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by

Yan-Ling Wang, Mao-Sheng Li, Zhu-Jun Zheng, and Shao-Ming Fei

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Yan-Ling Wang[1], Mao-Sheng Li[1], Zhu-Jun Zheng[1], Shao-Ming Fei[2,3]

[1]Department of Mathematics, South China University of Technology, Guangzhou 510640, P.R.China

[2]School of Mathematical Sciences, Capital Normal University, Beijing 100048, China

[3]Max-Planck-Institute for Mathematics in the Sciences, 04103 Leipzig, Germany

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Abstract We study the one-way local operations and classical communication (LOCC) problem. In $\mathbb{C}^d \otimes \mathbb{C}^d$ with $d \geq 4$, we construct a set of $3\lceil\sqrt{d}\rceil - 1$ one-way LOCC indistinguishable maximally entangled states which are generalized Bell states. Moreover, we show that there are four maximally entangled states which cannot be perfectly distinguished by one-way LOCC measurements for any dimension $d \geq 4$.

1 Introduction

In compound quantum systems, many global operators can not be implemented using only local operations and classical communication (LOCC). This reflects the fundamental feature of quantum mechanics called nonlocality. Meanwhile, the understanding of the limitation of quantum operators that can be implemented by LOCC is also one of the significant subjects in quantum information theory. And local distinguishability of quantum states plays an important role in exploring quantum nonlocality [1, 2]. In the bipartite case, Alice and Bob share a quantum system which is chosen from one of a known set of mutually orthogonal quantum states. Their goal is to identity the given state using only LOCC. The nonlocality of quantum information is therefore revealed when a set of orthogonal states can not be distinguished by LOCC. Moreover, the local distinguishability has been found practical applications in quantum cryptography primitives such as secret sharing and data hiding [3, 4].

The question of local discrimination of orthogonal quantum states has received considerable attentions in recent years [5-19]. It is well known that any two orthogonal maximally entangled states can be perfectly distinguished with LOCC [2]. In Refs.[8, 9], the authors proved that a set of $d+1$ or more maximally entangled states in $d \otimes d$ systems are not perfectly locally distinguishable. Hence it is interesting to ask whether there are locally indistinguishable sets consisting of d or fewer maximally entangled states in $d \otimes d$. For $d = 3$, Nathanson has shown that any three maximally entangled states can be perfectly distinguished [6]. Recently, the authors in [15, 17] considered one-way LOCC distinguishability and presented sets of d and $d-1$ indistinguishable maximally entangled states for $d = 5, ..., 10$. The problem remains open if there exists fewer than $d-1$ indistinguishable maximally entangled states for arbitrary dimension d. More recently, Nathanson shew that there exist triples of mutually orthogonal maximally entangled states in $\mathbb{C}^d \otimes \mathbb{C}^d$ which cannot be distinguished with one-way LOCC in any dimension d when d is even or $d \bmod 3 \equiv 2$ [16]. In addition, the authors in [18] gave a set with $\lceil \frac{d}{2} \rceil + 2$ maximally entangled states in $\mathbb{C}^d \otimes \mathbb{C}^d$ which is one-way LOCC indistinguishable, where $\lceil a \rceil$ means the least integer which is not less than a. And in [19], the authors presented sets with four and five maximally entangled states in $\mathbb{C}^{4m} \otimes \mathbb{C}^{4m}$ which is one-way LOCC indistinguishable but two-way distinguishable. Whether there are four or three one-way LOCC indistinguishable maximally entangled states in arbitrary dimension remains unknown.

In this paper, we give a positive answer to this question when the number of states in the set is four. For any dimension $d \geq 4$, we give a set of $3\lceil \sqrt{d} \rceil - 1$ one-way LOCC indistinguishable maximally entangled states. Moreover, we can find four maximally entangled states which cannot be perfectly distinguished by one-way LOCC measurements for any dimension $d \geq 4$.

2 Preliminaries

We first introduce some basic results that will be used in proving our theorems. Under the computational base $\{|ij\rangle\}_{i,j=0}^{d-1}$ of the tensor space of the d-dimensional Hilbert space

$\mathcal{H}_A \otimes \mathcal{H}_B$, the generalized Bell states in $\mathbb{C}^d \otimes \mathbb{C}^d$ are defined as follows:

$$|\psi_{nm}\rangle = I \otimes U_{nm}(\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|jj\rangle), \tag{1}$$

where $U_{mn} = X^m Z^n$ are generalized Pauli matrices constituting a basis of unitary operators, and $X|j\rangle = |j \oplus_d 1\rangle$, $Z|j\rangle = \omega^j|j\rangle$, $\omega = e^{\frac{2\pi\sqrt{-1}}{d}}$. We define $V_{mn} = U_{mn}^T$, where T stands for transpose. It is directly verified that $ZX = \omega XZ$.

Lemma 1. Suppose $U_{mn} = X^m Z^n, U_{m'n'} = X^{m'}Z^{n'}$, we have

$$U_{mn}U_{m'n'}^{\dagger} = \omega^{(m'-m)n'}U_{(m-m' \mathrm{mod}\ d)(n-n'\ \mathrm{mod}\ d)}.$$

Proof:

$$\begin{aligned}
U_{m'n'}^{\dagger}U_{mn} &= (X^{m'}Z^{n'})^{\dagger}(X^m Z^n)\\
&= (Z^{\dagger n'}X^{\dagger m'})(X^m Z^n)\\
&= (Z^{(d-1)n'}X^{(d-1)m'})(X^m Z^n)\\
&= (Z^{-n'}X^{-m'})(X^m Z^n)\\
&= Z^{-n'}X^{m-m'}Z^n\\
&= \omega^{(m'-m)n'}X^{m-m'}Z^{n-n'}\\
&= \omega^{(m'-m)n'}U_{(m-m' \mathrm{mod}\ d)(n-n'\ \mathrm{mod}\ d)}.
\end{aligned}$$

■

For the convenience of citation, we recall the results given in Refs.[16, 17].

Lemma 2. In $\mathbb{C}^d \otimes \mathbb{C}^d$, $N \leq d$ number of pairwise orthogonal maximally entangled states $|\psi_{n_i m_i}\rangle$, $i = 1, 2, \ldots, N$, taken from the set given in Eq. (1), can be perfectly distinguished by one-way LOCC $A \to B$, if and only if there exists at least one state $|\alpha\rangle \in \mathcal{H}_B$ for which the states $U_{n_1 m_1}|\alpha\rangle$, $U_{n_2 m_2}|\alpha\rangle, \ldots, U_{n_N m_N}|\alpha\rangle$ are pairwise orthogonal.

On the other hand, the set is perfectly distinguishable by one-way LOCC in the $B \to A$, if and only if there exists at least one state $|\alpha\rangle \in \mathcal{H}_A$ for which the states $V_{n_1 m_1}|\alpha\rangle, V_{n_2 m_2}|\alpha\rangle, \ldots, V_{n_N m_N}|\alpha\rangle$ are pairwise orthogonal.

Lemma 3. Given a set of states $S = \{|\psi_i\rangle = (I \otimes U_i)|\phi\rangle\} \subset \mathbb{C}^d \otimes \mathbb{C}^d$, with $|\phi\rangle$ the standard maximally entangled state. The elements of S can be perfectly distinguished with one-way LOCC if and only if there exists a set of states $\{|\phi_k\rangle\} \subset \mathbb{C}^d$ and a set of positive numbers $\{m_k\}$ such that $\sum_k m_k|\phi_k\rangle\langle\phi_k| = I_d$ and $\langle\phi_k|U_j^{\dagger}U_i|\phi_k\rangle = \delta_{ij}$.

In the following, we concentrated on the set of maximally entangled states. Any maximally entangled state in $\mathbb{C}^d \otimes \mathbb{C}^d$ can be written as $|\psi\rangle = (I \otimes U)|\psi_0\rangle$, where $|\psi_0\rangle = \frac{1}{\sqrt{d}}\sum_{i=1}^{d} |ii\rangle$, and U is a unitary matrix. Since there is a one to one correspondence between a maximally entangled state $|\psi_i\rangle$ and the unitary matrix U_i, we call the set of unitary matrices $\{U_i\}_{i=1}^d$ the defining unitary matrices of the set of maximally entangled states $\{|\psi_i\rangle\}_{i=1}^d$.

3 Sets of one-way LOCC indistinguishable states

The authors in [18] presented a set with $\lceil \frac{d}{2} \rceil + 2$ generalized Bell states in $\mathbb{C}^d \otimes \mathbb{C}^d$ which is one-way LOCC indistinguishable. In the following, at first we also consider the one-way distinguishability of generalized Bell states.

Theorem 1. In $\mathbb{C}^d \otimes \mathbb{C}^d$ $(d \geqslant 4)$, there exists an orthogonal set with $3\lceil\sqrt{d}\rceil - 1$ maximally entangled states which is one-way LOCC indistinguishable:
$\{|\psi_{00}\rangle, |\psi_{10}\rangle, \ldots, |\psi_{n-1,0}\rangle, |\psi_{2n-1,0}\rangle, |\psi_{3n-1,0}\rangle, |\psi_{4n-1,0}\rangle, \ldots, |\psi_{(n-1)n-1,0}\rangle, |\psi_{d-1,0}\rangle, |\psi_{n-1,1}\rangle,$
$|\psi_{2n-1,1}\rangle, |\psi_{3n-1,1}\rangle, |\psi_{4n-1,1}\rangle, \ldots, |\psi_{(n-1)n-1,1}\rangle, |\psi_{d-1,1}\rangle\}$, where $n = \lceil\sqrt{d}\rceil$.
The corresponding unitary matrices are given by
$\{U_{00},\ U_{10},\ \ldots,\ U_{n-1,0},\ U_{2n-1,0},\ U_{3n-1,0},\ U_{4n-1,0},\ \ldots,\ U_{(n-1)n-1,0},\ U_{d-1,0},\ U_{n-1,1},$
$U_{2n-1,1},\ U_{3n-1,1},\ U_{4n-1,1},\ \ldots,\ U_{(n-1)n-1,1},\ U_{d-1,1}\}$.

Proof: If $\{|\psi_{00}\rangle, |\psi_{10}\rangle, \ldots, |\psi_{n-1,0}\rangle, |\psi_{2n-1,0}\rangle, |\psi_{3n-1,0}\rangle, \ldots, |\psi_{(n-1)n-1,0}\rangle, |\psi_{d-1,0}\rangle, |\psi_{n-1,1}\rangle,$ $|\psi_{2n-1,1}\rangle, \ldots, |\psi_{(n-1)n-1,1}\rangle, |\psi_{d-1,1}\rangle\}$ can be one-way LOCC distinguished, then by lemma 2, $\exists\ |\alpha\rangle \neq 0 \in \mathbb{C}^d$, such that the set $\{U_{00}|\alpha\rangle, U_{10}|\alpha\rangle, \ldots, U_{n-1,0}|\alpha\rangle, U_{2n-1,0}|\alpha\rangle, U_{3n-1,0}|\alpha\rangle,$ $\ldots, U_{(n-1)n-1,0}|\alpha\rangle, U_{d-1,0}|\alpha\rangle, U_{n-1,1}|\alpha\rangle, U_{2n-1,1}|\alpha\rangle, \ldots, U_{(n-1)n-1,1}|\alpha\rangle, U_{d-1,1}|\alpha\rangle\}$ are mutually orthogonal.

From the orthogonality of $U_{00}|\alpha\rangle$ and $U_{10}|\alpha\rangle, U_{20}|\alpha\rangle, \ldots, U_{n-1,0}|\alpha\rangle$, we obtain

$$\begin{aligned}
\langle\alpha|U_{10}|\alpha\rangle &= \sum_{j=0}^{d-1}\omega^{j}\alpha_j\overline{\alpha}_j = 0,\\
\langle\alpha|U_{20}|\alpha\rangle &= \sum_{j=0}^{d-1}\omega^{2j}\alpha_j\overline{\alpha}_j = 0,\\
&\vdots\\
\langle\alpha|U_{n-1,0}|\alpha\rangle &= \sum_{j=0}^{d-1}\omega^{(n-1)j}\alpha_j\overline{\alpha}_j = 0.
\end{aligned}$$

Then by the orthogonality of $U_{2n-1,0}|\alpha\rangle$ and $U_{n-1,0}|\alpha\rangle, \ldots, U_{10}|\alpha\rangle, U_{00}|\alpha\rangle$, taking into account Lemma 1 we get

$$\begin{aligned}
\langle\alpha|U_{n-1,0}^{\dagger}U_{2n-1,0}|\alpha\rangle = \langle\alpha|U_{n,0}|\alpha\rangle &= \sum_{j=0}^{d-1}\omega^{nj}\alpha_j\overline{\alpha}_j = 0,\\
&\vdots\\
\langle\alpha|U_{10}^{\dagger}U_{2n-1,0}|\alpha\rangle = \langle\alpha|U_{2n-2,0}|\alpha\rangle &= \sum_{j=0}^{d-1}\omega^{(2n-2)j}\alpha_j\overline{\alpha}_j = 0,\\
\langle\alpha|U_{00}^{\dagger}U_{2n-1,0}|\alpha\rangle = \langle\alpha|U_{2n-1,0}|\alpha\rangle &= \sum_{j=0}^{d-1}\omega^{(2n-1)j}\alpha_j\overline{\alpha}_j = 0.
\end{aligned}$$

Similarly, from the orthogonality of $U_{3n-1,0}|\alpha\rangle, U_{4n-1,0}|\alpha\rangle, \ldots, U_{(n-1)n-1,0}|\alpha\rangle, U_{d-1,0}|\alpha\rangle$ and $U_{n-1,0}|\alpha\rangle, \ldots, U_{10}|\alpha\rangle, U_{00}|\alpha\rangle$, we have:

$$\sum_{j=0}^{d-1}\omega^{(2n)j}\alpha_j\overline{\alpha}_j = \sum_{j=0}^{d-1}\omega^{(2n+1)j}\alpha_j\overline{\alpha}_j = \cdots = \sum_{j=0}^{d-1}\omega^{(d-1)j}\alpha_j\overline{\alpha}_j = 0.$$

Putting the above $d-1$ equations together, we have

$$\sum_{j=0}^{d-1}\omega^{j}\alpha_j\overline{\alpha}_j = \sum_{j=0}^{d-1}\omega^{2j}\alpha_j\overline{\alpha}_j = \sum_{j=0}^{d-1}\omega^{3j}\alpha_j\overline{\alpha}_j = \cdots = \sum_{j=0}^{d-1}\omega^{(d-1)j}\alpha_j\overline{\alpha}_j = 0.$$

Solving these $d-1$ equations, we have $(\alpha_0\overline{\alpha}_0, \alpha_1\overline{\alpha}_1, \cdots, \alpha_{d-1}\overline{\alpha}_{d-1}) = \lambda(1, 1, \cdots, 1)$.

1) If $\lambda = 0$, then $(\alpha_0\overline{\alpha}_0, \alpha_1\overline{\alpha}_1, \cdots, \alpha_{d-1}\overline{\alpha}_{d-1}) = (0, 0, \cdots, 0)$, that is, $|\alpha\rangle = \mathbf{0}$.

2) If $\lambda \neq 0$, then for $\forall i, j$, we have $\alpha_i\overline{\alpha}_j \neq 0$. By the orthogonality of $U_{n-1,1}|\alpha\rangle$ and

$U_{n-1,0}|\alpha\rangle,\ldots,U_{20}|\alpha\rangle,U_{10}|\alpha\rangle,U_{00}|\alpha\rangle$ and Lemma 1, we have

$$\langle\alpha|U^{\dagger}_{n-1,0}U_{n-1,1}|\alpha\rangle = \langle\alpha|U_{01}|\alpha\rangle = \sum_{j=0}^{d-1}\omega^{0j}\alpha_j\overline{\alpha}_{j\oplus_d 1} = 0,$$
$$\vdots$$
$$\langle\alpha|U^{\dagger}_{10}U_{n-1,1}|\alpha\rangle = \langle\alpha|U_{n-2,1}|\alpha\rangle = \sum_{j=0}^{d-1}\omega^{(n-2)j}\alpha_j\overline{\alpha}_{j\oplus_d 1} = 0,$$
$$\langle\alpha|U^{\dagger}_{00}U_{n-1,1}|\alpha\rangle = \langle\alpha|U_{n-1,1}|\alpha\rangle = \sum_{j=0}^{d-1}\omega^{(n-1)j}\alpha_j\overline{\alpha}_{j\oplus_d 1} = 0.$$

By the orthogonality of $U_{2n-1,1}|\alpha\rangle,U_{3n-1,1}|\alpha\rangle,\ldots,U_{(n-1)n-1,1}|\alpha\rangle,U_{d-1,1}|\alpha\rangle$ and $U_{00}|\alpha\rangle$, $U_{10}|\alpha\rangle,U_{20}|\alpha\rangle,\ldots,U_{n-1,0}|\alpha\rangle$, we have

$$\sum_{j=0}^{d-1}\omega^{nj}\alpha_j\overline{\alpha}_{j\oplus_d 1} = \sum_{j=0}^{d-1}\omega^{(n+1)j}\alpha_j\overline{\alpha}_{j\oplus_d 1} = \cdots = \sum_{j=0}^{d-1}\omega^{(d-1)j}\alpha_j\overline{\alpha}_{j\oplus_d 1} = 0.$$

From the above equations, $(\alpha_0\overline{\alpha}_1,\alpha_1\overline{\alpha}_2,\cdots,\alpha_{d-1}\overline{\alpha}_0) = (0,0,\cdots,0)$ and $\alpha_i\overline{\alpha}_j \neq 0$ are contradictory. Therefore $\{|\psi_{00}\rangle,|\psi_{10}\rangle,\ldots,|\psi_{n-1,0}\rangle,|\psi_{2n-1,0}\rangle,|\psi_{3n-1,0}\rangle,\ldots,|\psi_{(n-1)n-1,0}\rangle$, $|\psi_{d-1,0}\rangle,|\psi_{n-1,1}\rangle,|\psi_{2n-1,1}\rangle,\ldots,|\psi_{(n-1)n-1,1}\rangle,|\psi_{d-1,0}\rangle\}$ cannot be one-way LOCC distinguished. ■

In the above discussions we restricted ourselves on the one-way LOCC undistinguished generalized Bell states. In the following we consider general orthogonal maximally entangled states that are indistinguishable under one-way LOCC.

Theorem 2. There exist four states of mutually orthogonal maximally entangled states in $\mathbb{C}^d\otimes\mathbb{C}^d$ which cannot be distinguished under one-way LOCC in any dimension $d \geq 4$.

Proof: Set $d = 2+r$, $r \geqslant 2$. Let P denote the $r\times r$ permutation matrix,

$$P = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}_{r\times r}.$$

$P^r = I$ is the $r\times r$ identity matrix. We prove the theorem by dealing with the following two cases:

Case 1: r is odd. Let

$$U_1 = \begin{bmatrix} \omega X & \\ & P \end{bmatrix}, \quad U_2 = \begin{bmatrix} \gamma Z & \\ & P^2 \end{bmatrix}, \quad U_3 = \begin{bmatrix} \sigma Y & \\ & P^{\frac{r+1}{2}} \end{bmatrix},$$

where ω, γ and σ are phases satisfying $|\omega| = |\gamma| = |\sigma| = 1$, $\overline{\gamma} \neq \pm i\overline{\omega}^2$, X, Y, Z are the Pauli matrices:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Let $|\psi_0\rangle$ be the standard maximally entangled state, $|\psi_0\rangle = \sum_{i=0}^{d-1} |ii\rangle$. We construct four maximally entangled states as follows:

$$\{|\psi_0\rangle, (I \otimes U_1)|\psi_0\rangle, (I \otimes U_2)|\psi_0\rangle, (I \otimes U_3)|\psi_0\rangle\} \subseteq \mathbb{C}^d \otimes \mathbb{C}^d.$$

One can check that these states are mutually orthogonal and maximally entangled. To show that these states cannot be distinguished under one-way LOCC, suppose that Alice performs an initial measurement $\mathbb{M}$ on her system and gets the measurement outcome corresponding to some operator M^T,

$$M = \begin{bmatrix} A & C^\dagger \\ C & B \end{bmatrix} \geqslant 0,$$

where A is a 2×2 matrix and B a $r \times r$ matrix. To be a perfect discrimination, we need $Tr(U_i M U_j^\dagger) = 0$ whenever $i \neq j$. The required orthogonal conditions imply that

$$Tr(MU_1) = \omega Tr(AX) + Tr(BP) = 0, \tag{2}$$

$$Tr(MU_2) = \gamma Tr(AZ) + Tr(BP^2) = 0, \tag{3}$$

$$Tr(MU_3) = \sigma Tr(AY) + Tr(BP^{\frac{r+1}{2}}) = 0, \tag{4}$$

$$Tr(U_2 M U_1^\dagger) = -i\overline{\omega}\gamma Tr(AY) + Tr(BP) = 0, \tag{5}$$

$$Tr(U_3 M U_1^\dagger) = -i\overline{\omega}\sigma Tr(AZ) + Tr(BP^{\frac{r-1}{2}}) = 0, \tag{6}$$

$$Tr(U_3 M U_2^\dagger) = -i\overline{\gamma}\sigma Tr(AX) + Tr(BP^{\frac{r-3}{2}}) = 0. \tag{7}$$

From equations (2) and (5), we have

$$\omega\, Tr(AX) + i\,\overline{\omega}\,\gamma\, Tr(AY) = 0.$$

Since A, B, X, Y, Z are all Hermitian and the product of two Hermitian matrices always has a real-valued trace, i.e. $Tr(AX)$ and $Tr(AY)$ are real. Clearly, $|Tr(AX)| = |Tr(AY)|$. If $Tr(AX) \neq 0$, then we have $i\overline{\omega}^2\gamma = -\frac{Tr(AX)}{Tr(AY)} = 1$ or -1. This is contradicted with $\overline{\gamma} \neq \pm i\,\overline{\omega}^2$. Hence we have $Tr(AX) = Tr(AY) = 0$. From equation (4) we obtain $Tr(BP^{\frac{r+1}{2}}) = 0$. Due to $P^r = I$ and the Hermitian of the matrix B, the equality $Tr(BP^{\frac{r-1}{2}}) = \overline{Tr(BP^{\frac{r+1}{2}})}$ holds, which gives rise to $Tr(BP^{\frac{r-1}{2}}) = 0$. Then by equation (6), we obtain $Tr(AZ) = 0$. Since the Pauli matrices form a basis for 2×2 Hermitian matrices, we are forced to conclude that $A = tI_2$ for some $t \geq 0$.

From lemma 3, to distinguish these states under one-way LOCC, Alice is required to have a complete measurement $\mathbb{M} = \{M_i\}$ consisting of rank one matrices. If A is a multiple of the identity matrix, then either $A = \mathbf{0}$ or else the rank of M is at least two. Thus, either $\mathbb{M}$ contains measurement operators of rank greater than one or else $\sum_i M_i \neq I$. In either case, $\mathbb{M}$ cannot be the first step towards a perfect one-way LOCC protocol.

Case 2: r is even. We set

$$U_1 = \begin{bmatrix} \omega X & \\ & P^2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} \gamma Z & \\ & P^4 \end{bmatrix}, \quad U_3 = \begin{bmatrix} \sigma Y & \\ & P^{\frac{r+3}{2}} \end{bmatrix}.$$

Similarly, one can get that $\{|\psi_0\rangle, (I \otimes U_1)|\psi_0\rangle, (I \otimes U_2)|\psi_0\rangle, (I \otimes U_3)|\psi_0\rangle\}$ cannot be perfectly distinguished by one-way LOCC, which completes the proof of the theorem. ■

4 Conclusion

we have studied further the one-way LOCC problem and presented a set of $3\lceil\sqrt{d}\rceil - 1$ one-way LOCC indistinguishable maximally entangled states which are all generalized Bell states. It should be noticed that if d is large enough, then the number $3\lceil\sqrt{d}\rceil - 1$ is much smaller than the number $\lceil\frac{d}{2}\rceil + 2$ in [18]. Moreover, we have also found four maximally entangled states which cannot be perfectly distinguished by one-way LOCC measurements for any dimension $d \geq 4$. For some particular dimension d, small one-way indistinguishable sets that contain only three states has been given in [16]. From our approach it would be desired that the results could be extended to the case of any $d \geq 4$.

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