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by

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Abstract

We investigate one-way quantum deficit for $2 \otimes d$ systems. Analytical expressions of one-way quantum deficit under both von Neumann measurement and weak measurement are presented. As an illustration, qubit-qutrit systems are studied in detail. It is shown that there exists non-zero one-way quantum deficits even quantum entanglement vanishes. Moreover, quantum deficit via weak measurement turns out to be weaker than that via von Neumann measurement. The dynamics of entanglement and one-way quantum deficit under dephasing channel is also investigated.

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INTRODUCTION

Quantum entanglement is one of the most important quantum correlations and plays a fundamental role in quantum information science [1]. Beyond entanglement, quantum discord [2, 3] plays a key role in some quantum speed-up for quantum information tasks [4], for instance, in assisted optimal state discrimination only one side discord is required, while the entanglement is not necessary [5, 6]. Other different measures in quantifying quantum correlations [7], such as one-way quantum deficit [8, 9], quantum dissonance [10], geometrical discord [11], measurement-induced nonlocality [12] have been also provided. Nonetheless, usually it is formidably difficulty to get analytical results for these quantum correlations. Analytical expressions of quantum discord [13–15] seem to be extremely hard due to the optimization involved [16, 17]. Only a few analytical results for the simplest two-qubit systems have been worked out [18–20].

Recently, in Ref. [21] the authors shew that analogous to quantum discord, one-way quantum deficit exhibits also frozen phenomenon. The one-way quantum deficit and quantum discord in XX spin chains have been investigated in [22]. And the explicit relationship between quantum discord and one-way quantum deficit has been studied in [23]. Similar to quantum discord, it is hard to derive analytical expressions of one-way quantum deficit of general two-qubit systems. The upper bound of one-way quantum deficit is shown to be the entropy of the measured subsystem [24]. Partial analytical expressions of one-way quantum deficit of five-parameter two-qubit X states have been provided in Ref [25].

In this manuscript, we study the one-way quantum deficit for $2\otimes d$ (qubit-qudit) systems. We provide the analytical results of one-way quantum deficit for a two-parameter class of states in $2 \otimes d$ quantum systems with $d \geq 3$. Moreover, we utilize the weak measurement [26] to investigate the one-way quantum deficit for the systems. Generally weak measurement exhibits amplifying roles [27]. However, we find the one-way quantum deficit via weak measurement is weaker than that via von Neumann measurement. We also study the decoherence of one-way quantum deficit via von Neumann measurement and weak measurement for qubit-qutrit systems.

ONE-WAY QUANTUM DEFICIT VIA VON NEUMANN MEASUREMENT

One-way quantum deficit is related to extracting work from a correlated system to a heat bath under local operations [8]. Consider Alice (A) and Bob (B) share a bipartite quantum system $\rho_{AB} \in \mathcal{H}^2 \otimes \mathcal{H}^d$ in 2 and d dimensional spaces $\mathcal{H}^2$ and $\mathcal{H}^d$, respectively. Let $\{P_i\}$ be local von Neumann (projective) measurement, $P_i P_j = \delta_{ij} P_i$, $\sum_i P_i = I$, with I the identity operator. The one-way quantum deficit is defined as the minimal increase of entropy after the projective measurement performing on the subsystem A [28],

$$\vec{\Delta}(\rho_{AB}) = \min_{\{P_j^A\}} S(\rho'_{AB}) - S(\rho_{AB}), \tag{1}$$

where $\rho'_{AB} = \sum_j (P_j^A \otimes I)\rho_{AB}(P_j^A \otimes I)$ is the state after measurement on A, $S(\rho) = -\mathrm{Tr}\rho\log_2\rho$ is the von Neumann entropy of the state ρ, and the minimum is taken over all possible projective measurements $\{P_j^A\}$. The one-way information deficit is non-negative and zero for classical-quantum correlated states.

A two-parameter family of states in $2 \otimes d$ quantum system was first introduced in [29],

$$\begin{aligned}\rho_{r,t} \quad &= r\sum_{i=0}^{1}\sum_{j=2}^{d-1}|ij\rangle\langle ij| + s(|\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-| \\ &+|\psi^+\rangle\langle\psi^+|) + t|\psi^-\rangle\langle\psi^-|,\end{aligned} \tag{2}$$

where $\{|ij\rangle : i = 0, 1, j = 2, 3, \ldots, d-1\}$ are orthonormal bases for the $2 \otimes d$ quantum systems and the four Bell bases are given as follows

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

The parameters satisfy $2(d-2)r+3s+t = 1$ with $0 \le r \le 1/(2d-4)$. It has been proven that any $2 \otimes d$ states can be transformed into $\rho_{r,t}$ with the help of local operations and classical communication (LOCC) [29]. The quantum discord for such states have been studied in Ref. [30].

Now, let us turn to calculate one-way quantum deficit for the state (2). We perform measurements on subsystem A by projective operators $P_k^A = |k'\rangle\langle k'|$, $k \in \{0, 1\}$, where

$$\begin{aligned}|0'\rangle &= \cos(\theta/2)|0\rangle - e^{-i\phi}\sin(\theta/2)|1\rangle, \\ |1'\rangle &= e^{i\phi}\sin(\theta/2)|0\rangle + \cos(\theta/2)|1\rangle.\end{aligned} \tag{3}$$

The projective measurement bases are described by the angles θ and ϕ of the Bloch sphere, with $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi)$.

To treat one-way quantum deficit, the key problem is to minimize the first term in (1). However, the eigenvalues of $\sum_j (P_j^A \otimes I)\rho_{AB}(P_j^A \otimes I)$ do not contain the measurement parameters θ and ϕ. That is to say, for $2 \times d$ systems, the one-way quantum deficit is independent of projective measurements and we do not need to do the minimization. Namely, the result is optimal under any projective measurements. The eigenvalues of the post measured state are given by $\{s, s, \frac{s+t}{2}, \frac{s+t}{2}, r, r, \cdots, r\}$. Taking into account that $S(\rho) = -[3s \log_2 s + t \log_2 t + 2(d-2)r \log_2 r]$, we obtain the analytical expressions of one-way quantum deficit for $2 \otimes d$ states,

$$\vec{\Delta} = s \log_2 2s + t \log_2 2t - (s+t) \log_2(s+t). \tag{4}$$

It turns out that one-way quantum deficit for $2 \otimes d$ states is the same as the quantum discord of the two-parameter states [30].

ONE-WAY QUANTUM DEFICIT VIA WEAK MEASUREMENT

Weak measurement was formulated in Ref. [26] by using the pre and post-selected quantum systems. In Ref. [31] the authors constructed weak measurement operators,

$$q(+x) = \sqrt{\frac{1 - \tanh[x]}{2}} M_0 + \sqrt{\frac{1 + \tanh[x]}{2}} M_1,$$
$$q(-x) = \sqrt{\frac{1 + \tanh[x]}{2}} M_0 + \sqrt{\frac{1 - \tanh[x]}{2}} M_1,$$

where x is a parameter describing the strength of the measurement, M_0 and M_1 are the two orthogonal projectors satisfying $M_0 + M_1 = I$ and $q(+x)^\dagger q(+x) + q(-x)^\dagger q(-x) = I$. Much attention has been paid to weak measurement both theoretically and experimentally [32].

Now we study one-way quantum deficit under weak measurement. Instead of projective measurement, under weak measurement the post measured state has the form $\rho' = \sum_{+x,-x} [q(x) \otimes I] \cdot \rho \cdot [q(x) \otimes I]^\dagger$. The eigenvalues of this state is given by $\{\frac{1}{2}(s + t + (s-t)\mathrm{sech}[x], \frac{1}{2}(s + t - (s-t)\mathrm{sech}[x], s, s, r, r, \cdots, r\}$. Thus, the one-way quantum deficit via weak measurement is given by

$$\vec{\Delta}_w = -\sum_{i=0,1} \Lambda_i \log_2 \Lambda_i + s \log_2 s + t \log_2 t, \tag{5}$$

where $\Lambda_i = \frac{1}{2}(s + t + (-1)^i (s - t)\text{sech}[x]$.

We have derived the analytical formulae of one-way quantum deficit under projective measurement and weak measurement, respectively. It is observed that the analytical expressions of one-way quantum deficit under weak measurement or projective measurement are independent of the dimension d. In the following we investigate the relationship between quantum entanglement and one-way quantum deficit, as well as their evolution under noisy channel.

ENTANGLEMENT, ONE-WAY QUANTUM DEFICIT IN QUBIT-QUTRIT SYSTEMS

We consider qubit-qutrit systems ($d = 3$). The qubit-qutrit states are given by

$$\begin{aligned} \sigma_{r,t} &= r(|02\rangle\langle 02| + |12\rangle\langle 12|) + s(|\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-| \\ &+|\psi^+\rangle\langle\psi^+|) + t|\psi^-\rangle\langle\psi^-|. \end{aligned} \tag{6}$$

The geometric discord of such states under various noise channels have been studied in Ref. [33].

For $2 \otimes 3$ systems the positive partial transposition (PPT) criterion is the necessary and sufficient condition for separability [34, 35]. We use the *negativity* N as the measure of entanglement [29],

$$N(\sigma) = \max\{0, \|\sigma^{T_B}\|_1 - 1\}, \tag{7}$$

where T_B stands for the partial transpose with respect to the subsystem B, and σ^{T_B} is the partial transposes state of σ, $\|\sigma\|_1 = \text{Tr}[\sqrt{\sigma^\dagger \sigma}]$ denotes the trace norm of σ. For the qubit-qutrit state $\sigma_{r,t}$ the negativity is given by $N(\sigma_{r,t}) = \max\{0, 2(r + t) - 1\}$.

Take $s = 0.15$. The relationship between the negativity and one-way quantum deficit is shown in Fig.1. The state is separable for $t \leq 0.45$ and entangled for $t > 0.45$. For separable states, the one-way quantum deficit via projective and weak measurements could be still greater than zero. The weak quantum deficit (dashed blue line) is weaker than one-way quantum deficit via von Neumann measurement (solid orange line).

Now we consider decoherence of qubit-qutrit systems under dephasing channels. After

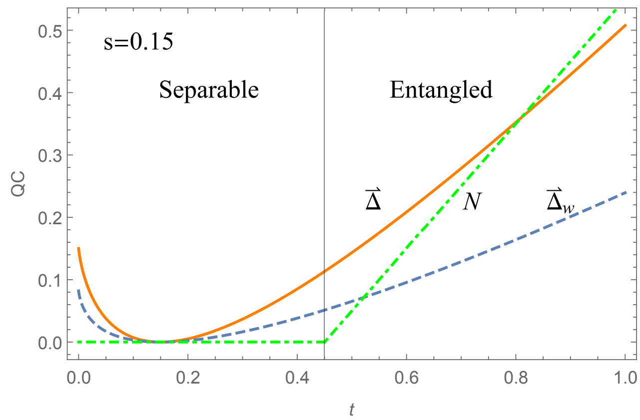

FIG. 1: (Color online) Solid orange line for one-way quantum deficit via projective measurement $\vec{\Delta}$, dashed blue line for weak quantum deficit $\vec{\Delta}_w$, and dotted-dashed green line for negativity N.

the dephasing channels the qubit-qutrit state $\sigma_{r,t}$ is transformed to be

$$\sigma'_{r,t} = \sum_{i=0}^{1}\sum_{j=0}^{2} E_i \otimes F_j \cdot \sigma_{r,t} \cdot E_i^\dagger \otimes F_j^\dagger,$$

where

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma_A} \end{pmatrix}, \quad E_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\gamma_A} \end{pmatrix},$$

and

$$F_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\gamma_B} & 0 \\ 0 & 0 & \sqrt{1-\gamma_B} \end{pmatrix}, \quad F_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{\gamma_B} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$F_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{\gamma_B} \end{pmatrix}.$$

The parameters $\gamma_A = 1 - e^{-\tau\Gamma_A}$ and $\gamma_B = 1 - e^{-\tau\Gamma_B}$, with $\Gamma_{A(B)}$ the decay rate of the subsystem $A(B)$ and $\gamma_{A(B)} \in [0,1]$.

The one-way quantum deficit of state $\sigma'_{r,t}$ can be calculated directly from the optimal projective measurement (3) with $\theta = 0$ and arbitrary ϕ, which is given by

$$\vec{\Delta}(\sigma'_{r,t}) = \sum_{j=0}^{1} \lambda_j \log_2 \lambda_j - (s+t)\log_2 \frac{1}{2}(s+t). \tag{8}$$

where

$$\lambda_j = \frac{1}{2}[s + t + (-1)^j (s - t)\sqrt{(1-\gamma_A)(1-\gamma_B)}].$$

Similarly, under this decoherence channel the one-way quantum deficit via weak measurement is given by

$$\vec{\Delta}_w(\sigma'_{r,t}) = \sum_{j=0}^{1}[\eta_j \log_2 \eta_j - \xi_j \log_2 \xi_j], \tag{9}$$

where

$$\eta_j = \frac{1}{2}[s + t + (-1)^j (s - t)\sqrt{(1-\gamma_A)(1-\gamma_B)}],$$

and

$$\xi_j = \frac{1}{2}[s + t + (-1)^j (s - t)\mathrm{sech}[x]\sqrt{(1-\gamma_A)(1-\gamma_B)}].$$

The negativity of $\sigma'_{r,t}$ has the form

$$N(\sigma'_{r,t}) = \max\left\{0, \frac{1}{3}[2(2r + t - 1) + (2r + 4t - 1)\sqrt{(1-\gamma_A)(1-\gamma_B)}]\right\}. \tag{10}$$

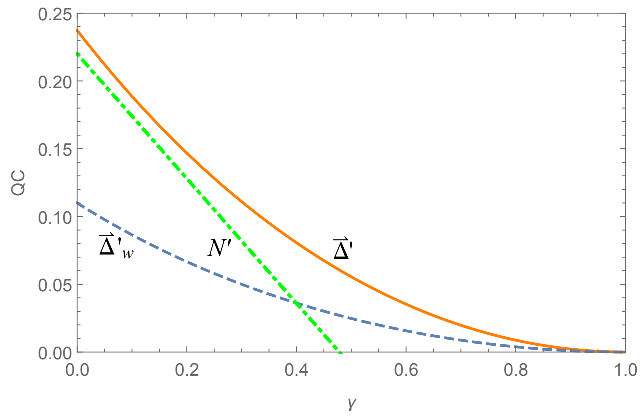

FIG. 2: (Color online) Under dephasing channels, the quantum correlation for qubit-qutrit systems. The decoherence of one-way quantum deficit via projective measurement $\vec{\Delta}'$ is depicted by solid orange line. Weak quantum deficit $\vec{\Delta}'_w$ is described in dashed blue line. The dotted-dashed green line denotes negativity N'. We suppose $\gamma_A = \gamma_B = \gamma$ and set $r = 0.03, s = 0.12, t = 0.58, x = 0.8$.

The dynamics of the system under dephasing channel can be seen in Fig. 2. In finite time, entanglement sudden death happens (dotted-dashed green line), while one-way quantum deficit under projective or weak measurements vanish gradually. Moreover, under the whole decoherence dynamical process the weak quantum deficit is also always weaker than the one-way quantum deficit via projective measurement.

CONCLUSIONS

We have extended previous studies on one-way quantum deficit for two-qubit systems to the case of $2 \otimes d$ systems. We have provided analytical expressions of one-way quantum deficit under both projective measurement and weak measurement. It has been shown that there still exits non-zero one-way quantum deficit for separable states. In particular, we have investigated the quantum entanglement (negativity) and quantum deficits for qubit-qutrit systems. It has been found that the one-way quantum deficit via weak measurement is weaker than the one under projective measurement. Under the decoherence of dephasing channel, one sees the entanglement sudden death, while one-way quantum deficits do not vanish suddenly. Our results could help to understand the one-way quantum deficit. Such approach may be also used to investigate quantum correlations for multipartite systems.

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