, rue Joliot Curie, 91190, Gif-sur-Yvette, France (email:Silviu.Niculescu@lss.supelec.fr).*

Abstract: Study of traffic flow dynamics has attracted considerable attention for decades. One way to thoroughly understand traffic flow is to study how each vehicle in traffic interacts with each other, through drivers' acceleration/deceleration decisions. Such decisions are strongly influenced by reaction delays of drivers, which arise naturally due to drivers' physiology, perception, and motor programming. In this article, we consider how the memory of drivers, modeled here with distributed delays, affects the decision making process in a car following scenario, in which each driver aims to keep a fixed time-headway with respect to the preceding vehicle. Taking a frequency domain analysis, we reveal the effects of the parameters of the driver memory model on the asymptotic stability features of the arising inter-vehicle spacing propagation dynamics.

Keywords: traffic flow, stability, slinky effects, driver memory.

1. INTRODUCTION

The dynamics of traffic flow behavior has attracted considerable attention in research communities since the 1930s (Helbing, 2001), with the aim to understand the root causes of traffic jam formations and accidents, and thereby to optimize vehicle flow in cities while preventing congestion, improve signalization to effectively channel traffic flow, reduce accidents and thus casualties, and reduce congestion to ultimately minimize emissions and detriments to environment (Treiber and Kesting, 2013). For this objective, physicists and engineers developed numerous mathematical models to model traffic flow from various perspectives, primarily from macroscopic and microscopic points-of-views. While macroscopic models capture features such as density and flow rate, microscopic models focus mainly on the characteristics of individual vehicle dynamics and their collective behavior.

The presence of human drivers makes the traffic flow dynamics complicated. While human drivers have exceptional sensory and decision making capabilities, they also have low bandwidths, can be easily challenged in their decisions when faced with complex scenarios, and cannot execute their decision rapidly due to their physiology, which impose reaction delays (Chandler et al., 1958; Green, 2000). The effects of delays have been recognized in this context, and broadly studied especially by incorporating the delay parameter conveniently within microscopic models (Helbing, 2001; Sipahi and Niculescu, 2010). In this direction, many studies focus on a discrete-delay model, with delay $\tau > 0$, where human decisions are formed based on a stimulus that occurred at a point of time in the past at time $t - \tau$. These studies also investigated the strategy of multiple vehicle following and analyzed extensively linear and nonlinear microscopic models by way of simulations and/or analytical tools, to explain traffic flow patterns, including jam formations. See (Bando and Hasebe, 1998; Helbing, 2001; Orosz et al., 2004; Orosz and Stepan, 2004; Treiber et al., 2006; Sipahi and Niculescu, 2006a) and the references therein.

One of the main objectives in the cited references is to investigate how humans with their decisions may maintain asymptotic stability (AS) in the vehicle formations despite their delayed reactions. Technically speaking, AS refers to exponential decay of the response of the system states (velocity and position of vehicles) in time against impulsive perturbations. Despite the simplicity of the mathematical models, assessment of AS is not a trivial task in the presence of delays. This is mainly because the arising mathematics with delays becomes infinite dimensional requiring nontrivial developments for the analysis of and synthesis for dynamical systems affected by delays (Stépán, 1989).

This article is inspired by a mathematical model based on Pipes's work (Pipes, 1953), which has attracted further attention in the literature due to its simplicity, as well as its validity in matching experiments performed with human drivers (Bose and Ioannou, 2003). This model assumes that vehicles follow each other on a single-lane as drivers in each vehicle aim to maintain zero relative velocity with respect to the preceding vehicle by applying a force input to the vehicle. The driver's reactions are however time delayed, and hence every decision is formulated based on what has happened at time $t - \tau$. This model has been studied by considering both constant-headway/velocity control, see e.g. (Helbing, 2001; Bando and Hasebe, 1998), and constant time-headway control decisions of human drivers (Bose and Ioannou, 2003), including (Sipahi and Niculescu, 2006c,b,a, 2008; Sipahi et al., 2009). Moreover, the authors expanded this model for the fixed-headway control strategy by incorporating drivers' memory effects modeled by distributed delays (Sipahi et al., 2007). Such models are able to capture continuous experience of driving, and are hence expressed as a collection of infinitely many stimuli distributed over the history with a finite horizon.

A natural next research problem is to then investigate how constant time-headway driving strategy coupled with drivers' memory effects together affect AS properties of Pipes's model. This article is focused on these analyses, and their parametric study. Consistent with the previous studies cited above, here we shall focus on time-invariant models and on the fate of the equilibrium dynamics of traffic flow through linear analysis. To answer these non-trivial questions regarding AS, the well-known *frequency-sweeping* technique (Chen and Latchman, 1995; Sipahi et al., 2011) will be adopted.

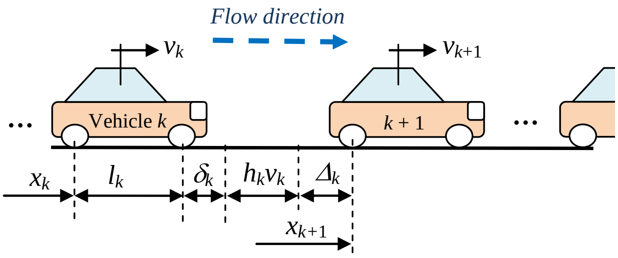

Fig. 1. Platoon of vehicles, inspired from (Bose and Ioannou, 2003).

To the best of our knowledge, the above described analysis including constant-time headway driving strategy and driver memory effects has not been studied in the open literature. Mathematical modeling and the preliminaries are presented in Section II. Analysis of AS is performed in Section III along with illustrative examples and discussions. Conclusions end the article in Section IV.

2. MATHEMATICAL MODELING

Mathematical modeling and the pertaining discussions in this section are borrowed/inspired from Bando and Hasebe (1998); Bose and Ioannou (2003); Sipahi and Niculescu (2006a), however, the model is extended based on our main interest in studying the effects of driver memory in connection with constant time-headway driving strategy. As motivated in the previous section, here we start with Pipes model

$$\dot{v}_k(t) = \alpha_k(v_{k+1}(t-\tau_k) - v_k(t-\tau_k)), \;\; k = 1, \ldots, n, \quad (1)$$

where v_k is the velocity of the k^{th} vehicle, see Fig 1, $\tau_k > 0$ is the constant delay, n is the number of vehicles and the weighting $\alpha_k > 0$ can be seen as a measure of driver aggressiveness per unit mass. The above differential equation describes that driver k aims to vanish the velocity error $v_{k+1}(t) - v_k(t)$ by penalizing it using the gain α_k. Furthermore, since a driver's sensing and decision execution are not instantaneous, decisions performed at time t are based on the velocity error sensed with a delay τ_k. This model, which is a continuous-time deterministic microscopic car following model, considers that vehicles travel on a single-lane with the aim to keep their velocities constant, without changing lanes.

By incorporating a general memory effect, f_k, into the system (1), we arrive at the following model:

$$\dot{v}_k(t) = \alpha_k \int_0^\infty f_k(r)(v_{k+1}(t-r) - v_k(t-r))dr, \quad (2)$$

where we assume that the delay kernel $f(r)$ is a measurable function of exponential order, with r in the units of time. When $f(r)$ is a Dirac delta function, one recovers from (2) the discrete delay model in (1).

In this article, the delay kernel is taken as a normalized uniform distribution function, given by,

$$f_k(r) = \begin{cases} 1/\tau_{k,mw} & \tau_{k,dt} < r < \tau_{k,dt} + \tau_{k,mw} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where $\tau_{k,dt}$ is the elapsed dead time before any incoming stimulus can be perceived by the driver, and $\tau_{k,mw}$ is the memory window size of driver k. This model can be seen as an average of the information available in the memory.

Letting next $V_k(s)$ and $F_k(s)$ be the Laplace transforms of v_k and f_k, respectively, with s being the Laplace variable, and combining (2) and (3) in Laplace domain, one obtains the following transfer function $G_k(s)$ between the velocities of two consecutive vehicles,

$$G_k(s) = \frac{V_k(s)}{V_{k+1}(s)} = \frac{\alpha_k F_k(s)}{s + \alpha_k F_k(s)}, \quad (4)$$

which was studied by the authors for its stability properties (Sipahi et al., 2007). Specifically, in the cited work, the delay kernel was taken to be identical for all the drivers, that is, $f(r) = f_k(r)$, $\tau_{dt} = \tau_{k,dt}$, $\tau_{mw} = \tau_{k,mw}$, and with the knowledge of

$$F(s) = \frac{e^{-s\tau_{dt}}(1 - e^{-s\tau_{mw}})}{s\tau_{mw}}, \quad (5)$$

the characteristic equation of the system was studied for linear stability of traffic flow with vehicles configured in both a chain and ring configuration.

2.1 *Spacing Dynamics with Constant-Time Headway*

Constant-time headway strategy considers that driver k aims to perform control to maintain a constant headway with respect to the preceding vehicle by an amount that is proportional to the speed of the vehicle v_k and elapsed time $h_k > 0$. Under this definition, the headway expression is formulated as,

$$\delta_k(t) = x_{k+1}(t) - x_k(t) - l_k - \Delta_k - h_k v_k(t),$$

where vehicle length l_k and minimum safe headway Δ_k do not contribute to AS analysis as they are constants. Considering this driving strategy along with the model in (2), headway propagation dynamics between consecutive pairs of vehicles can be studied. Letting $D_k(s)$ be the Laplace transform of $\delta_k(t)$, it is then possible to extract the transfer function $D_k(s)/D_{k+1}(s)$, following (Bose and Ioannou, 2003), as

$$\frac{D_k(s)}{D_{k+1}(s)} = \hat{G}_k(s) = \frac{1 - G_k - sh_kG_k}{1 - G_{k+1} - sh_{k+1}G_{k+1}} G_{k+1}, \quad (6)$$

where $G_k(s)$ is given in (4).

2.2 *Preliminaries for Analyzing Asymptotic Stability (AS)*

Analysis of AS requires the study of the characteristic roots of the system, which are the zeros of the characteristic function $\Psi(s)$. In the case when $\Psi(s)$ represents a retarded-type time-delay system, then it is known that the supremum of the real part of these roots, also known as spectral abscissa function, exhibits continuity with respect to system parameters and delays

(Datko, 1985). Hence, the stability of a retarded-type system is determined by its point spectrum, and similar to ordinary differential equations, a loss or acquisition of the exponential stability of the trivial solution of such systems is associated with the characteristic roots on the imaginary axis, $s = j\omega$ (Michiels and Niculescu, 2007; Gu and Niculescu, 2003).

In other words, for retarded-type systems, the change of stability is possible only by a root $j\omega$ "crossing" the imaginary axis of the complex plane as a parameter of interest, e.g., the delay, changes. System and/or delay parameters that correspond to all such crossings will then partition the parameter space into regions characterized by two properties: (i) the number of strictly unstable roots of $\Psi(s) = 0$ is constant for all the parameters located inside a region, and (ii) for each parameter value on the boundary of such regions, there exists at least one characteristic root located on the imaginary axis. The regions corresponding to the case when there are no unstable roots define the stability regions. These are the fundamental steps behind root continuity arguments (Datko, 1985) and τ-decomposition/D-subdivision theorems (Neimark, 1949).

In addition to the above principles, for neutral-type systems, stability is determined not only by the point spectrum similar to retarded-type systems, but also by the essential spectrum. Essential spectrum is associated with a particular delay-difference equation, whose stability is necessary for the neutral-type system to exhibit stability. Stability of this spectrum can be lost under infinitesimally small delays even if the spectrum exhibits stability for zero delays. This feature of the spectrum is related to small-delay stabilizability condition, as extensively studied (Hale and Verduyn Lunel, 1993; Avellar and Hale, 1980).

3. MAIN RESULTS

Here a frequency sweeping framework inspired from (Chen and Latchman, 1995) is adopted to AS analysis of the dynamics at hand. This framework allows to develop certain geometric arguments in the interpretation of AS, leading to a practical stability analysis method, while also laying out the connections to prior work for comparison purposes. Specifically, we investigate how AS is affected in the delay parameter, and how driver aggressiveness influence AS.

3.1 Interconnection Schemes

Recall that detection of imaginary zeros $s = j\omega$ of the characteristic function Ψ is the starting point of the stability analysis. Here, Ψ is found from the denominator of (6), and its root at $s = 0$ is a removable singularity, and hence will be neglected Sipahi et al. (2007). This root is related to the translational dynamics of the chain of vehicles as a rigid body. Accordingly, Ψ is given for each k as

$$\Psi_k = (s+\alpha_k F(s))(s+\alpha_{k+1}F(s))(1-\alpha_{k+1}h_{k+1}F(s)), \quad (7)$$

which, after simplifications, suggests that the linear stability analysis of the entire chain of vehicles requires the stability analysis of the following interconnection schemes for all k:

Interconnection scheme 1 is associated with the stability of each vehicle affected by decisions made by the drivers under the proposed memory model:

$$P_k(s) \cdot Q(s) = -1, \quad k = 1, \ldots, n, \quad (8)$$

where

$$P_k(s) = \alpha_k \frac{e^{-s\tau_{dt}}}{s}, \qquad Q(s) = \frac{1 - e^{-s\tau_{mw}}}{s\tau_{mw}}. \quad (9)$$

Interconnection scheme 2 is associated with the stability of the dynamics arising as per constant-time headway driving strategy, and affected by a driver's memory:

$$R_k \cdot Q(s) = 1, \quad k = 1, \ldots, n, \quad (10)$$

where R_k is given by

$$R_k = \alpha_k h_k e^{-s\tau_{dt}}. \quad (11)$$

Here, we remark that the stability properties of Interconnection scheme 1 was studied in (Sipahi et al., 2007), while the stability of the second interconnection needs to be developed. For this development, we shall take advantage of the similarities between the two schemes, as explained next.

3.2 Detecting the Stability Switching Curves

In principle, the settings that partition the parameter space into stable and unstable regions will be detected by implementing a frequency sweeping technique on both interconnection schemes, at the stability switching of system's characteristic roots $s = j\omega$. The advantage in formulating the stability problem through such interconnections is also beneficial, since the effects of α_k and/or τ_{dt} can be decoupled from the operator $Q(s)$, which is only a parameter of memory window size.

Analysis of Interconnection scheme 1: If/when the stability of interconnection scheme 1 is lost, the following must hold as per the above discussions:

$$P_k(j\omega) \cdot Q(j\omega) = -1. \quad (12)$$

Remark 1. (Sipahi et al. (2007)). The zeros of the characteristic function $\Psi_k = P_k \cdot Q + 1$ obtained from (12) exhibit continuity with respect to delay parameters including around the origin of the parameter space. The spectrum of this interconnection hence resembles that of retarded-type time-delay systems (Stépán, 1989).

One can next develop the magnitude and argument conditions on (12), to construct an approach to compute the parameter settings on α_k, τ_{dt} and τ_{mw} such that the system can be in transition from stability to instability; see details in (Sipahi et al., 2007). To summarize, notice that the magnitude condition can be developed independent of τ_{dt},

$$|P_k(j\omega)|^2|Q(j\omega)|^2 = 1 \Rightarrow \frac{\alpha_k^2}{\omega^2}\frac{\sin^2(u)}{u^2} = 1, \quad (13)$$

where $u = \tau_{mw}\omega/2$, which is non-negative without loss of generality. From the above equation, given α_k, one can sweep u, solve ω next, and then find $\tau_{mw} = 2u/\omega$.

One next uses the identity $\angle(1-e^{-j\omega\tau_{mw}}) = \tan^{-1}(\cot(u)) = \pi/2 - u$ to formulate the phase condition on (12), to then express dead-time delay as

$$\tau_{dt} = \frac{1}{\omega}\left(\angle(1 - \cos(u) + j\sin(u)) + 2\pi\ell\right), \ell \in \mathbb{Z},$$
$$\Rightarrow \tau_{dt} = \frac{1}{\omega}\left(\pi/2 - u + 2\pi\ell\right), \quad (14)$$

Since u and ω are known, τ_{dt} can be computed.

Following the above line of logic, one simply needs to sweep u, and compute all the parameters of interest that will form the boundaries dividing the parameter space into stable and unstable regions.

Property 1. (Sipahi et al. (2007)). There exists only one stability region in $\tau_{dt} - \tau_{mw}$ space for a fixed k, and this region is connected to the origin of this space. Since the origin is included in all the stability regions for all k, the existence of a common stability region is guaranteed.

Analysis of Interconnection scheme 2: The approach developed above can be adapted to this interconnection scheme as follows. Firstly, if a stability switching occurs, then the following equation must hold as per the magnitude condition:

$$|R_k \cdot Q(j\omega)| = 1 \Rightarrow (\alpha_k h_k)^2 \frac{\sin^2(u)}{u^2} = 1. \tag{15}$$

Lemma 1. The interconnection scheme does not exhibit imaginary axis crossings independent of memory dead-time and memory window size if and only if $|R_k| = \alpha_k h_k < 1$.

Proof 1. It follows from the fact that $|Q(j\omega)|^2 < 1, \forall u \in \mathbb{R}$ and hence when $|R_k| < 1$, the interconnection scheme does not have a solution. This is true for all delays since the magnitude condition is independent of memory dead-time τ_{dt} and memory window size τ_{mw}.

In the case when $|R_k| \geq 1$, the following developments will be needed. Let $|R_k| \geq 1$ be given. Then, from (15), at least one solution to u always exists. Define the set of such solutions with

$$U = \{u_1, \ldots, u_\mu\},$$

where μ is the number of solutions, and for each $u = u_p \in U$, one organizes (10) as follows

$$e^{-j\omega\tau_{dt}} = \frac{j\omega\tau_{mw}}{\alpha_k h_k(1 - e^{-j\omega\tau_{mw}})}. \tag{16}$$

Different from the interconnection scheme 1, in the above equation on the right hand side, we have an $s = j\omega$ term instead of $s^2 = -\omega^2$ term in the numerator, and an additional h_k term in the denominator. The discrepancy due to the s term will reduce phase by $\pi/2$ yielding

$$\tau_{dt} = \frac{1}{\omega}\left(-u + 2\pi\ell\right), \tag{17}$$

where it is assumed that $\omega > 0$ without loss of generality, and since τ_{mw}, h_k and α_k are positive real quantities, these parameters do not appear in the above equation.

3.3 Stability Analysis

Readers are referred to (Sipahi et al., 2007) for detailed discussions on the stability analysis of interconnection scheme 1. For interconnection scheme 2, different from the interconnection scheme 1, this analysis requires several non-trivial steps, as detailed next.

Notice that, since ω does not appear in (15) as a free parameter, in contrast to (13), the solution of ω in cannot be directly obtained. This prompts the observation that a solution to τ_{dt} can always be obtained for any ω. Indeed, from (17), it is easy to show that a feasible and finite $u \in U$ for any choice of ω will correspond to a delay value τ_{dt}. Indeed, in the extreme case when ω is indefinitely large, $\omega \to \infty$, an infinite number of imaginary axis crossings can exist for $\tau_{dt} \to 0^+$.

The above observation indicates that the spectrum of the dynamics in interconnection scheme 2 may have characteristically different behavior, resembling that of "neutral class" time-delay systems (Hale and Verduyn Lunel, 1993). In such systems, the stability of the dynamics is not only determined by the point spectrum but also by the essential spectrum of the system, as explained in Section 2.2.

Notice that the dynamics associated with interconnection scheme 2 does not fit to the standard neutral-class systems, yet the above observations show evidence that this dynamics can exhibit a spectrum similar to neutral-type time-delay systems. To obtain further perspective on this, and to find out any such similarity, let us investigate the interconnection for $\tau_{mw} \to 0^+$. In this case, we have

$$\lim_{\tau_{mw}\to 0^+} ((1 - e^{-s\tau_{mw}})/\tau_{mw}) \approx s, \tag{18}$$

and hence the characteristic equation is approximated as

$$\tilde{\Psi}_k = s(1 - \alpha_k h_k e^{-s\tau_{dt}}) = 0, \tag{19}$$

which now fits to a standard neutral-type time-delay system. In this case, stability of the essential spectrum is determined by the following difference equation,

$$\mathcal{D} = 1 - \alpha_k h_k e^{-s\tau_{dt}} = 0, \tag{20}$$

which is known to be stable for $\tau_{dt} \geq 0$ if and only if $|\alpha_k h_k| < 1$, and unstable for $\tau_{dt} > 0$ if $|\alpha_k h_k| \geq 1$ (Hale and Verduyn Lunel, 1993).

Finally, one can study the stability of the interconnection around the origin of the delay parameter space, taking also the limit $h \to 0^+$. In this case, using

$$\lim_{\tau_{dt}\to 0^+} \left(e^{-s\tau_{dt}}\right) \approx 1 - s\tau_{dt}, \tag{21}$$

modifies (20) as

$$\tilde{\Psi}_k = s - \alpha_k h_k(1 - s\tau_{dt})s = 0. \tag{22}$$

The above approximated characteristic equation indicates that a pole at $s = 0$ will always exist for small delays, and moreover, another pole will arise at

$$s \to \frac{\alpha_k h_k - 1}{\alpha_k h_k \tau_{dt}}, \tag{23}$$

which destabilizes the interconnection with a pole at $s = 0$ when $\alpha_k h_k = 1$, and with an unbounded unstable pole $|s| \to \infty$ when $\alpha_k h_k > 1$.

In view of the above findings, the stability analysis of interconnection scheme 2 can be summarized as follows:

a) For $|R_k| = \alpha_k h_k < 1$, the interconnection does not possess any roots on the imaginary axis as per Lemma 1, and since it maintains its stability for infinitesimally small delays, it remains stable for all delays.
b) For $|R_k| = \alpha_k h_k \geq 1$, the interconnection is unstable for all delays. This is due to the fact that the essential spectrum of the interconnection for small delays behaves analogous to neutral-type time-delay systems, causing the system to always remain unstable.

Remark 2. Notice that due to the formulation of memory effects, one has the delay terms as coefficients in the arising characteristic equation. Such a formulation is rather non-standard as majority of the existing stability analysis techniques focused on characteristic equations in which delays appear only in exponential functions (Sipahi et al., 2011). One key difference here is that when the delay appears only in exponential functions, the solutions of the critical delays can be generated following a periodicity rule, that is, if τ is a root of the characteristic equation at $s = j\omega$, then $\tau_\ell = \tau + 2\pi\ell/\omega$ are also solutions, since $e^{-j\tau\omega} = e^{-j\tau_\ell\omega}$. Nevertheless, when the delay appears also as

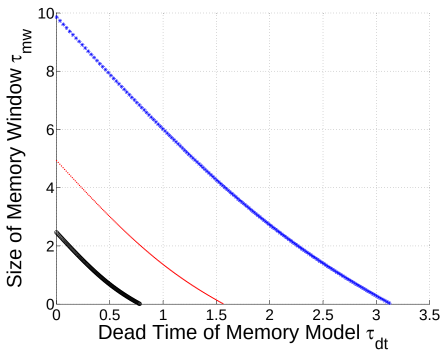

Fig. 2. Interconnection scheme 1: Points on the stability switching curves for $\alpha_k = 0.5$ (thick blue), $\alpha_k = 1$ (thin red), and $\alpha_k = 2$ (thicker black). Stability region is bordered by these curves as well as the τ_{dt} and τ_{mw} axes.

a coefficient in the characteristic equation, as is the case with τ_{mw} here, then the periodicity property is lost along the axis of τ_{mw} , and hence a specific approach must be developed to address the arising stability problem. See for example (Kokame et al., 2001; Beretta and Kuang, 2002; Qiao and Sipahi, 2014) for studies where delay appears explicitly as a parameter in the characteristic equation.

3.4 Illustrative Examples

Stability of Interconnection Scheme 1: Stability of this interconnection follows from (Sipahi et al., 2007). For completeness, the results of the stability analysis are presented. For this, three cases are considered with $\alpha_k = 0.5$, $\alpha_k = 1$, and $\alpha_k = 2$, where the latter case is borrowed from the cited study for easy comparison. Following the discussions from Subsection 3.2, we first compute the points on the stability switching boundaries by sweeping the scaled frequency parameter u. These points lie on some curves that encapsulate the stability region together with the axes of the delay parameters, τ_{dt} and τ_{mw}, as shown in Figure 2. From the cited study, we know that the stability region presented in this figure is the only one and there exists no other stable regions in the parameter space.

Stability of Interconnection Scheme 2: In order to be consistent, here we take $\alpha_k = 1$, and present two cases, one in which the stability of interconnection is always guaranteed ($|R_k| < 1$), and the other where it is unstable ($|R_k| \geq 1$) for any delays. To support the results, spectrum of the case studies is also provided with the help of QPMR toolbox (Vyhlídal and Zítek, 2009). For the stable case, we pick $h_k = 0.5$, and for the unstable case we have $h_k = 2$. In both cases, we let the memory window size be $\tau_{mw} = 0.001$ and memory dead time $\tau_{dt} = 0.3$. QPMR toolbox reveals Figures 3-4 for these cases, and the results are consistent with the predictions.

Discussions: Several key messages are obtained from the above results. When a driver is more aggressive with larger α_k, then to be able to accommodate stability, the size of the memory window and dead time shrinks. In some sense, to maintain stability, relatively more aggressive drivers should rely on only more recent historical events, but not too outdated events. Conversely, a less aggressive driver can take benefit of rich information available in a much wider memory even if the events are relatively more outdated. Furthermore, since $\alpha_k h_k$ must remain less than one for the interconnection scheme

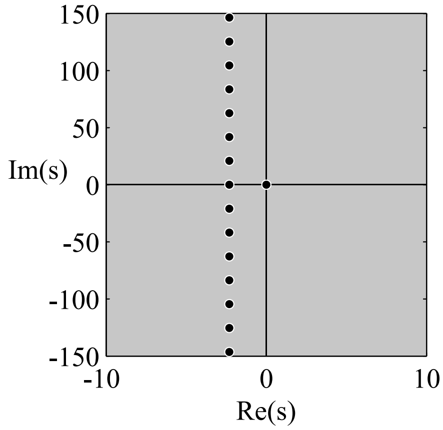

Fig. 3. The spectrum of interconnection scheme 2 for $\tau_{mw} = 0.001$, $\tau_{dt} = 0.3$, and $|R_k| = \alpha_k h_k = 0.5$. As expected, the spectrum of the dynamics exhibits stable behavior, except the invariant zero root. Computation: QPMR toolbox.

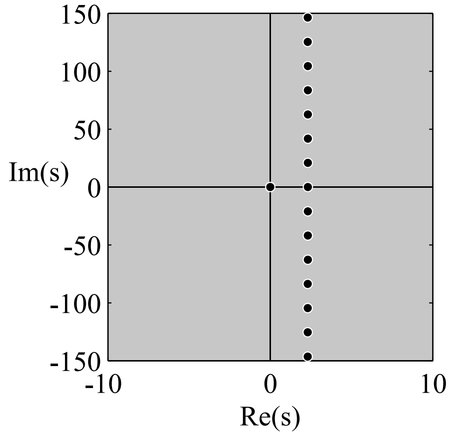

Fig. 4. The spectrum of interconnection scheme 2 for $\tau_{mw} = 0.001$, $\tau_{dt} = 0.3$, and $|R_k| = \alpha_k h_k = 2$. As expected, the spectrum of the dynamics exhibits a neutral-type behavior with a chain of poles arranged alongside the imaginary axis. Computation: QPMR toolbox.

2 to be stable, larger α_k in aggressive driving strongly limits the largest time-headway h_k that can be selected without losing stability. At high cruising speeds, such a driving strategy could be dangerous since the headway between two vehicles would not be large enough to prevent a possible collision.

4. CONCLUSIONS

Linear stability of a class of microscopic car following dynamics with human memory effects is studied in this paper. The analysis is based on two dynamical systems represented by unique interconnection schemes, one of which exhibits interesting properties in its spectrum, similar to neutral-type time delay systems. A frequency domain technique is adapted here to perform the analysis, by which the parametric settings of the driver memory model and driving strategy are laid out.

5. ACKNOWLEDGMENTS

FMA acknowledges the support of the European Union's 7th Framework Programme under grant #318723 (MatheMACS). Authors acknowledge Mr. Adrian Ramírez of Department of

Automatic Control, CINVESTAV-IPN, México for producing Figures 3-4 using the QPMR computation package (Vyhlídal and Zítek, 2009).

REFERENCES

Avellar, C.E. and Hale, J.K. (1980). On the zeros of exponential polynomials. *Journal of Mathematical Analysis and Applications*, 73(2), 434–452.

Bando, M. and Hasebe, K. (1998). Analysis of optimal velocity model with explicit delay. *Physical Review E*, 58(5), 5429–5435.

Beretta, E. and Kuang, Y. (2002). Geometric stability switch criteria in delay differential systems with delay dependent parameters. *SIAM Journal on Mathematical Analysis*, 33(5), 11441165.

Bose, A. and Ioannou, P. (2003). Analysis of traffic flow with mixed manual and semiautomated vehicles. *IEEE Transactions on Intelligent Transportation Systems*, 4(4), 173–188.

Chandler, R., Herman, R., and Montroll, E. (1958). Traffic dynamics: Analysis of stability in car following. *Operations Research*, 7.

Chen, J. and Latchman, H.A. (1995). Frequency sweeping tests for asymptotic stability independent of delay. *IEEE Transactions on Automatic Control*, 40(9), 1640–1645.

Datko, R. (1985). Remarks concerning the asymptotic stability and stabilization of linear delay differential equations. *Journal of Mathematical Analysis and Applications*, 111(2), 571–584.

Green, M. (2000). "How Long Does It Take to Stop?" methodological analysis of driver perception-brake times. *Transactions on Human Factors*, 2, 195–216.

Gu, K. and Niculescu, S.I. (2003). Survey on recent results in the stability and control of time-delay systems. *ASME Journal of Dynamic Systems, Measurement, and Control*, 125(2), 158–165.

Hale, J.K. and Verduyn Lunel, S.M. (1993). *An Introduction to Functional Differential Equations*, volume 99 of *Applied Mathematical Sciences*. Springer-Verlag, New York.

Helbing, D. (2001). Traffic and related self-driven many-particle systems. *Review of Modern Physics*, 73, 1067–1141.

Kokame, H., Hirata, K., Konishi, K., and Mori, T. (2001). Difference feedback can stabilize uncertain steady states. *IEEE Trans. Automatic Control*, 46, 1908–1913.

Michiels, W. and Niculescu, S.I. (2007). *Stability and Stabilization of Time-Delay Systems: An Eigenvalue-Based Approach*. SIAM Advances in Design and Control, Philadelphia, PA, USA.

Neimark, J.I. (1949). D-subdivision and spaces of quasi-polynomials (in Russian). *Prikladnaya Matematika i Mekhanika*, 13, 349–380.

Orosz, G. and Stepan, G. (2004). Hopf bifurcation calculations in delayed systems with translational symmetry. *Journal of Nonlinear Science*, 14(6), 505–528.

Orosz, G., Wilson, R., and Krauskopf, B. (2004). Global bifurcation investigation of an optimal velocity traffic model with driver reaction time. *Physical Review E*, 70(2), 026207.

Pipes, L. (1953). An operational analysis of traffic dynamics. *Journal of Applied Physics*, 24, 271281.

Qiao, W. and Sipahi, R. (2014). Delay-dependent coupling for a multi-agent lti consensus system with inter-agent delays. *Physica D: Nonlinear Phenomena*, 267, 112–122.

Sipahi, R., Atay, F.M., and Niculescu, S.I. (2007). Stability of traffic flow behavior with distributed delays modeling the memory effects of the drivers. *SIAM Journal on Applied Mathematics*, 68(3), 738–759.

Sipahi, R. and Niculescu, S.I. (2006a). Analytical stability study of a deterministic car following model under multiple delay interactions. In *6th IFAC Workshop on Time-Delay Systems*. L'Aquila, Italy.

Sipahi, R. and Niculescu, S.I. (2006b). Slow time-varying delay effects - robust stability characterization of deterministic car following models. In *IEEE International Conference on Control Applications*. Munich, Germany.

Sipahi, R. and Niculescu, S.I. (2006c). Some remarks on the characterization of delay interactions in deterministic car following models. In *MTNS*. Kyoto, Japan.

Sipahi, R. and Niculescu, S.I. (2008). Chain stability in traffic flow with driver reaction delays. In *American Control Conference*. Seattle, WA.

Sipahi, R. and Niculescu, S.I. (2010). *Deterministic Time-Delayed Traffic Flow Models: A Survey*, chapter Complex Time-Delay Systems, 297–322. Springer-Verlag, Berlin.

Sipahi, R., Niculescu, S.I., Abdallah, C.T., Michiels, W., and Gu, K. (2011). Stability and stabilization of systems with time delay, limitations and opportunities. *IEEE Control Systems Magazine*, 31(1), 38–65.

Sipahi, R., Niculescu, S.I., and Delice, I.I. (2009). Asymptotic stability of constant time headway driving strategy with multiple driver reaction delays. In *Proceedings of the American Control Conference*, 4898–4903. St. Louis, MO.

Stépán, G. (1989). *Retarded Dynamical Systems: Stability and Characteristic Functions*, volume 210 of *Pitman Research Notes in Mathematics Series*. Longman Scientific & Technical, co-publisher John Wiley & Sons, Inc., New York.

Treiber, M. and Kesting, A. (2013). *Traffic Flow Dynamics; Data, Models and Simulation*. Springer-Verlag.

Treiber, M., Kesting, A., and Helbing, D. (2006). Delays, inaccuracies and anticipation in microscopic traffic models. *Physica A*, 360(1), 71–88.

Vyhlídal, T. and Zítek, P. (2009). Mapping based algorithm for large-scale computation of quasi-polynomial zeros. *IEEE Transactions on Automatic Control*, 54(1), 171–177.